21 Transmission strategies for the MIMO MAC

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21.1. Introduction

In wireless point-to-point links, one applies multiple antennas to increase the spectral efficiency and the performance of wireless systems [1, 2]. On the other hand, in multiuser scenarios, multiple antennas at the base or even at the mobiles require the development of new transmission strategies in order to achieve the benefits of using the spatial domain. In multiple-input multiple-output (MIMO) multiple access channels (MAC), the optimum transmission strategy depends on the objective function, the power constraints, the channel statistics or the channel realization, the type of channel state information (CSI), and the SNR range.

The analysis of multiuser MIMO systems is very important since usually more than one user are involved in cellular as well as ad hoc systems. Up to now, only little has been found out about MIMO multiuser systems. The achievable rates and the transmission strategy depend at least on the following.

(i) *Structure of the wireless MIMO system.* In the common cellular approach, many mobiles share one base station which controls the scheduling and transmission strategies, for example, power control in a centralized manner. In cellular systems the inter- and intracell interference can be controlled by spectrum and time allocation. In MIMO systems an additional dimension, namely the space, is available for allocation purposes.

(ii) *Transmit strategies.* Obviously, the transmit strategies of the participating mobiles influence the achievable rate and the properties of the complete system. In turn, the transmit strategies depend on the type of CSI at the transmitter, that is, the more CSI is known about the own channel as well as about the other users and the interference, the more adaptive and smart transmission strategies can be applied. If no CSI is available at the transmitter, it is the best to use multiuser space-time (-spreading) codes.

(iii) *Receiver strategies.* Different decoding and detection strategies can be used at the receiver. The range leads from single-user detection algorithms which treat the other users as a noise up to linear and even nonlinear multiuser detection algorithms. Of course, the receiver architecture depends on the type of CSI, too.

(iv) *System parameters.* In general, an important factor is the scenario in which the wireless system works. In home or office scenarios, the system parameter heavily differs from parameters in public access, hot spots, or high velocity scenarios. User parameters, resource parameters, and especially channel parameters have to be taken into account. The achievable performance and throughput depend on those system parameters.

The optimization problems which arise in multiuser MIMO wireless systems are divided into two classes. In the first one, the objective function measures a global performance criteria of the system. In order to increase the throughput of the MIMO MAC, the sum capacity can be maximized [3, 4, 5, 6] or the normalized mean-square error can be minimized [7]. The solution of this class optimization problems leads to transmission strategies which can be unfair for some users. If users experience poor channel conditions for long periods of time, they are not allowed to transmit. Therefore, the other class of optimisation problems deals with the fulfilment of rate [8], SINR [9], or MSE [10] requirements with minimal power. We study performance criteria for one user subject to fulfilment constraints. In order to solve this class of problems, it is necessary to understand the geometry of the achievable rate, SINR, or MSE region. In both classes of optimization problems, the constraints can be either individual power constraints of each user or a sum power constraint. The second class of programming problems are nonconvex nonlinear programming problems which are notoriously complicate to analyze. The large number of degrees of freedom in the temporal as well as the spatial domain increases the number of parameters which can be controlled. In order to simplify the analysis, it is of advantage to divide the programming problem into parts which can be solved in an iterative fashion.

In this chapter, we motivate and analyze important representative problems of both classes. The development from the single-antenna MAC to the MIMO MAC is shown and the differences and common ground between the single-antenna and the multiple-antenna cases are stressed. Furthermore, we focus on the connections between the different objective functions and their corresponding programming problems. We show which results of recent literature can be reused and which results must not be reused. Finally, we illustrate the optimum transmission strategies by examples.

21.2. Preliminaries

21.2.1. System model

Consider the multiple access channel in Figure 21.1. The communication channel between each user and the base station is modelled by a quasistatic block flat fading MIMO channel.

We have K mobiles with n_T antennas. We can easily extend the results to the case in which every mobile has a different number of transmit antennas. The base

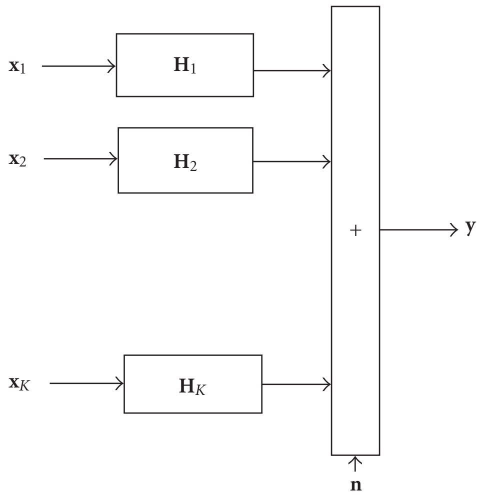

FIGURE 21.1. MIMO MAC system.

station owns n_R receive antennas. In the discrete time model, the received vector $\mathbf{y}$ at one time slot at the base station can be described by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n} \tag{21.1}$$

with the receiver noise $\mathbf{n} \in \mathbb{C}^{n_R \times 1}$ which is additive white Gaussian noise (AWGN), flat fading channel matrices $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$, and transmit signals $\mathbf{x}_k \in \mathbb{C}^{n_T \times 1}$. We assume uncorrelated noise with covariance $\sigma_n^2 \mathbf{I}_{n_R}$. The inverse noise power is denoted by $\rho = 1/\sigma_n^2$.

