Research Article

Constraints on a Vacuum Energy from Both SNIa and CMB Temperature Observations

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We investigate the cosmic thermal evolution with a vacuum energy which decays into photon at the low redshift. We assume that the vacuum energy is a function of the scale factor that increases toward the early universe. We put on the constraints using recent observations of both type Ia supernovae (SNIa) by Union-2 compilation and the cosmic microwave background (CMB) temperature at the range of the redshift \(0.01 < z < 3\). From SNIa, we find that the effects of a decaying vacuum energy on the cosmic expansion rate should be very small but could be possible for \(z < 1.5\). On the other hand, we obtain the severe constraints for parameters from the CMB temperature observations. Although the temperature can be still lower than the case of the standard cosmological model, it should only affect the thermal evolution at the early epoch.

1. Introduction

One of the biggest cosmological mysteries is the accelerating cosmic expansion which was discovered by the observations of distant type Ia supernovae (SNIa) starting from more than 10 years ago. For the origin of the accelerating expansion, the following possibilities have been proposed: the modified gravity, such as \(f(R)\) gravity [1, 2], brane-world cosmology [3], inhomogeneous cosmology [4, 5], and existence of unknown energy called as dark energy which is equivalent to a cosmic fluid with a negative pressure.

It is strongly suggested that the dark energy amounts to 70% of the total energy density of the universe from the astronomical observations such as SNIa [6–8], the anisotropy of cosmic microwave background (CMB) [9–11], and the baryon acoustic oscillation [12]. Although there are various theoretical models of dark energy (details are shown in a review [13, 14]), its physical nature is still unknown. Considering above-referenced observations, we can not exclude the constant \(\Lambda\) term which is the most simple model of dark energy.

The interacting dark energy with CMB photon has been discussed, whereas decaying vacuum into photon is related to a primordial light-element abundances [15] and the CMB intensity [16]. From the point of the thermodynamical evolution in the universe, a decaying vacuum energy modifies the temperature-redshift relation [17–19]. This modification affects the cosmic thermal evolution after a hydrogen recombination [20], such as formation of molecules [21, 22] and the first star [21]. In addition to this model, the angular power spectrum of CMB could be also modified [23]. In the previous analysis, thermal history at the higher redshift has been studied. However, as seen in [20, 21], the time variation of temperature at low redshift differs from the model without a decaying vacuum.

In the present study, we update the observational consistency of a vacuum energy (hereafter we denote as \(\Lambda\)) coupled with CMB photon. Hereafter, we call this model D\(\Lambda\)CDM (model). To examine the consistency with observations at low redshift, we focus on the cosmic evolution comparing both the type Ia supernovae and the redshift dependence of CMB temperatures. This is because the time dependence of the CMB temperature for a D\(\Lambda\)CDM differs from CDM model with a constant \(\Lambda\) term of which hereafter we call SACDM (model). In Section 2, formulation of D\(\Lambda\)CDM is reviewed. In Section 3, the \(m-z\) relation is investigated for
DACDM in a flat universe. Parameters inherent in this model are constrained from the CMB temperature observations in Section 4. Concluding remarks are given in Section 5.

2. Dynamics of the Decaying $\Lambda$ Model

The Einstein’s field equation is written as follows (e.g., [24]):
\[
R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = 8\pi GT_{\mu\nu},
\]
(1)
where $G$ is the gravitational constant and $T_{\mu\nu}$ is the energy momentum tensor. If we assume the perfect fluid, $T_{\mu\nu}$ is written as follows:
\[
T_{\mu\nu} = \text{diag}(-\rho, p, p, p).
\]
(2)
Here $\rho$ and $p$ are the energy density and the pressure, respectively. Note that we choose the unit of $c = 1$.

