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Research Article

Stability Criteria of Interval Time-Varying Delay Systems and Their Application

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The stability for a class of uncertain linear systems with interval time-varying delays is studied. Based on the delay-dividing approach, the delay interval is partitioned into two subintervals. By constructing an appropriate Lyapunov-Krasovskii functional and using the convex combination method and the improved integral inequality, the delay-dependent stability criteria with less conservation are derived. Finally, some numerical examples are given to show the effectiveness and superiority of the proposed method.

1. Introduction

Time delay arises in many systems like manufacturing, telecommunications, chemical industry, power, transportation, and so on. It is generally regarded as a main source of instability and poor performance, which has a negative impact on the performance of the system [1–3]. Therefore, the stability of time-delay systems has always been the focus of attention [4–20].

In recent years, some scholars have put forward many effective methods in order to reduce the conservation of the existing results, solve the time-delay problem of system, and make the system more stable. For example, in the process of analyzing the time-delay system, the free weighting matrix method is used in [5], which reduces the conservation of the fixed weighting matrix. In [6], the delay-dividing approach is adopted. However, too many partition intervals increase the computational complexity and simulation time, which lead to the decrease of the system operating efficiency. In the construction of functional, [8] is introduced to the triple-integral terms. The conclusion shows that there is no obvious decrease in the conservation of the results after adding the item. When dealing with integral terms which are generated in the process of functional derivation, one common point of the above reference is the use of Jensen's inequality. Jensen's inequality is simple and convenient, but it has a certain conservation. Integral inequality of the Wirtinger type is introduced by [9]. Under the premise that conservation of the results is not affected, the number of decision variables to be used is small. But the method is mainly used in such case that time delay is not decomposed. Therefore, it is meaningful to obtain less conservative stability criterion by combining the delay-dividing approach and integral inequality.

Motivated by the above research, this paper considers the problem of delay-dependent stability for uncertain systems with interval time-varying delay. By constructing an appropriate Lyapunov-Krasovskii functional and using the convex combination method and the improved integral inequality, a new less conservative delay-dependent stability criterion is proposed. The proposed method is verified by the classical numerical examples and applied to the WSCC 3-machine 9-bus system. The results suggest that the proposed method is less conservative than some known results.

2. Description of Linear Uncertain Time-Delay Systems

Consider the following uncertain time-delay system:

$$\dot{x}(t) = (A + \Delta A(t))\, x(t) + (B + \Delta B(t))\, x(t - h(t)), \qquad t > 0,$$

$$x(t) = \varphi(t), \quad \forall t \in [-h_2, 0], \tag{1}$$

where $x(t) \in R^n$ is the state vector of system and $\varphi(t)$ is defined as the continuous initial real function on the interval $[-h_2, 0]$. Time-delay function $h(t)$ is differentiable and satisfies the following conditions:

$$\begin{gathered} 0 \le h_1 \le h(t) \le h_2, \\ \dot{h}(t) \le d < \infty, \\ \forall t > 0, \end{gathered} \tag{2}$$

where h_1, h_2, d are constants; A, B are real constant matrices with corresponding dimensions; $\Delta A(t)$, $\Delta B(t)$ are uncertain parameter matrices with appropriate dimensions and denote uncertainty of the time-varying, satisfying the following conditions:

$$[\Delta A(t) \quad \Delta B(t)] = DF(t)[E_1 \quad E_2], \tag{3}$$

where D, E_1, E_2 are real constant matrices of appropriate dimensions; $F(t)$ is a uncertain matrix with measurable element and satisfies $F^T(t)F(t) \le I$.

Time-delay interval is divided into two sections, namely, $[h_1, h_2] = [h_1, h] \cup [h, h_2]$, where

$$h = \lambda h_1 + (1-\lambda) h_2, \quad 0 < \lambda < 1, \ h(t) \in [h_1, h_2]. \tag{4}$$

3. Delay-Dependent Stability Theorem and Main Results

Firstly, some related lemmas are given in this section.

Lemma 1 (see [11]). *For any positive-definite matrix $W \in R^{n\times n}$, scalar $\gamma > 0$ and the integral term $-\gamma \int_{t-\gamma}^{t} \dot{x}^{\mathrm{T}}(s) W \dot{x}(s) ds$ of vector function $\dot{x}(t) : [-\gamma, 0] \to R^n$ has definition; the following inequality holds:*

$$\begin{aligned} &-\gamma \int_{t-\gamma}^{t} \dot{x}^{\mathrm{T}}(s) W \dot{x}(s) ds \\ &\le \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -W & W \\ * & -W \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\gamma) \end{bmatrix}. \end{aligned} \tag{5}$$