Equation (21.1) can be rewritten in compact form as

$$\mathbf{y} = \widehat{\mathbf{H}}\widehat{\mathbf{x}} + \mathbf{n} \tag{21.2}$$

with $\widehat{\mathbf{H}} = [\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_K]$ and $\widehat{\mathbf{x}} = [\mathbf{x}_1^T, \dots, \mathbf{x}_K^T]^T$. We collect the transmit covariance matrices in

$$\widehat{\mathbf{Q}} = \begin{pmatrix} \mathbf{Q}_1 & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{Q}_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Q}_K \end{pmatrix}. \tag{21.3}$$

21.2.2. Performance metrics

Under the assumption that the receiver knows the channel realization $\mathbf{H}_k$, the mutual information for user k is given by

$$
\begin{aligned}
I(\mathbf{y}; \mathbf{x}_k | \mathbf{H}_k) &= \log\det\left(\mathbf{I} + \rho \sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H\right) \\
&\quad - \log\det\left(\mathbf{I} + \rho \sum_{l=1,\, l\neq k}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H\right)
\end{aligned}
\tag{21.4}
$$

with SNR ρ and transmit covariance matrices $\mathbf{Q}_k$. The transmit signals of the users are assumed to be zero-mean independent complex Gaussian distributed with covariance matrix $\mathbf{Q}_k$. This probability density function (pdf) maximizes the individual mutual information of each user. Obviously, the individual mutual information of user k depends on the multiuser interference and noise, that is, it is a function of all transmission matrices $\mathbf{H}_k$ between the users and the base, the SNR ρ and the transmit strategies $\mathbf{Q}_k$ of all users,

$$
R_k(\mathcal{Q}, \mathcal{H}, \rho) = \log\det\left(\frac{\mathbf{I} + \rho \sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H}{\mathbf{I} + \rho \sum_{l=1,\, l\neq k}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H}\right) \tag{21.5}
$$

with the set of covariance matrices $\mathcal{Q}$ and the set of channel realizations $\mathcal{H}$

$$
\mathcal{Q} = \{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_K\}, \qquad \mathcal{H} = \{\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_K\}. \tag{21.6}
$$

The achievable rate of user k is denoted by R_k. It is possible that the receiver first detects the signals of a set of users and subtracts them from the received signal before detecting the user k. As long as the users transmit at a rate smaller than or equal to their achievable rate, their signals are detected with arbitrary small probability of error and are therefore correctly subtracted. We assume that the signals of users 1 to $k-1$ are correctly subtracted. In this case, the individual mutual information of user k is given by

$$
R_k^{\mathrm{SIC}}(\mathcal{Q}, \mathcal{H}, \rho) = \log\det\left(\frac{\mathbf{I} + \rho \sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H}{\mathbf{I} + \rho \sum_{l=k+1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H}\right). \tag{21.7}
$$

The receiver starts with user one, detects its data, and subtracts it from the received signal. The received signal for user one is interfered by all other users. Then the second user is detected and subtracted. The second user gets interference from all but the first user. This procedure continues until the last user is detected without any interference. This approach is called successive interference cancellation (SIC). Usually, one assumes that the data of all users are detected without any errors because the users transmit with rate below their capacity.

If we assume that the receiver detects the user signals in a linear fashion, the optimal choice is the linear multiuser MMSE receiver. If we apply the linear MMSE receiver, the performance metric is the *normalized MSE* [11]. The linear MMSE receiver weights the received signal vector $\mathbf{y}$ by the Wiener filter

$$\hat{x}_k = \mathbf{Q}_k \mathbf{H}_k^H \left(\sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H + \sigma_n^2 \mathbf{I} \right)^{-1} \mathbf{y}. \tag{21.8}$$

The covariance matrix of the estimation error $\mathbf{K}_\epsilon$ is given by

$$\mathbf{K}_\epsilon = \mathbb{E}_{\mathbf{H}}\left[(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^H \right]. \tag{21.9}$$

The *normalized MSE* is defined as the trace of the normalized covariance matrix of the estimation error in (21.9). The corresponding performance metric is the individual normalised MSE of user k which is given by

$$\begin{aligned} \mathrm{MSE}_k &= \mathrm{tr}\,(\mathbf{Q}^{-1/2} \mathbf{K}_\epsilon \mathbf{Q}^{-1/2}) \\ &= n_T - \mathrm{tr} \left(\rho \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H \left(\rho \sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H + \mathbf{I} \right)^{-1} \right). \end{aligned} \tag{21.10}$$

In contrast to the capacity, it is not possible to perform SIC without error propagation, since the argument of error free reception is missing. Therefore, each user k experiences interference from all other users. The achievable MSE region is given by all MSE tuples $(m_1, \ldots, m_K)$ for which $(m_1 \geq \mathrm{MSE}_1, \ldots, m_K \geq \mathrm{MSE}_K)$ holds.