The equation of motion is obtained with use of the Friedmann-Robertson-Walker metric of the homogeneous and isotropic principle:
\[
ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],
\]
(3)
where $a(t)$ is the scale factor and $k$ is the specific curvature constant. This leads to the Friedmann equations:
\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2},
\]
(4)
From the conservation’s law of energy, we can obtain the equation of the energy density:
\[
\dot{\rho} = -3(1 + w) \frac{\dot{a}}{a} \rho,
\]
(5)
where $w$ is the coefficients of the equation of state, $p = w\rho$, and defined by
\[
w = \begin{cases} 
1/3 & \text{for photon and neutrino,} \\
0 & \text{for baryon and cold dark matter,} \\
-1 & \text{for vacuum energy.}
\end{cases}
\]
(6)
We take the component of the energy density as
\[
\rho = \rho_{\gamma} + \rho_{\nu} + \rho_{m} + \rho_{\Lambda},
\]
(7)
where $\gamma$, $\nu$, $m$, and $\Lambda$ indicate photon, neutrino, nonrelativistic matter (baryon plus cold dark matter), and the vacuum, respectively. Here we neglect the energy contribution of $\Lambda$ on other components except for photon: the $\rho_{\gamma}$ and $\rho_{m}$ evolves as $\rho_{\gamma} \propto a^{-4}$ and $\rho_{m} \propto a^{-3}$, respectively. From (5), the equation of the photon coupled with $\Lambda$ is written as
\[
\dot{\rho}_{\gamma} + 4H\rho_{\gamma} = -\dot{\rho}_{\Lambda},
\]
(8)
Let us define the parameter of the energy density as follows:
\[
\Omega_i \equiv \frac{\rho_i}{\rho_{cr}} = \frac{8\pi G}{3H_0^2} \rho_i,
\]
(9)
where $\rho_{cr}$ is the critical density defined by the present Hubble constant $H_0$. We can rewrite (8) as follows (for details also see [22, 23]):
\[
\frac{d\Omega_{\gamma}}{da} + 4 \frac{\Omega_{\gamma}}{a} = -\frac{d\Omega_{\Lambda}}{da}.
\]
(10)
Models with time-dependent $\Lambda$-term have been studied as summarized in [25, 26]. We adopt the energy of the vacuum which varies with the scale factor [20–23]:
\[
\Omega_{\Lambda}(a) = \Omega_{\Lambda 1} + \Omega_{\Lambda 2} a^{-m},
\]
(11)
where $\Omega_{\Lambda 1}$, $\Omega_{\Lambda 2}$, and $m$ are constants. Note that the present value of $\Omega_{\Lambda}$ is expressed by $\Omega_{\Lambda 0} = \Omega_{\Lambda 1} + \Omega_{\Lambda 2}$.

Integrating (10) with (11), we obtain the photon energy density as a function of $a$ [22]:
\[
\Omega_{\gamma} = \begin{cases} 
\Omega_{\gamma 0} + \alpha (a^{-m} - 1) & (m \neq 4), \\
\Omega_{\gamma 0} + 4\Omega_{\Lambda 2} \ln a & (m = 4),
\end{cases}
\]
(12)
where $\Omega_{\gamma 0} = 2.471 \times 10^{-5} h^{-2} (T_{\gamma 0}/2.725 \text{ K})^4$ is the present photon energy density, $h$ is the normalized Hubble constant ($H_0 = 100 h \text{ km/sec/Mpc}$), and $T_{\gamma 0}$ is the CMB temperature at the present epoch. We define $\alpha$ as $\alpha = m\Omega_{\Lambda 2}/(4 - m)$.

Since we assume the flat geometry ($k = 0$) in this work, we can write the condition of $\Omega_{\Lambda}$ as follows:
\[
\Omega_{\gamma 0} + \Omega_{\Lambda 1} + \Omega_{\Lambda 2} = 1.
\]
(13)
From now on, we adopt the present cosmological parameters: Hubble constant $h = 0.738$ [27, 28], and the present density parameter of matter $\Omega_m = 0.2735$ [11]. The present temperature is $T_{\gamma 0} = 2.725 \text{ K}$ observed by COBE [29]. In our study, we search the parameter regions in the range of $\Omega_{\Lambda 2}$ and $m$: $10^{-7} < \Omega_{\Lambda 2} < 10^{-2}$ and $0 < m < 4$. The range of $m$ has already been obtained from the previous analysis [23].

Here we note the range of $\Omega_{\Lambda 2}$ in the present analysis. Kimura et al. [20] found that the radiation temperature (i.e., the energy density of radiation) in DCDM model becomes numerically negative at a point of $a < 1$ when $m$ and/or $\Omega_{\Lambda 2}$ is too large. To avoid this unreasonable situation, that is, $T_{\gamma} < 0$, Nakamura et al. [23] imposed the limit of $\Omega_{\Lambda 2}$ as follows:
\[
\alpha < \Omega_{\gamma 0} & (m \neq 4),
\]
(14)
\[
\Omega_{\Lambda 2} < \Omega_{\gamma 0}/92 & (m = 4).
\]
From the condition (14), we obtain the limit of $\Omega_{\Lambda 2}$ as $4.9 \times 10^{-7}$ for $m = 4$. We can consider that the effect of $\Omega_{\Lambda 2}$ is negligible when $\Omega_{\Lambda 2}$ is smaller than $10^{-7}$.