Lemma 2 (see [12]). *Suppose $h_1 \le h(t) \le h_2$, where $h(t) : R_+ \to R_+$. Then, for any symmetric matrix $R = R^{\mathrm{T}} > 0$, the following integral inequality holds:*

$$\begin{aligned} &-\int_{t-h_2}^{t-h_1} y^{\mathrm{T}}(s) R y(s) ds \le \delta^{\mathrm{T}}(t) \Big\{ (h_2 - h(t)) T R^{-1} T^{\mathrm{T}} \\ &+ (h(t) - h_1) Y R^{-1} Y^{\mathrm{T}} + [0 \quad Y \quad T] + [0 \quad Y \quad T]^{\mathrm{T}} \Big\} \\ &\cdot \delta(t), \end{aligned} \tag{6}$$

where $\delta(t) = [\int_{t-h_1}^{t} y^{\mathrm{T}}(s) ds \quad \int_{t-h(t)}^{t-h_1} y^{\mathrm{T}}(s) ds \quad \int_{t-h_2}^{t-h(t)} y^{\mathrm{T}}(s) ds]^{\mathrm{T}}$, $T = [T_1^{\mathrm{T}} \quad T_2^{\mathrm{T}} \quad T_3^{\mathrm{T}}]^{\mathrm{T}}$, $Y = [Y_1^{\mathrm{T}} \quad Y_2^{\mathrm{T}} \quad Y_3^{\mathrm{T}}]^{\mathrm{T}}$.

Lemma 3 ([13] Schur complement). *For given real matrices Ω_1, Ω_2, Ω_3 of appropriate dimensions, satisfying $\Omega_1 = \Omega_1^{\mathrm{T}}$, $\Omega_2 = \Omega_2^{\mathrm{T}}$, then the following conditions are equivalent:*

(a) $\begin{bmatrix} \Omega_1 & \Omega_3 \\ \Omega_3^{\mathrm{T}} & \Omega_2 \end{bmatrix} < 0$;

(b) $\Omega_1 < 0$, $\Omega_2 - \Omega_3^{\mathrm{T}} \Omega_1^{-1} \Omega_3 < 0$;

(c) $\Omega_2 < 0$, $\Omega_1 - \Omega_3 \Omega_2^{-1} \Omega_3^{\mathrm{T}} < 0$.

Lemma 3 is mainly used to transform nonlinear matrix inequalities into linear matrix inequalities.

Lemma 4 (see [14]). *Given matrices U, W, and $Q = Q^{\mathrm{T}}$ of appropriate dimensions, then $Q + UF(t)W + W^{\mathrm{T}} F^{\mathrm{T}}(t) U^{\mathrm{T}} < 0$, for all $F(t)$ satisfying $F(t)^{\mathrm{T}} F(t) \le I$, if and only if there exists a positive number $\varepsilon > 0$ such that*

$$Q + \varepsilon^{-1} U U^{\mathrm{T}} + \varepsilon W^{\mathrm{T}} W < 0. \tag{7}$$

Lemma 4 is mainly used to deal with uncertainty matrix.

Lemma 5 (see [15]). *Suppose $h_1 \le h(t) \le h_2$, where $h(t) : R_+ \to R_+$. Then, for any constant matrices Ξ_1, Ξ_2, and Ω with proper dimensions, the following matrix inequality*

$$\Omega + (h(t) - h_1) \Xi_1 + (h_2 - h(t)) \Xi_2 < 0 \tag{8}$$

holds, if and only if

$$\begin{gathered} \Omega + (h_2 - h_1) \Xi_1 < 0, \\ \Omega + (h_2 - h_1) \Xi_2 < 0. \end{gathered} \tag{9}$$

Proof. Substituting $\theta = (h(t) - h_1)/(h_2 - h_1)$, then $0 \le \theta \le 1$ and $1 - \theta = (h_2 - h(t))/(h_2 - h_1)$. Therefore, we have

$$\begin{aligned} &\theta [\Omega + (h_2 - h_1) \Xi_1] + (1-\theta) [\Omega + (h_2 - h_1) \Xi_2] \\ &\quad = \Omega + (h(t) - h_1) \Xi_1 + (h_2 - h(t)) \Xi_2. \end{aligned} \tag{10}$$