Using the individual rate or the individual MSE, each user can require its quality-of-service (QoS) by giving a minimum rate r_k or a maximum MSE m_k which has to be achieved. The problem of the fulfilment of service requirements consists of computing a transmit strategy which fulfills for all $1 \leq k \leq K$ that $R_k \geq r_k$ or $\mathrm{MSE}_k \leq m_k$ by minimizing the individual $p_k \leq P_k$ or sum transmit power $\sum_{k=1}^{K} p_k \leq P$. The transmit power p_k of user k corresponds to the trace of its transmit covariance matrix $p_k = \mathrm{trace}(\mathbf{Q}_k)$.

Another performance metric is the sum of the individual performance metrics. The sum capacity is simply defined as the sum of the individual capacities $\sum_{k=1}^{K} R_k^{\mathrm{SIC}}(\mathcal{Q}, \mathcal{H}, \rho)$, that is, with SIC, we obtain

$$C(\mathcal{Q}, \mathcal{H}, \rho) = \log\det \left(\mathbf{I} + \rho \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H \right). \tag{21.11}$$

The normalized sum MSE is defined in the same manner, that is, $\mathrm{MSE} = \sum_{k=1}^{K} \mathrm{MSE}_k$ and it is given by

$$\mathrm{MSE}(\mathcal{Q}, \mathcal{H}, \rho) = K n_T - n_R + \mathrm{tr} \left(\left[\rho \sum_{k=1}^{n_T} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H + \mathbf{I} \right]^{-1} \right). \tag{21.12}$$

The sum capacity and the sum MSE describe the performance of the complete MAC. The system throughput can be measured by the sum capacity in (21.11) or by the sum MSE (21.12).

21.2.3. Assumptions and constraints

We assume that the transmitter as well as the receiver know the channel perfectly. This ideal leads to an upper bound on the achievable performance. The channel between each mobile and the base station is frequency flat. The coherence time of the channel is large enough

(i) to encode over a sufficiently large number of blocks for achieving approximately the capacity conditioned on one channel realization if the capacity is considered as the performance metric, or

(ii) to transmit one symbol which could be even a space-time symbol if the MSE is considered as the performance metric.

SIC without error propagation at the base station is assumed for capacity optimization. For MSE minimization, no SIC is performed.

The transmit power of the mobiles can be constrained in various ways depending on the scenario considered. The most constrained scenario corresponds to a power constraint on each single antenna of each mobile. This constraint is relevant from a transmit antenna amplifier point of view.

Less restricted constraints are individual power constraints of the users, that is,

$$p_k = \operatorname{tr}(\mathbf{Q}_k) \leq p_k^{\max}. \tag{21.13}$$

Individual power constraints are important in regard to public health conditions.

A less restricted constraint is a sum power constraint for all mobiles in one cell, that is,

$$\sum_{k=1}^{K} p_k = \sum_{k=1}^{K} \operatorname{tr}(\mathbf{Q}_k) \leq P. \tag{21.14}$$

The sum power constraint is important if the power can be distributed across the users in one cell, but the cell sum power is limited in order to keep the intercell interference under control. In addition to this, the sum power constraint can be motivated by the downlink transmission in which the base station has a power constraint.

Usually, two different temporal power allocation constraints are applied on top of the sum, individual, and antenna constraint, namely, the short-term and long-term power constraint. The short-term power constraint operates on the power allocated to the transmitted signal vector over one constant channel realization, that is, the individual short-term power constraint is the one given above in (21.13). If it is allowed to distribute the power amount over ergodic many channel fading blocks, the transmit policy is a function of the channel realization $\mathbf{H}_k$.

The long-term individual power constraint for user k is given by

$$\mathbb{E}_{\mathbf{H}_k}\left[\operatorname{tr}\mathbf{Q}_k(\mathbf{H}_k)\right] \le P_k \tag{21.15}$$

and the corresponding long-term sum power constraint is given by

$$\sum_{k=1}^{K}\mathbb{E}_{\mathbf{H}_k}\left[\operatorname{tr}\mathbf{Q}_k(\mathbf{H}_k)\right] \le P. \tag{21.16}$$

The optimum transmit policy under short-term and long-term power constraints differs in the additional degree of freedom for the long-term power constraint which leads to some kind of temporal water-filling with perfect CSI at transmitter and receiver. Since we are interested in the impact of the spatial dimension (and not in the temporal), we assume only short-term power constraints in this section.