3. SNIa Constraints

The cosmic distance measures depend sensitively on the spatial curvature and expansion dynamics of the models. Therefore, the magnitude-redshift relation for distant standard candles are proposed to constrain the cosmological parameters.
To calculate the effects of the cosmic expansion at low-$z$, we calculate the theoretical distance modules,

$$\mu_{th} = m_B - M = 5 \log d_L + 25,$$

(15)

where $m_B$, $M$, and $d_L$ are the apparent and absolute magnitudes, and the luminosity distance, respectively. Here $d_L$ is related to the radial distance $r$ in the metric as follows [24]:

$$d_L = (1 + z)r.$$  

(16)

We can obtain $r$ as follows:

$$r = \int \frac{dt}{a(t)} = \frac{1}{H_0} \int \frac{dz}{E(z)},$$

(17)

where $z$ is the redshift defined by $a = 1/(1 + z)$. Now $E(z)$ is defined as

$$E(z)^2 = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_m(1 + z)^3 + \Omega_{\Lambda 1} + \Omega_{\Lambda 2}(1 + z)^m.$$  

(18)

Then, (16) is rewritten as follows:

$$d_L = \left(1 + \frac{z}{H(z)} E(z) \right).$$

(19)

As a consequence, the theoretical distance modules are calculated by (15) and (19).

Recently, the Supernovae Cosmology Project (SCP) collaboration released their Union2 sample of 557 SNIa data [8]. The Union2 compilation is the largest published and spectroscopically confirmed SNIa sample. The observations are in the redshift range of $0.01 < z < 2$. For the SNIa data set, we evaluate $\chi^2$ value of the distance modules,

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{\mu_{th,i} - \mu_{obs,i}}{\sigma_{\mu,obs,i}^2 + \sigma_{v,i}^2}\right)^2,$$

(20)

where $\mu_{obs,i}$ and $\mu_{th,i}$ are the observed and the theoretical values of the distance modules. $\sigma_{v}$ is the dispersion in the redshift due to the peculiar velocity $v$ is written as:

$$\sigma_{v} = \left(\frac{v}{E(z)}\right).$$

(21)

We adopt $v = 300 \text{ km sec}^{-1}$ [40]. The total number of the united sample $N$ is 557 for the present analysis.

We confirm that $\Omega_{\Lambda 2}$ has done some contribution to change the $\mu - z$ relation. When $\Omega_{\Lambda 2}$ increases, the expansion rate in the universe decreases, which is similar to the behavior of the matter dominant universe in the Friedmann model.

Figure 1 indicates the relation between the distance modules and the redshift against the SNIa observations in terms of $\Lambda$CDM and D$\Lambda$CDM with several values of $m$ for a fixed value of $\Omega_{\Lambda 2} = 10^{-2}$. As the value of $m$ increases, $\mu$ tends to decrease because of the increasing cosmic expansion rate. We recognize that $m$ is not seriously effective to change the $\mu - z$ relation. Even if we chose the larger value of $m > 4$, the effect is still small.

Figure 2 shows the allowed parameter region due to the $\chi^2$ fitting of (20) in D$\Lambda$CDM from SNIa constraints. We obtain the minimum value of $\chi^2 = 677$ (the reduced $\chi^2$ is defined to be $\chi^2_r = \chi^2/N \simeq 1.215$, where $N$ is the degree of freedom). The best fit parameter region of $\Omega_{\Lambda 2}$ for $m < 4$ is investigated as $\Omega_{\Lambda 2} < 4.5 \times 10^{-3}$ at 1$\sigma$ confidence level (C.L.).

Since the parameter region is very loose compared with the previous analysis [22, 23], this result implies that the decaying-$\Lambda$ has minor effects on the expansion rate at $z < 2$. 

**Figure 1:** Illustration of the magnitude-redshift relation in $\Lambda$CDM model ($m = 0$) and D$\Lambda$CDM with $\Omega_{\Lambda 2} = 10^{-2}$ compared with SNIa observations (cross-shaped ones with error bars). Note that the theoretical curve for $m = 3$ has no difference with that of $m = 0$.

**Figure 2:** Constraints on the $m - \Omega_{\Lambda 2}$ plane from the recent observations of CMB. The lines indicate upper bounds of 1, 2, and 3$\sigma$ confidence level. The dashed and solid lines are the limits deduced from SNIa and CMB temperature observations. The shaded region shows the excluded region obtained from [23].
following the Stefan-Boltzmann Law, recent temperature observations. It has been shown that the decaying-Λ term affects the photon temperature evolution at the early epoch [20]. In this section, we study about the consistency of DACDM with recent temperature observations.

The temperature evolution is obtained from (10). Following the Stefan-Boltzmann Law, $\rho_T \propto T^{4}_\gamma$, the relation between the photon temperature and the cosmological redshift is obtained as follows [22, 23]:

$$T_\gamma(z) = T_\gamma(0) \left(1 + \frac{\Omega_{\gamma A}}{\Omega_{\gamma 0}}\right)^{1/4},$$

$$\Omega_{\gamma A} \equiv \begin{cases} 
\alpha (1 + z)^{m-4} - 1 & (m \neq 4), \\
-4\Omega_{A2} \ln(1 + z) & (m = 4).
\end{cases}$$

From (22), the temperature-redshift relation at higher-z approaches that of SCDM model, $T_\gamma \propto (1 + z)$. At lower-z, the temperature evolution deviates from the proportional relation.