According to convex combination method, the conclusion is proved. □

Theorem 6. *For any given constant $0 \le h_1 < h_2$, $0 < \lambda < 1$, if there are positive-definite matrices P, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, M_1, M_2, M_3, positive numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and matrices T_i, Y_i $(i = 1, 2, 3)$ of appropriate dimensions, satisfying the following matrix inequality:*

$$\begin{gathered} \begin{bmatrix} \Omega^0 + \Omega^i & \Phi & \delta_i \overline{T}^i \\ * & \Psi^i & 0 \\ * & * & -\delta_i M_{i+1} \end{bmatrix} < 0, \\ \begin{bmatrix} \Omega^0 + \Omega^i & \Phi & \delta_i \overline{T}^i \\ * & \Psi^i & 0 \\ * & * & -\delta_i M_{i+1} \end{bmatrix} < 0, \\ i = 1, 2 \end{gathered} \tag{11}$$

then system (1) is asymptotically stable, where

$$\Omega^0 = \begin{bmatrix} \Omega_{11}^0 & PB & M_1 & 0 & 0 \\ * & \Omega_{22}^0 & 0 & 0 & 0 \\ * & * & \Omega_{33}^0 & 0 & 0 \\ * & * & * & \Omega_{44}^0 & 0 \\ * & * & * & * & \Omega_{55}^0 \end{bmatrix},$$

$$\Omega^1 = \begin{bmatrix} \varepsilon_1 E_1^{\mathrm{T}} E_1 & \varepsilon_1 E_1^{\mathrm{T}} E_2 & 0 & 0 & 0 \\ * & \varepsilon_1 E_2^{\mathrm{T}} E_2 & \Omega_{23}^1 & \Omega_{24}^1 & 0 \\ * & * & \Omega_{33}^1 & \Omega_{34}^1 & 0 \\ * & * & * & \Omega_{44}^1 - \dfrac{M_3}{\delta_2} & \dfrac{M_3}{\delta_2} \\ * & * & * & * & \dfrac{-M_3}{\delta_2} \end{bmatrix},$$

$$\Omega^2 = \begin{bmatrix} \varepsilon_2 E_1^{\mathrm{T}} E_1 & \varepsilon_2 E_1^{\mathrm{T}} E_2 & 0 & 0 & 0 \\ * & \varepsilon_2 E_2^{\mathrm{T}} E_2 & 0 & \Omega_{24}^2 & \Omega_{25}^2 \\ * & * & \dfrac{-M_2}{\delta_1} & \dfrac{M_2}{\delta_1} & 0 \\ * & * & * & \Omega_{44}^2 - \dfrac{M_2}{\delta_1} & \Omega_{45}^2 \\ * & * & * & * & \Omega_{55}^2 \end{bmatrix},$$

$$\Phi^{\mathrm{T}} = \begin{bmatrix} MA & MB & 0 & 0 & 0 \\ D^{\mathrm{T}} P & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi^1 = \begin{bmatrix} -M & MD \\ 0 & -\varepsilon_1 I \end{bmatrix},$$

$$\Psi^2 = \begin{bmatrix} -M & MD \\ 0 & -\varepsilon_2 I \end{bmatrix},$$

$$\Omega_{11}^0 = PA + A^{\mathrm{T}} P + Q_3 + Q_4 + Q_5 + Q_6 - M_1,$$

$$\Omega_{22}^0 = (d-1) Q_6 + T_2 + T_2^{\mathrm{T}} - Y_2 - Y_2^{\mathrm{T}},$$

$$\Omega_{33}^0 = Q_2 - Q_3 - M_1,$$

$$\Omega_{44}^0 = Q_1 - Q_2 - Q_5,$$

$$\Omega_{55}^0 = -Q_1 - Q_4,$$

$$\Omega_{23}^1 = \Omega_{24}^2 = -Y_1^{\mathrm{T}} + Y_2 + Y_1^{\mathrm{T}},$$

$$\Omega_{24}^1 = \Omega_{25}^2 = -Y_3 + T_2 + Y_3^{\mathrm{T}},$$

$$\Omega_{33}^1 = \Omega_{44}^2 = Y_1 + Y_1^{\mathrm{T}},$$

$$\Omega_{34}^1 = \Omega_{45}^2 = Y_3^{\mathrm{T}} + T_1,$$

$$\Omega_{44}^1 = \Omega_{55}^2 = T_3 + T_3^{\mathrm{T}},$$

$$M = h_1^2 M_1 + \delta_1 M_2 + \delta_2 M_3,$$

$$\begin{aligned} \delta_1 &= h - h_1 = \lambda (h_2 - h_1), \\ \delta_2 &= h_2 - h = (1-\lambda)(h_2 - h_1), \\ \overline{T}^1 &= \begin{bmatrix} 0 & T_2^{\mathrm{T}} & T_1^{\mathrm{T}} & T_3^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}}, \\ \overline{Y}^1 &= \begin{bmatrix} 0 & Y_2^{\mathrm{T}} & Y_1^{\mathrm{T}} & Y_3^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}}, \\ \overline{T}^2 &= \begin{bmatrix} 0 & T_2^{\mathrm{T}} & 0 & T_1^{\mathrm{T}} & T_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \overline{Y}^2 &= \begin{bmatrix} 0 & Y_2^{\mathrm{T}} & 0 & Y_1^{\mathrm{T}} & Y_3^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \end{aligned} \tag{12}$$