21.3. Sum performance optimization

The first problem is as follows. *Assume that the channel realizations* $\mathbf{H}_k$ *are known and fixed. Solve the sum capacity optimization under the sum power constraint, that is,*

$$\begin{gathered}\max\log\det\left(\mathbf{I}+\rho\sum_{k=1}^{K}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right)\\ \textit{subject to}\\ \sum_{k=1}^{K}\operatorname{tr}\mathbf{Q}_k\le P,\quad \mathbf{Q}_k\succeq 0,\ 1\le k\le K.\end{gathered} \tag{21.17}$$

The second problem is given as follows. *Assume that the channel realizations* $\mathbf{H}_k$ *are known and fixed. Solve the normalized sum* MSE[1] *minimization under the sum power constraint, that is,*

$$\begin{gathered}\min\operatorname{tr}\left(\left[\mathbf{I}+\rho\sum_{k=1}^{K}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right]^{-1}\right)\\ \textit{subject to}\\ \sum_{k=1}^{K}\operatorname{tr}\mathbf{Q}_k\le P,\quad \mathbf{Q}_k\succeq 0,\ 1\le k\le K.\end{gathered} \tag{21.18}$$

The optimal transmit strategies in sum capacity maximization as well as sum MSE minimization have a very interesting intrinsic structure. This leads to one algorithmic structure which solves both programming problems. We start with

[1]For convenience, we omitted the constant terms $Kn_T - n_R$ in the normalized sum MSE.

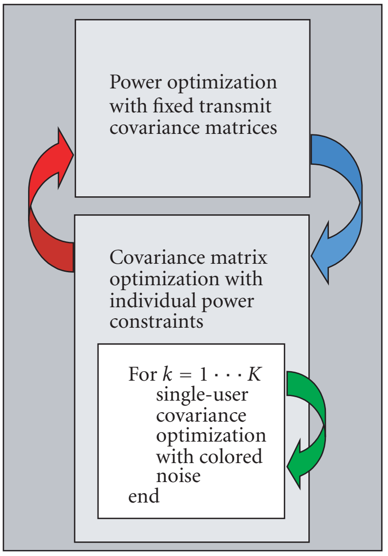

FIGURE 21.2. Sum performance optimization algorithm.

a top-down approach and present the signal processing structure which achieves the sum capacity or minimizes the normalized sum MSE, at first. The original problem of transmit strategy optimization is decomposed into two subproblems, namely the power allocation and the covariance matrix optimization under individual power constraints. This scheme is illustrated in Figure 21.2.

The transmit strategies of the K users are divided into two parts, namely, the power allocation and the transmit covariance matrix optimization for fixed power allocation. The outer loop is between power allocation $p_1, \dots, p_K$ and covariance matrix $\mathbf{Q}_1, \dots, \mathbf{Q}_K$ optimization under individual power constraints. The covariance matrix optimization can be decomposed into an inner loop in which single-user covariance matrix optimization with respect to the effective channel is performed. In the case in which the sum performance is measured by the sum capacity, the inner single-user waterfilling algorithm can be derived in closed form [4]. Then, the covariance matrix optimisation corresponds to iterative waterfilling. In the following, the two parts of the iterative algorithm are described in more detail.

21.3.1. Power optimization with fixed transmit covariance matrices

The programming problem for fixed transmit covariance matrices $\mathbf{Q}_1, \dots, \mathbf{Q}_K$ reduces to a convex vector-valued optimization problem [12], that is, for the sum capacity optimization. *The channel realizations* $\mathbf{H}_k$ *of all users* k *are assumed to be known. Keep the transmit covariance matrices fixed* $\mathbf{Q}'_1, \mathbf{Q}'_2, \dots, \mathbf{Q}'_K$. *Distribute a* *fixed amount of transmit power P across the mobiles, that is, solve*

$$\begin{gathered}\max \log\det\left(\mathbf{I}+\rho\sum_{k=1}^{K}p_k\mathbf{H}_k\mathbf{Q}'_k\mathbf{H}_k^H\right)\\ \textit{subject to}\\ \sum_{k=1}^{K}p_k\le P,\quad p_k>0,\ 1\le k\le K\end{gathered}\tag{21.19}$$

or for sum MSE minimization, *solve*

$$\begin{gathered}\min \operatorname{tr}\left(\left[\mathbf{I}+\rho\sum_{k=1}^{K}p_k\mathbf{H}_k\mathbf{Q}'_k\mathbf{H}_k^H\right]^{-1}\right)\\ \textit{subject to}\\ \sum_{k=1}^{K}p_k\le P,\quad p_k>0,\ 1\le k\le K.\end{gathered}\tag{21.20}$$

Since, the programming problems in (21.19) and (21.20) are convex problems, they can be effectively solved by convex optimization techniques like interior point methods [12]. Especially for the sum capacity optimization, a tool called MAXDET [13] can be used. By analyzing the necessary and sufficient Karush-Kuhn-Tucker (KKT) optimality conditions, the optimal power allocation p_1, $\dots, p_K$ can be characterized in the following way: for small SNR values, that is, for small P, only one user is supported, that is, $p_k = P$ and $p_{l\neq k} = 0$. If we increase the available transmit power (and the SNR), more and more users obtain partial transmit power. The first user which is supported has the maximum channel matrix eigenvalue, that is, $\lambda_{\max}(\mathbf{H}_k\mathbf{H}_k^H) \ge \lambda_{\max}(\mathbf{H}_{l\neq k}\mathbf{H}_{l\neq k})$. This has been shown in [7] for the sum MSE minimization and in [14] for the optimal transmit covariance matrices.