In the meanwhile, recently, the temperatures at higher-z are observed using the molecules such as the fine structure of excitation levels of the neutral or ionized carbon [30–34] and the rotational excitation of CO [35–37] and Sunyaev-Zel’dovich (S-Z) effect [38, 39]. Furthermore, there are a lot of temperature observations for the lower-z owing to the S-Z effect as shown in Table 1. Therefore, we expect to constrain more severely the allowable region on the $m - \Omega_{A2}$ plane from the new temperature observations.

Figure 3 illustrates the temperature evolution in SCDM and DACDM model with $\Omega_{A2} = 10^{-2}$. It is shown that the larger value of $m$ results in the lower-$T_\gamma$. When we adopt $m = 0.15$, the theoretical curve of $T_\gamma$ should be inconsistent with observations even for the temperature of the largest uncertainty [32]. We can conclude that the deviation from SACDM model might be small for $m < 0.1$.

Using the recent observations of the CMB temperature, whose accuracy has been improved quantitatively, we can put severe constraints on the temperature evolution in DACDM model. We calculate $\chi^2$-analysis as follows:

$$\chi^2 = \sum_i \frac{(T_\gamma(z) - T_{\text{obs},i})^2}{\sigma_i^2},$$

where $T_\gamma(z)$ is theoretical temperature calculated by (22) and $T_{\text{obs},i}$ the observational data as shown in Table 1. $\sigma_i$ is the uncertainty of the observation. $N = 25$ is the number of the observational data. Note that the result of Srianand et al. [32] does not have best-fit value. Therefore, we assume the best-fit value is the same as the mean value, $T_{\text{Srianand}} = 10.0 \pm 4.0$ K.

In Figure 2, we show the limits in the $m - \Omega_{A2}$ plane calculated by (23). The shaded region indicates the parameter regions which should be excluded and are from (14). We can obtain the allowed parameter range as follows:

$$\Omega_{A2} < 6.1 \times 10^{-4} \text{ at } 1\sigma \text{ C.L., } 1.7 \times 10^{-3} \text{ at } 2\sigma \text{ C.L.}$$

![Figure 3](image_url)
Although temperature constraints give the upper bound, we can obtain the parameter range which is more severe than that obtained in the previous temperature constraint [22].

Finally, we identify the influence of temperature observations at $z < 0.6$ by S-Z effect. We also calculate the $\chi^2$-fitting (23) using observations at $z > 0.6$ and $z < 0.6$, separately. As the results, we obtain the upper bound of $1\sigma$ ($2\sigma$) C.L. as follows:

$$\Omega_{A2} < 2.8 \times 10^{-3} \ (4.7 \times 10^{-3}) \quad \text{by S} - \text{Z at} \ z < 0.6,$$

$$\Omega_{A2} < 5.5 \times 10^{-4} \ (1.9 \times 10^{-3}) \quad \text{by CIB, CO at} \ z > 0.6. \ (25)$$

These results suggested that the temperature observations at a higher-$z$ are important to constrain $\Lambda$CDM model.

5. Concluding Remarks

We have investigated the consistency of the decaying-$\Lambda$ model with both the $\mu - z$ relation of SNIa and CMB temperature. We obtain the upper bound of our model parameters from both observations. First, we have constrained our model from SNIa: we can obtain the upper bound of parameters, which region is wider compared with previous constraints [22, 23]. From this result, a decaying-$\Lambda$ has minor effects on the cosmic expansion at low-$z$ and its parameters cannot be well constrained.

On the other hand, we can obtain more severe parameter constraints from CMB temperature. We acquire the limit of $\Omega_{A2} < 1.7 \times 10^{-3} \ (2\sigma$ C.L. From this result, the temperature can become lower by a few percentages. That should affect the hydrogen recombination. We note that we cannot get the bound of $m$. Because change in $\Omega_{A2}$ cancels the effects of the variation of $m$.

Our parameter is mainly obtained by the observations at $z > 2$. It has been suggested that the observations of the temperature at higher redshift is important to constrain the cosmological models with energy flow into photon [23]. The upper limit of $\Omega_{A2}$ obtained from the present constraints is still larger than the previous constraints of $\Omega_{A2} < 1.7 \times 10^{-4}$ at 95.4% C.L. [23]. The analysis using the CMB temperature fluctuation of the recent observation seems to be unable to give the tight constraints. We plan to perform more detailed analysis, for instance using CMB anisotropy of the newest WMAP data [10, 11].

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