Proof. The Lyapunov-Krasovskii functional is constructed as follows:

$$\begin{aligned} V(x(t), t) = {} & x^{\mathrm{T}}(t) P x(t) + \int_{t-h_2}^{t-h} x^{\mathrm{T}}(s) Q_1 x(s)\, ds \\ & + \int_{t-h}^{t-h_1} x^{\mathrm{T}}(s) Q_2 x(s)\, ds \\ & + \int_{t-h_1}^{t} x^{\mathrm{T}}(s) Q_3 x(s)\, ds \\ & + \int_{t-h_2}^{t} x^{\mathrm{T}}(s) Q_4 x(s)\, ds \\ & + \int_{t-h}^{t} x^{\mathrm{T}}(s) Q_5 x(s)\, ds \\ & + \int_{t-h(t)}^{t} x^{\mathrm{T}}(s) Q_6 x(s)\, ds \\ & + h_1 \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) M_1 \dot{x}(s)\, ds\, d\theta \\ & + \int_{-h}^{-h_1} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) M_2 \dot{x}(s)\, ds\, d\theta \\ & + \int_{-h_2}^{-h} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) M_3 \dot{x}(s)\, ds\, d\theta, \end{aligned} \tag{13}$$

where $h = \lambda h_1 + (1-\lambda) h_2$, $0 < \lambda < 1$.

The derivative of $V(x(t), t)$ along trajectories of systems (1) is

$$\begin{aligned} & \dot{V}(x(t), t) \\ & \quad = 2x^{\mathrm{T}}(t) P \dot{x}(t) + x^{\mathrm{T}}(t) (Q_3 + Q_4 + Q_5 + Q_6) x(t) \\ & \qquad + x^{\mathrm{T}}(t-h_1)(Q_2 - Q_3) x(t-h_1) \\ & \qquad + x^{\mathrm{T}}(t-h)(Q_1 - Q_2 - Q_5) x(t-h) \\ & \qquad - x^{\mathrm{T}}(t-h_2)(Q_1 + Q_4) x(t-h_2) \\ & \qquad - (1 - \dot{h}(t)) x^{\mathrm{T}}(t-h(t)) Q_6 x(t-h(t)) \\ & \qquad + \dot{x}^{\mathrm{T}}(t) (h_1^2 M_1 + \delta_1 M_2 + \delta_2 M_3) \dot{x}(t) \end{aligned}$$

$$
\begin{aligned}
&-h_1\int_{t-h_1}^{t}\dot{x}^{\mathrm{T}}(s)M_1\dot{x}(s)\,ds\\
&-\int_{t-h}^{t-h_1}\dot{x}^{\mathrm{T}}(s)M_2\dot{x}(s)\,ds\\
&-\int_{t-h_2}^{t-h}\dot{x}^{\mathrm{T}}(s)M_3\dot{x}(s)\,ds.
\end{aligned}
\tag{14}
$$

By Lemma 1, we obtain

$$
\begin{aligned}
&-h_1\int_{t-h_1}^{t}\dot{x}^{\mathrm{T}}(s)M_1\dot{x}(s)\,ds\\
&\quad\le\begin{bmatrix}x(t)\\ x(t-h_1)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-M_1 & M_1\\ * & -M_1\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h_1)\end{bmatrix}.
\end{aligned}
\tag{15}
$$

Function $h(t)$ is time-varying and for $\forall t\ge 0$, $h(t)$ must fall in $[h_1,h]$ or $[h,h_2]$, so we need to discuss two cases.

The first case, when $h(t)\in[h_1,h]$, then

$$
\begin{aligned}
&\int_{t-h_2}^{t-h}\dot{x}^{\mathrm{T}}(s)M_3\dot{x}(s)\,ds\\
&\quad\le\begin{bmatrix}x(t)\\ x(t-h_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\dfrac{-M_3}{\delta_2} & \dfrac{M_3}{\delta_2}\\ * & \dfrac{-M_3}{\delta_2}\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h_2)\end{bmatrix}.
\end{aligned}
\tag{16}
$$