21.3.2. Transmit covariance matrix optimization under individual power constraints

We have the following two problems. For sum capacity optimization: *in order to maximize the sum capacity for fixed and known channel realizations* $\mathbf{H}_k$, *and for fixed power allocation* $p_1, \dots, p_K$, *find the optimal transmit covariance matrices* $\mathbf{Q}_1^*, \dots, \mathbf{Q}_K^*$, *that is, solve*

$$\begin{gathered}\max \log\det\left(\mathbf{I}+\rho\sum_{k=1}^{K}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right)\\ \textit{subject to}\\ \operatorname{tr}\mathbf{Q}_k\le p_k,\quad \mathbf{Q}_k\succeq 0,\ 1\le k\le K\end{gathered}\tag{21.21}$$

or for sum MSE minimization:

$$\max \operatorname{tr}\left(\left[\mathbf{I}+\rho\sum_{k=1}^{K}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right]^{-1}\right)$$
subject to
$$\operatorname{tr}\mathbf{Q}_k \leq p_k, \quad \mathbf{Q}_k \succeq 0,\ 1 \leq k \leq K. \tag{21.22}$$

The optimization problems in (21.21) and (21.22) are convex with respect to the transmit covariance matrices $\mathbf{Q}_1,\dots,\mathbf{Q}_K$. However, since the objective function is matrix valued, the optimization is not as easy as in the power allocation case. Therefore, the optimization is further split into a series of single-user optimization problems which are to be solved one after the other. In each single-user optimization step, the other users and their received signals $\mathbf{H}_l\mathbf{Q}_l\mathbf{H}_l^H$ are treated as additional colored noise. This reduces the complexity of the algorithm from simultaneously optimizing K transmit covariance matrices to K succeeding single transmit covariance matrix optimizations. Especially for a large number of users K in the cell, the complexity is reduced. Hence, we arrive at the following two single-user optimization problems with colored noise. We treat the two cases sum capacity optimization and sum MSE minimization separately. For the kth user, we write the noise plus interference as

$$\mathbf{Z}_k = \mathbf{I}+\rho\sum_{\substack{l=1\\ l\neq k}}^{K}\mathbf{H}_l\mathbf{Q}_l\mathbf{H}_l^H. \tag{21.23}$$

21.3.2.1. Sum capacity

In the case in which the performance metric is the sum capacity, the single-user problem which is iteratively solved is the waterfilling with respect to the effective channel $\mathbf{Z}_k^{-1/2}\mathbf{H}_k$. For each user $k \in [1\cdots K]$, we solve the optimization problem

$$\max\log\det\left(\mathbf{I}+\rho\overbrace{\mathbf{Z}_k^{-1/2}\mathbf{H}_k}^{\tilde{\mathbf{H}}_k}\mathbf{Q}_k\mathbf{H}_k^H\mathbf{Z}_k^{-1/2}\right)$$
subject to
$$\operatorname{tr}\left(\mathbf{Q}_k\right) \leq p_k, \quad \mathbf{Q}_k \succeq 0. \tag{21.24}$$

The next result shows that the single-user covariance optimizations for all users $1 \leq k \leq K$ in (21.24) mutually solve the optimization problem (21.21). This result corresponds to [4, Theorem 3].

If all covariance matrices $\mathbf{Q}_k^*$ *mutually solve the optimization problem in* (21.24) *for* $\mathbf{Z}_k$ *in* (21.23), *then they solve optimization problem in* (21.21) *for the sum capacity, too.*

The result follows from the fact that the optimization problem in (21.24) has the same optimality conditions as the original problem in (21.21).

In order to prove convergence of the iterative single-user water filling, note that the objective in (21.24) differs from the objective in (21.21) only by a constant which is independent of $\mathbf{Q}_k$. Therefore, in each single-user water filling step the sum capacity is increased. The channel matrix $\widetilde{\mathbf{H}}_k = \mathbf{Z}_k^{-1/2}\mathbf{H}_k$ in (21.24) is the effective channel which is weighted by the inverse noise plus interference. The iterative single-user performance algorithm in (21.24) solves the original optimization problem in (21.21).

21.3.2.2. Sum MSE

In the case in which the performance metric is the sum MSE, the single-user problem which is iteratively solved is the original sum MSE problem for fixed transmit strategies of the other users. For each user $k \in [1 \cdots K]$, we solve the optimization problem

$$\begin{aligned} &\min \operatorname{tr}\left(\left[\mathbf{I} + \mathbf{Z}_k + \rho \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H\right]^{-1}\right) \\ \text{subject to} \quad & \\ &\operatorname{tr}(\mathbf{Q}_k) \leq p_k, \quad \mathbf{Q}_k \succeq 0. \end{aligned} \tag{21.25}$$

The next result shows that the single-user covariance optimizations for all users $1 \leq k \leq K$ in (21.25) solve the optimization problem (21.22).

If all covariance matrices $\mathbf{Q}_k^*$ mutually solve the optimization problem in (21.25) *for*

$$\mathbf{Z}_k = \sigma_n^2 \mathbf{I} + \sum_{l=1,\, l\neq k}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H, \tag{21.26}$$

then they solve optimization problem in (21.22)*, too.*

Note that the single-user optimization problem in (21.24) has an interesting interpretation: assume the single-user MSE optimization with colored noise $\mathbf{Z}_k = \mathbf{U}_Z \boldsymbol{\Lambda}_Z \mathbf{U}_Z^H$. We can write

$$\begin{aligned} \operatorname{tr}\left(\left[\mathbf{Z}_k + \rho \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H\right]^{-1}\right) &= \operatorname{tr}\left(\boldsymbol{\Lambda}_Z \left[\mathbf{I} + \rho \widetilde{\mathbf{H}}_k \mathbf{Q}_k \widetilde{\mathbf{H}}_k^H\right]^{-1}\right) \\ &= \sum_{l=1}^{n_R} \lambda_Z^{-1}(l) \left(\left[\mathbf{I} + \rho \widetilde{\mathbf{H}}_k \mathbf{Q}_k \widetilde{\mathbf{H}}_k^H\right]^{-1}\right)_{l,l}. \end{aligned} \tag{21.27}$$

The channel matrix $\widetilde{\mathbf{H}}_k = \mathbf{Z}_k^{-1/2}\mathbf{H}_k$ in (21.27) is the weighted effective channel. The iterative single-user MSE algorithm solves the original optimization problem in (21.22). However, in contrast to the iterative water filling algorithm, we cannot derive a simple algorithm which solves the single-user MSE problem because of the dependence on the noise eigenvalues in (21.27).

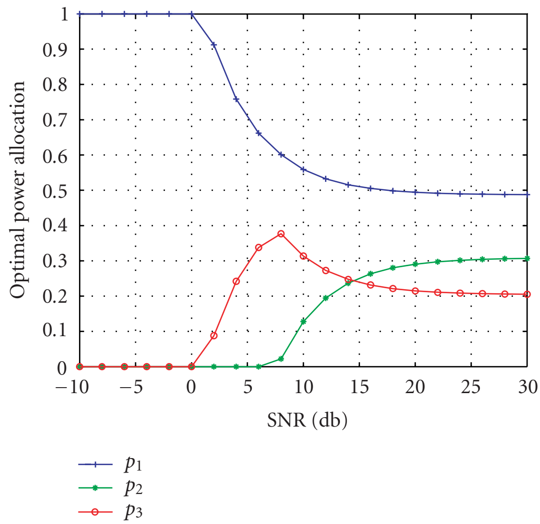

FIGURE 21.3. Example MIMO MAC, power allocation over SNR for sum capacity maximization, for MIMO $K = 3, n_t = 2, n_r = 2$.

21.3.3. Illustration of iterative algorithm

In Figure 21.3, the optimal power allocation for the three users of a MIMO system with three users with two transmit antenna each and a base with two receive antennas, is shown.

In Figure 21.3, it can be observed that SNR values up to 0 dB, only one user is active (user one). Then for SNR values up to 7 dB two users are active. For SNR values above 7 dB all three users are active. In contrast to the SIMO scenario, for SNR values approaching infinity, equal power allocation is not optimal. It is worth mentioning that the second user who is active for SNR values greater than zero, gets less transmit power than the third active user for SNR values above 15 dB. The roles of the users change in MIMO MAC due to the additional degree of freedom in choosing the transmit covariance matrices. Furthermore, the optimal power allocation does not converge to equal power allocation for SNR approaching infinity.

21.4. Performance region analysis

In contrast to the sum performance of multiuser MIMO systems, very little is known up to now about the complete performance region and how to achieve required points in the interior of the region. The difference between the sum performance and the complete region will be illustrated by an example for the capacity region and the sum capacity. We focus in this section on the mutual information as the performance metric. However, the results can be applied to other performance metrics as the individual MSEs or SINRs as well.

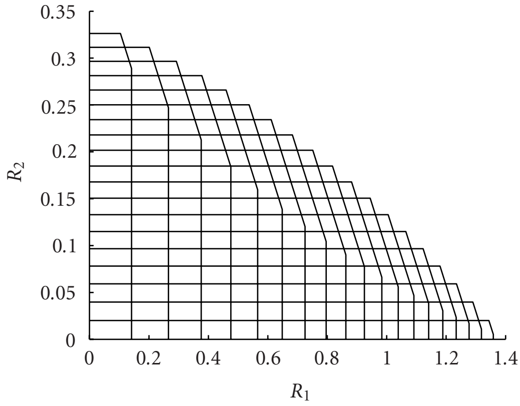

Figure 21.4. Example of a MAC capacity region.