Using Lemma 2, it can be obtained that

$$
\begin{aligned}
&-\int_{t-h}^{t-h_1}\dot{x}^{\mathrm{T}}(s)M_2\dot{x}(s)\,ds\le 2\delta_1^{\mathrm{T}}(t)\\
&\quad\cdot T\left[x(t-h(t))-x(t-h)\right]+2\delta_1^{\mathrm{T}}(t)\\
&\quad\cdot Y\left[x(t-h_1)-x(t-h(t))\right]+\delta_1^{\mathrm{T}}(t)\\
&\quad\cdot\left\{(h-h(t))TM_2^{-1}T^{\mathrm{T}}+(h(t)-h_1)YM_2^{-1}Y^{\mathrm{T}}\right\}\\
&\quad\cdot\delta_1(t),
\end{aligned}
\tag{17}
$$

where, $\delta_1(t)=\left[x^{\mathrm{T}}(t-h_1)\ \ x^{\mathrm{T}}(t-h(t))\ \ x^{\mathrm{T}}(t-h)\right]^{\mathrm{T}}$, $T=\left[T_1^{\mathrm{T}}\ \ T_2^{\mathrm{T}}\ \ T_3^{\mathrm{T}}\right]^{\mathrm{T}}$, $T=\left[Y_1^{\mathrm{T}}\ \ Y_2^{\mathrm{T}}\ \ Y_3^{\mathrm{T}}\right]^{\mathrm{T}}$.

According to (13)–(17), we have

$$
\begin{aligned}
&\dot{V}(x(t),t)\le\eta^{\mathrm{T}}(t)\Big[\Xi_1+\Pi^{\mathrm{T}}M\Pi\\
&\quad+(h-h(t))\overline{T}^1M_2^{-1}\overline{T}^{1\mathrm{T}}+(h(t)-h_1)\overline{Y}^1M_2^{-1}\overline{Y}^{1\mathrm{T}}\Big]\\
&\quad\cdot\eta(t),
\end{aligned}
\tag{18}
$$

where, $\eta^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-h(t))\ \ x^{\mathrm{T}}(t-h_1)\ \ x^{\mathrm{T}}(t-h)\ \ x^{\mathrm{T}}(t-h_2)]^{\mathrm{T}}$, $\Theta_1=PDF(t)E_1+E_1^{\mathrm{T}}F^{\mathrm{T}}(t)D^{\mathrm{T}}P$, $\Theta_2=PDF(t)E_2$, $\Xi_1=\begin{bmatrix}\Omega_{11}^0+\Theta_1 & PB+\Theta_2 & M_1 & 0 & 0\\ * & \Omega_{22}^0 & \Omega_{23}^1 & \Omega_{24}^1 & 0\\ * & * & \Omega_{33}^0+\Omega_{33}^1 & \Omega_{34}^1 & 0\\ * & * & * & \Omega_{44}^0+\Omega_{44}^1-M_3/\delta_2 & M_3/\delta_2\\ * & * & * & * & \Omega_{55}^0-M_3/\delta_2\end{bmatrix}$, $\Pi=\left[A+DF(t)E_1\ \ B+DF(t)E_2\ \ 0\ \ 0\ \ 0\right]$, and the definitions of symbols Ω_{11}^0, Ω_{22}^0, Ω_{33}^0, Ω_{44}^0, Ω_{55}^0, Ω_{23}^1, Ω_{24}^1, Ω_{33}^1, Ω_{34}^1, Ω_{44}^1, M, $\overline{T}^1$, $\overline{Y}^1$ are the same as (11).

By Lemma 3, if $\Xi_1+\Pi^{\mathrm{T}}M\Pi+(h-h(t))\overline{T}^1M_2^{-1}\overline{T}^{1\mathrm{T}}+(h(t)-h_1)\overline{Y}^1M_2^{-1}\overline{Y}^{1\mathrm{T}}<0$, then

$$
\begin{aligned}
&\begin{bmatrix}\Xi_1 & \Pi^{\mathrm{T}}M\\ * & -M\end{bmatrix}+(h-h(t))\overline{T}^1M_2^{-1}\overline{T}^{1\mathrm{T}}\\
&\quad+(h(t)-h_1)\overline{Y}^1M_2^{-1}\overline{Y}^{1\mathrm{T}}<0.
\end{aligned}
\tag{19}
$$

In view of Lemma 4 and Schur complement, (19) can be expressed as

$$
\begin{aligned}
&\Gamma^1+(h-h(t))\overline{T}^1M_2^{-1}\overline{T}^{1\mathrm{T}}+(h(t)-h_1)\overline{Y}^1M_2^{-1}\overline{Y}^{1\mathrm{T}}\\
&\quad<0,
\end{aligned}
\tag{20}
$$

where $\Gamma^1=\begin{bmatrix}\Omega^0+\Omega^1 & \Phi\\ * & \Psi^1\end{bmatrix}$, and the definitions of symbols Ω^0, Ω^1, Φ, Ψ^1 are the same as (11).