In Figure 21.4, an example capacity region of a MAC is shown. It can be observed that the "atom" element which constitutes the whole region is one pentagon. For each power allocation p_1, p_2 and fixed pair of transmit covariance matrices $\mathbf{Q}_1$, $\mathbf{Q}_2$, the rate tupels inside one pentagon can be achieved. Since the base station performs SIC, there are two permutation orders in which the users signals can be decoded. The edge points in the interior of the capacity region correspond exactly with the two different decoding orders. For fixed transmit covariance matrices of user one $\mathcal{Q}_1$ and of user two $\mathcal{Q}_2$, respectively, the two rate tupels with the sum rate $R_s = \log\det(\mathbf{I} + \rho\mathbf{H}_1\mathcal{Q}_1\mathbf{H}_1^H + \rho\mathbf{H}_2\mathcal{Q}_2\mathbf{H}_2^H)$ are given by

$$\begin{aligned}
\left(R_1^1, R_2^1\right) &= \left(\log\det\left(\mathbf{I} + \rho\mathbf{H}_1\mathcal{Q}_1\mathbf{H}_1^H\right), R_s - R_1^1\right),\\
\left(R_1^2, R_2^2\right) &= \left(R_s - R_2^2, \log\det\left(\mathbf{I} + \rho\mathbf{H}_1\mathcal{Q}_1\mathbf{H}_1^H\right)\right).
\end{aligned} \tag{21.28}$$

In (R_1^1, R_2^1), the second user is decoded first without errors and subtracted since he is communicating with rate less than or equal to his capacity. For K users the resulting region is no longer a two-dimensional pentagon but a polymatroid. Polymatroids were introduced by Edmonds in [15]. This important structure of the achievable region has been used in [16, 17] for analyzing the ergodic and delay-limited capacity region for SISO multiuser channels under long-term power constraints.

In order to illustrate the recent results on the capacity region of multiple-antenna multiple-user channels and their achievability, we will assume in the following that the K users have rate requirements[2] $\gamma_1, \dots, \gamma_K$ which has to be fulfilled.

[2]In general, the users have some QoS requirements depending on their service. The QoS requirement can be expressed in terms of rate requirements, SINR requirements, MSE requirements, or even in bit error rate (BER) requirements. We will focus on rate requirements.

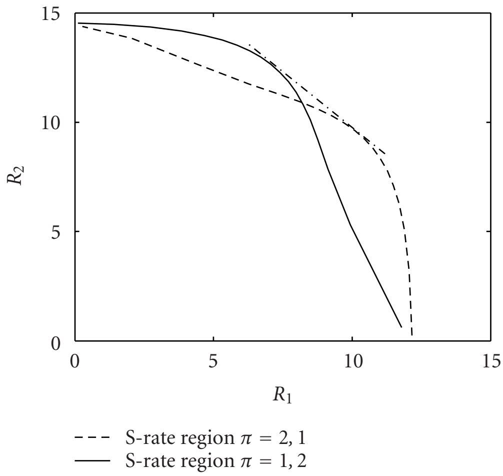

FIGURE 21.5. Example of a MIMO MAC capacity region.

The rate requirements should be fulfilled with minimum total transmit power, that is,

$$\min P \text{ subject to} \quad R_k \geq \gamma_k \quad \forall 1 \leq k \leq K. \tag{21.29}$$

It is well known for the SISO MAC that the region created by all achievable polymatroids is itself convex. In multiple antenna systems this in general is not the case. We show a typical MIMO MAC capacity region in Figure 21.5.

In Figure 21.5, it can be observed that the union of the achievable rate regions with decoding orders $\pi = \{1, 2\}$ and $\pi = \{2, 1\}$ is not convex. The line on which the sum capacity is achieved is missing (point-dashed line). Using the standard time sharing argument, the convex hull of the both so-called spatial (S) rate regions is the complete capacity region [18]. It turns out that all points on these S-rate regions can be achieved by convex optimization. However the points under the sum capacity area are only achievable by a linear combination of $K!$ corners of the sum capacity area. An efficient algorithm which computes the set of optimal transmit covariance matrices that achieve given performance requirements with minimal sum transmit power is still missing.

21.4.1. SISO MAC

First, we consider the SISO MAC channel with perfect CSI at the receiver and at the transmitter. The scalar channels of the users are given by $h_1, \dots, h_K$. The following approach solves the programming problem in (21.29).

(1) First, order the users according to their channel realizations in descending order, with permutation π, that is, $h_{\pi_1} \leq h_{\pi_2} \leq \cdots \leq h_{\pi_K}$.

(2) Start with the first user and allocate power p_{π_1}, such that his rate requirement γ_{π_1} is fulfilled with identity

$$p_{\pi_1} = \frac{2^{\gamma_{\pi_1}} - 1}{\rho h_{\pi_1}}. \tag{21.30}$$

(3) Treat the interference which is created by undetected users as noise and go on for the next user in a similar manner.

In the SISO MAC the performance metrics of user k rate R_k, SINR_k, MSE_k are closely related by

$$R_k = \log\left(1 + \mathrm{SINR}_k\right) = \log\left(\frac{1}{\mathrm{MSE}_k}\right). \tag{21.31}$$

The described procedure solves the general problem of QoS requirement fulfillment. This result is included in [16, 17] for the multiple access channel and in [19, 20] for the broadcast channel with long-term power constraint and the optimization with respect to ergodic and delay-limited and outage capacity. If a long-term power constraint is applied, the power is distributed across the users as well as across time.

21.4.2. SIMO MAC

In the SIMO case, we concentrate on the performance metric SINR which is closely related to the rate and MSE (compare to (21.31)). Therefore, the time sharing argument cannot be applied, since we have stringent delay constraint. The results from this section can be found in [21, 22]. The SINR for the SIMO MAC with beamforming vectors $\mathbf{u}_1, \dots, \mathbf{u}_K$ and power allocation $p_1, \dots, p_K$ is given by

$$\mathrm{SINR}_k = \frac{p_k \mathbf{u}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{Z}_k \mathbf{u}_k} \tag{21.32}$$

with $\mathbf{Z}_k = \sum_{l=1}^{k-1} p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma_n^2 \mathbf{I}$ as interference plus noise for the kth user. It is assumed that the base station performs SIC beginning with user K, then $K-1$ and so on. For fixed transmit powers $p_1, \dots, p_K$ the optimal beamformers are scaled MMSE receivers, that is,

$$\mathbf{u}_k^* = \frac{\mathbf{Z}_k^{-1}\mathbf{h}_k}{\|\mathbf{Z}_k^{-1}\mathbf{h}_k\|^2}. \tag{21.33}$$

It remains to find the optimal power allocation that achieves the targets $\gamma_1, \dots, \gamma_K$. The cascaded interference structure facilitates efficient computation of the optimal power allocation. Start with $k = 1$ and solve

$$\mathbf{p}_k^* \mathbf{h}_k^H \mathbf{Z}_k^{-1} \mathbf{h}_k = \gamma_k \tag{21.34}$$

for all $k = 1, 2, \ldots, K$. The optimal decoding order is not known in general. It depends on the channel realizations as well as on the correlation between them. The interested reader is referred to [22, Section III].

21.4.3. MIMO MAC

In the MIMO case the problem in (21.29) has to be solved for rates $R_k^{\text{SIC}}(\mathcal{Q}, \mathcal{H}, \rho)$ defined in (21.7). The approach in [18, Section IV B] provides the complete achievable S-rate region for the MIMO MAC. By solving the following optimization problem:

$$\max_{\mathcal{Q}} \sum_{k=1}^{K} q_k R_k^{\text{SIC}}(\mathcal{Q}, \mathcal{H}, \rho) \tag{21.35}$$

under the constraint that the sum transmit power is constraint and all transmit covariance matrices are positive definite for a decoding order which corresponds to $q_1 \geq q_2 \geq \cdots \geq K$. The optimization problem in (21.35) is convex and can be efficiently solved by convex optimization methods [12].

In [7] it has been shown that the two-user region of unachievable individual MSEs is convex. It follows that the two-user region over $1 - \text{MSE}_1$ and $1 - \text{MSE}_2$ is convex, too. Because no SIC can be applied for MSE optimization, the optimization in (21.35) can be performed in order to achieve all points on the boundary of the achievable MSE region. The question remains, whether the K user region of unachievable individual MSEs is convex, or not. In general, it is a philosophical question what kind of performance measure to use and how to scale it properly.

Multiuser MIMO performance analysis and fulfillment of QoS requirement is a very active research area with many interesting puzzles and problems.

21.5. Open problems and further research topics

This section provided an overview of recent results in the area of information theoretic and performance analysis of multiple-user multiple-antenna systems. Recently, one of the big open problems in information theory was solved, namely, the capacity region of the nondegraded MIMO broadcast channel [23, 24, 25]. Nevertheless, there are many open problems in this area and we list a few of them.

(i) How to find all points on the boundary of the capacity region? This has been discussed in Section 21.4.

(ii) What happens if the perfect CSI assumption is relaxed? The case in which the mobiles have partial CSI, for example, covariance knowledge is discussed in [26, 27].

(iii) Duality theory: the capacity region of the MIMO MAC and the MIMO BC are equal under perfect CSI at transmitters and receivers [28]. Therefore, the results for the MIMO MAC can be transferred to the MIMO BC. What happens without the perfect CSI assumption?

(iv) Incorporating cross layer design issues, the queues, their arrival rates, and length at the mobiles, have to be taken into account. One established performance metric is the stability of the overall wireless communication system. Recently, many results were presented regarding this interesting topic [29, 18, 30].

Abbreviations

MIMO	Multiple-input multiple-output
MAC	Multiple access channels
CSI	Channel state information
SNR	Signal-to-noise ratio
SINR	Signal-to-interference and noise ratio
MSE	Mean Squared error
AWGN	Additive white gaussian noise
pdf	Probability density function
SIC	Successive interference cancellation
MMSE	Minimum mean square error
QoS	Quality-of-Service
BER	Bit error rate
SISO	Single-input single-output
SIMO	Single-input multiple-output
BC	Broadcast channel

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