Besides, based on the Lemma 5, (20) is equivalent to

$$
\begin{aligned}
&\Gamma^1+(h-h_1)\overline{T}^1M_2^{-1}\overline{T}^{1\mathrm{T}}<0,\\
&\Gamma^1+(h-h_1)\overline{Y}^1M_2^{-1}\overline{Y}^{1\mathrm{T}}<0.
\end{aligned}
\tag{21}
$$

For (21), using Schur complement, we can get

$$
\begin{aligned}
&\begin{bmatrix}\Omega^0+\Omega^1 & \Phi & \delta_1\overline{T}^1\\ * & \Psi^1 & 0\\ * & * & -\delta_1M_2\end{bmatrix}<0,\\
&\begin{bmatrix}\Omega^0+\Omega^1 & \Phi & \delta_1\overline{Y}^1\\ * & \Psi^1 & 0\\ * & * & -\delta_1M_2\end{bmatrix}<0.
\end{aligned}
\tag{22}
$$

Namely, when $i=1$, (22) are equivalent to linear matrix inequalities (11); therefore $\dot{V}(x(t),t)<0$. Then system (1) is asymptotically stable.

The second case, when $h(t)\in[h,h_2]$, then

$$
\begin{aligned}
&-\int_{t-h}^{t-h_1}\dot{x}^{\mathrm{T}}(s)M_2\dot{x}(s)\,ds\\
&\quad\le\begin{bmatrix}x(t-h_1)\\ x(t-h)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\dfrac{-M_2}{\delta_1} & \dfrac{M_2}{\delta_1}\\ * & \dfrac{-M_2}{\delta_1}\end{bmatrix}\begin{bmatrix}x(t-h_1)\\ x(t-h)\end{bmatrix}.
\end{aligned}
\tag{23}
$$

By Lemma 2, we have

$$
\begin{aligned}
&-\int_{t-h_2}^{t-h}\dot{x}^{\mathrm{T}}(s)M_3\dot{x}(s)\,ds\le 2\delta_2^{\mathrm{T}}(t)\\
&\quad\cdot T\left[x(t-h(t))-x(t-h_2)\right]+2\delta_2^{\mathrm{T}}(t)\\
&\quad\cdot Y\left[x(t-h)-x(t-h(t))\right]+\delta_2^{\mathrm{T}}(t)\\
&\quad\cdot\left\{(h_2-h(t))TM_3^{-1}T^{\mathrm{T}}+(h(t)-h)YM_3^{-1}Y^{\mathrm{T}}\right\}\\
&\quad\cdot\delta_2(t),
\end{aligned}
\tag{24}
$$

where $\delta_2(t)=\left[x^{\mathrm{T}}(t-h)\ \ x^{\mathrm{T}}(t-h(t))\ \ x^{\mathrm{T}}(t-h_2)\right]^{\mathrm{T}}$.

Similar to the first case, we can obtain

$$\dot{V}(x(t),t) \le \eta^{\mathrm{T}}(t)\left[\Gamma^2 + (h-h(t))\overline{T}^2 M_3^{-1}\overline{T}^{2\mathrm{T}} + (h(t)-h_1)\overline{Y}^2 M_3^{-1}\overline{Y}^{2\mathrm{T}}\right]\eta(t), \tag{25}$$

where $\Gamma^2 = \begin{bmatrix} \Omega^0+\Omega^2 & \Phi \\ * & \Psi^2 \end{bmatrix}$, and the definitions of symbols Ω^0, Ω^2, Φ, Ψ^2 are the same as (11).

Similarly,

$$\begin{bmatrix} \Omega^0+\Omega^2 & \Phi & \delta_2\overline{T}^2 \\ * & \Psi^2 & 0 \\ * & * & -\delta_2 M_3 \end{bmatrix} < 0, \qquad \begin{bmatrix} \Omega^0+\Omega^2 & \Phi & \delta_2\overline{Y}^2 \\ * & \Psi^2 & 0 \\ * & * & -\delta_2 M_3 \end{bmatrix} < 0. \tag{26}$$

Namely, when $i = 2$, (26) are equivalent to linear matrix inequalities (11), hence $\dot{V}(x(t),t) < 0$. Then system (1) is asymptotically stable. Combined with the above two cases, as long as the linear matrix inequality (11) is satisfied, system (1) is asymptotically stable. The proof is now completed. □

When the time-delay function $h(t)$ is not differentiable or the time-delay-variation rate d is unknown, in Lyapunov-Krasovskii functional (13), removing integral terms $\int_{t-h(t)}^{t} x^{\mathrm{T}}(s)Q_6x(s)ds$, the following conclusion can be obtained by using the time-delay segmentation technique.

Corollary 7. *For any given constant $0 \le h_1 < h_2$, $0 < \lambda < 1$, if there are positive-definite matrices P, Q_1, Q_2, Q_3, Q_4, Q_5, M_1, M_2, M_3, positive numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, and matrices T_i, Y_i $(i = 1,2,3)$ of appropriate dimensions, satisfying the following matrix inequality*

$$\begin{bmatrix} \Omega^{0\prime}+\Omega^i & \Phi & \delta_i\overline{Y}^i \\ * & \Psi^i & 0 \\ * & * & -\delta_i M_{i+1} \end{bmatrix} < 0, \qquad \begin{bmatrix} \Omega^{0\prime}+\Omega^i & \Phi & \delta_i\overline{Y}^i \\ * & \Psi^i & 0 \\ * & * & -\delta_i M_{i+1} \end{bmatrix} < 0, \qquad i = 1,2, \tag{27}$$

then system (1) is asymptotically stable, where

$$\Omega^{0\prime} = \begin{bmatrix} \Omega_{11}^{0\prime} & PB & M_1 & 0 & 0 \\ * & \Omega_{22}^{0\prime} & 0 & 0 & 0 \\ * & * & \Omega_{33}^0 & 0 & 0 \\ * & * & * & \Omega_{44}^0 & 0 \\ * & * & * & * & \Omega_{55}^0 \end{bmatrix},$$

$$\Omega_{11}^{0\prime} = PA + A^{\mathrm{T}}P + Q_3 + Q_4 + Q_5 - M_1, \qquad \Omega_{22}^{0\prime} = T_2 + T_2^{\mathrm{T}} - Y_2 - Y_2^{\mathrm{T}}. \tag{28}$$

The definitions of other symbols are the same as (11).

Remark 8. The delay-dividing technique requires $0 < \lambda < 1$, and the maximum allowable delay bound is related to the accuracy of λ. For given λ, according to Theorem 6 and Corollary 7, we can obtain the corresponding maximum allowable delay bound, and maximum value of these is taken as the maximum allowable delay bound of system. By improving the accuracy of λ, the maximum allowable delay bound of system can be increased, and the time-delay information is more fully utilized. Furthermore, the conservation of system will reduce and the computational complexity will increase.

4. The Analysis of Simulation Examples

In this section, the validity of Theorem 6 and Corollary 7 is verified by the classical numerical examples; then they are applied to the WSCC 3-machine 9-bus system for example analysis.

Example 1. Consider the uncertain linear system (1) described by the matrices as

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}. \tag{29}$$

When the accuracy of λ is 0.1, $h_1 = 0$ and time-delay-variation rates d are equal to 0.3, 0.5, and 0.9, respectively; the maximum allowable delay bound h_2 for system (1) is obtained by using Theorem 6. The results are shown in Table 1; it is clear that the proposed stability criterion is less conservative than those in [16–18].

Example 2. Consider the time-delay system (1) with the following parameters:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$

$$B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$D = E_1 = E_2 = 0. \tag{30}$$

When the accuracy of λ is 0.01, $h_1 = 1$ and time-delay-variation rates d are equal to 0.5 and 0.9, respectively; delay bound h_2 for ensuring stability of system (1) is obtained by using Theorem 6. When d is unknown, delay bound h_2 is got by using Corollary 7. The results are shown in Table 2. Compared with [16, 19, 20], the method of this paper is less conservative.

Example 3. Consider WSCC 3-machine 9-bus system, the wiring diagram of the system is shown in Figure 1. Generator G1 is infinite bus, and there is a time delay in the loop controlled by G3. Branches and parameter nodes are detailed in [21].

The generator equation of system can be expressed as

$$\begin{aligned}
\dot{\delta}_i &= \omega_i - \omega_s, \\
2H_i\dot{\omega}_i &= P_{mi} - \left(E'_{qi} - X'_{di}I_{di}\right) I_{qi} - \left(E'_{di} - X'_{qi}I_{qi}\right) I_{di} \\
&\quad - D_i\left(\omega_i - \omega_s\right), \\
T'_{d0i}\dot{E}'_{qi} &= -E'_{qi} - \left(X_{di} - X'_{di}\right) I_{di} + E_{fdi}, \\
T'_{q0i}\dot{E}'_{di} &= -E'_{di} - \left(X_{qi} - X'_{qi}\right) I_{qi}, \\
T_{Ai}\dot{E}_{fdi} &= -E_{fdi} + K_{Ai}\left(V_{refi} - V_{Gi}\right).
\end{aligned} \tag{31}$$

By calculating, the state variables at the equilibrium point are obtained:

$$\begin{aligned}
&\left[\, \delta_2 \;\; \omega_2 \;\; E'_{q2} \;\; E'_{d2} \;\; E_{fd2} \;\; \delta_3 \;\; \omega_3 \;\; E'_{q3} \;\; E'_{d3} \;\; E_{fd3} \,\right] \\
&\quad = \left[-0.2525 \;\; 0 \;\; 1.0983 \;\; -0.2923 \;\; 2.3339 \;\; -0.2590 \;\; 0 \;\; 1.1208 \;\; -0.2857 \;\; 2.7257\right].
\end{aligned} \tag{32}$$

The coefficient matrices in the system model are as follows:

$$\mathbf{A}_0 = \begin{bmatrix}
0 & 377.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.1421 & -0.0039 & -0.0249 & -0.1097 & 0 & 0.1009 & 0 & 0.1202 & 0.0594 & 0 \\
-0.0096 & 0 & -0.2233 & 0.0536 & 0.1667 & 0.1549 & 0 & 0.4965 & 0.0116 & 0 \\
-1.817 & 0 & 0.2657 & -5.023 & 0 & 0.9126 & 0 & 0.2903 & 0.7403 & 0 \\
-257.8 & 0 & -2146 & 361.7 & -50.0 & -191.1 & 0 & -1017 & 88.58 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 377.0 & 0 & 0 & 0 \\
0.2157 & 0 & 0.2061 & 0.1216 & 0 & -0.3470 & -0.0083 & -0.0708 & -0.2916 & 0 \\
0.1444 & 0 & 0.3780 & 0.0173 & 0 & -0.0057 & 0 & -0.1092 & 0.0248 & 0.125 \\
2.3717 & 0 & 0.4298 & 1.828 & 0 & -5.548 & 0 & -0.2416 & -14.26 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2358 & 829.7 & -50.0
\end{bmatrix},$$
$$\mathbf{A}_\tau = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-275.0 & 0 & -879.6 & 9.670 & 0 & -139.9 & 0 & -43.92 & -342.3 & 0
\end{bmatrix}. \tag{33}$$

Table 1: Maximum allowable delay bound for $h_1 = 0$ and different d.

Method	d		
	0.3	0.5	0.9
[14]	1.0280	0.9322	0.7590
[15]	1.0281	0.9561	0.8919
[13] Theorem 1 ($\lambda = 0.5$)	1.0943	1.0043	0.9131
Theorem 6 ($\lambda = 0.5$)	1.1415	1.1387	1.1388

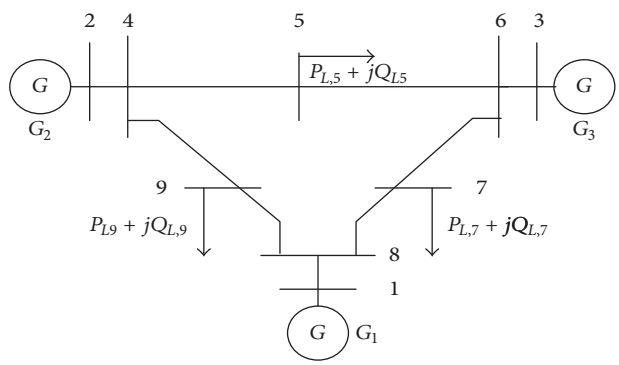

Figure 1: Wiring diagram of WSCC 3-machine 9-bus system.

Table 2: Maximum allowable delay bound for $h_1 = 1$ and different d.

Method	d		
	0.5	0.9	Unknown
[16]	2.07	1.74	1.74
[17]	2.15	2.12	2.12
[13] Theorem 1	($\lambda = 0.31$) 2.2792	($\lambda = 0.46$) 1.9200	($\lambda = 0.46$) 1.9200
Theorem 6 and Corollary 7	($\lambda = 0.49$) 2.4626	($\lambda = 0.52$) 2.4453	($\lambda = 0.52$) 2.4453

Table 3: Maximum allowable delay bound for $h_1 = 0$ and different d.

Method	d		
	0.3	0.5	0.9
[15]	0.2692	0.2499	0.1779
[13] Theorem 1	($\lambda = 0.1$) 0.2688	($\lambda = 0.3$) 0.2575	($\lambda = 0.4$) 0.1814
Theorem 6	($\lambda = 0.4$) 0.3596	($\lambda = 0.4$) 0.3568	($\lambda = 0.4$) 0.3568

When the accuracy of λ is 0.1, $h_1 = 0$ and time-delay variation rates d are equal to 0.3, 0.5, and 0.9, respectively; the maximum allowable delay bound h_2 is obtained by using [16, 18] and Theorem 6. The results are shown in Table 3. Obviously, our criterion leads to much less conservative results.

5. Conclusion

The paper investigates the stability of uncertain linear systems with interval time-varying delay. According to the delay-dividing approach, the delay interval is partitioned into two subintervals and a new Lyapunov-Krasovskii functional is constructed, which makes use of the information on the some delayed sufficiently. The delay-dependent stability criteria are presented by using convex combination technique and improved integral inequality. In the example of WSCC 3-machine 9-bus system, the calculation results show that the upper bound of time delay is larger than that of the previous references. Therefore, the delay-dependent stability criterion presented in this paper is less conservative. Based on the work of this paper, then the analytical method of the uncertain time-delay systems can be extended to the nonlinear uncertain time-delay systems, and the stability of nonlinear uncertain time-delay systems will be analyzed.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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