Research Article
Solar Field Mapping and Dynamo Behavior

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We discuss the importance of the Sun’s large-scale magnetic field to the Sun-Planetary environment. This paper narrows its focus down to the motion and evolution of the photospheric large-scale magnetic field which affects many environments throughout this region. For this purpose we utilize a newly developed Netlogo cellular automata model. The domain of this algorithmic model is the Sun’s photosphere. Within this computational space are placed two types of entities or agents; one may refer to them as bluebirds and cardinals; the former carries outward magnetic flux and the latter carries out inward magnetic flux. One may simply call them blue and red agents. The agents provide a granularity with discrete changes not present in smooth MHD models; they undergo three processes: birth, motion, and death within the photospheric domain. We discuss these processes, as well as how we are able to develop a model that restricts its domain to the photosphere and allows the deeper layers to be considered only through boundary conditions. We show the model’s ability to mimic a number of photospheric magnetic phenomena: the solar cycle (11-year) oscillations, the Waldmeier effect, unipolar magnetic regions (e.g. sectors and coronal holes), Maunder minima, and the march/rush to the poles involving the geometry of magnetic field reversals. We also discuss why the Sun sometimes appears as a magnetic monopole, which of course requires no alteration of Maxwell’s equations.

1. Introduction
Magnetohydrodynamical (MHD) modeling within our Sun-Planet system is fraught with difficulties due to the diversity of environments within the vast dynamical systems influenced by the Sun’s radiative and particulate emissions. This may be well understood, when one considers the environmental changes that occur throughout the different gases and plasmas within our solar system. From the hot dense interior of our star to the low beta plasmas in the distant interplanetary regime, densities change from values in excess of $10^{26}$ particles/cm$^3$ in the center of the Sun to simply a few particles/cm$^3$ in the nearby solar wind, outside the Earth’s magnetosphere. A number of different systems embedded within the solar heliosphere are affected by changes occurring in the Sun’s photosphere owing to both radiative transfer and particulates and their associated solar activity magnetically controlled responses. Such aspects are called “space weather,” when the Sun’s activity affects our terrestrial environment. The remarkable variations owing to solar activity, at times in excess of 100%, originate in the Sun’s outer layers, relatively near the photosphere (e.g., the chromosphere and corona), and these remarkable and astonishing changes pervade much of the heliosphere. It is this paper’s purpose to provide and illustrate the usefulness of a new tool that may help track changes in the photospheric magnetic field, once provided information about new active regions (sources of new photospheric magnetic field) which bubble up from below, into the photosphere. Thus, this paper discusses a new tool that may be able to predict photospheric “weather” by calculating temporal changes in the Sun’s large-scale magnetic field structures. To gain traction against the myriad of complexities that a complete treatment of the entire solar interior seems to require, we employ a novel approach that reduces the complexity by focusing predominantly on the photospheric surface. This methodology appears to work. We will discuss why this reduction in the domain of our model is allowed, and how it works. We also explain how the photospheric mapping model calculates photospheric field changes. We shall also
show that this model appears to work as we observe its ability to calculate many known photospheric field behaviors.

To gain an appreciation of the workings of this model, let us begin by considering the workings of simpler models of this type. Our particular model will be working with the calculation of magnetic field motions on the Sun’s surface, the photosphere, given a complete description of the existing magnetic fields at a certain time, and any changes associated with new field introduced into the photosphere. This is the equivalent of a Markov process. Namely, knowing the state of a system at time, \( t \), one then calculates the system state at time \( t + \Delta t \). Thus the method we outline is a kind of evolutionary model, namely, how a quantity (the magnetic field in the photosphere) evolves in time. For a much simpler one dimensional case, one may think of this as being a kind of “ballistic trajectory model,” namely, how a cannon shell moves in a ballistic trajectory. This is done by the changes in the \( x \), \( y \), and \( z \) components of its position and how these are affected by its current state: knowing its position, velocity, and the gravity field it finds in its local environment. This illustrates how simple our model might behave.

Interestingly, the current method, and the observed behaviors of solar magnetism are more weather-like in terms of exhibiting chaos in their behavior. That is, the subsequent behaviors have “nonlinear interactions.” For example, motions of field elements can result in their removal from the photosphere. This occurs in our model when two opposite field elements “collide.” It does not matter to the model, whether the collision results in the fields being pulled below the photosphere, or rather being ejected into the solar wind. They simply disappear from the domain of our model, the photosphere. We call this collision process between field entities, death, as a simple descriptor occurring when opposite sign field agents get too close to each other. A consequence of this nonlinear process is simple to understand for anyone who plays pool or billiards. A near miss of two balls results in quite a different behavior compared with a glancing strike. The model is ultra sensitive to small deviations of its initial conditions. This results in a “state trajectory” being highly sensitive to initial conditions, as in weather systems. We discuss a number of recurrent patterns of field structures in our model that appear similar to field structures observed by many solar physicists. The most famous of these patterns is the 11-year solar cycle. Other familiar patterns such as unipolar magnetic regions (UMRs, [1]) are also discussed.

In addition to serving as the best observed source of solar magnetism, it is no coincidence that the Sun’s photosphere happens to radiate 99+% of the Sun’s energy into space. This makes the photospheric surface most significant to solar structures at a given time, but also where our knowledge of the solar structure is best “known,” or “tied down.” It is both a physical source of solar and interplanetary phenomena, plus a source of knowledge about happenings in the interplanetary environment. Hence it is no surprise that one may use the photospheric observed fields to serve as entities to fix our model’s boundary conditions on. One may simply call this “being pragmatic.” Hence, the model in this paper concentrates on understanding the motions of magnetic fields within this unique surface, the Sun’s photosphere.

The photosphere is quite different from our more familiar terrestrial surfaces, such as the oceanic surface. The photospheric boundary is less well-defined or sharp by having a significant scale height, within which its properties change. This makes it somewhat less than a true mathematical boundary, where one, for example, finds space cleanly divided in two by, for example, a plane located at \( z = 0 \). The most correct feature we are familiar with, on Earth, of its properties would be a cloud, owing to its surface properties gradually changing, or having depth. For the photosphere of the Sun, one finds properties typically change on scales near a scale-height of \( \sim 100 \text{ km} \). Seemingly large, this distance is sufficiently small (a part per million) compared with the Sun-Earth distance, for the Sun to display a sharp disk when it appears in the sky. It is also sharp; roughly a part per 10,000 compared with the solar radius. This sharpness allows one to consider the photosphere as a 2D surface, rather than a 3D volume.

Another surprising feature of this remarkable outer boundary of the Sun, is that its pressure is markedly less than our more familiar 1 bar, or 1000 milli-bars of atmospheric pressure. The photosphere has a density similar to the Earth’s thin ionosphere. It is shocking that so thin a layer allows the photosphere to transform its particulate convective flux into radiative electromagnetic flux, such that virtually all of its energy flux radiates freely into space. This transformation is all the more remarkable, because of the wondrous energy flux involved: There are two main aspects which allows this transformation: the enormous turbulent velocities, several km/sec, near the sonic speed, and the enormous gravity of the photosphere, \( \sim 27x \) the Earth’s. These properties result in an enormous atomic collision frequency within the photosphere and yield the tiny scale-height of the photosphere yielding a sharp boundary. It is this sharp boundary that allows our model to work within the Sun’s relatively sharp 2D layer, as opposed to being dependant upon the 3D volume of the solar interior. Given these aspects, ascending in height from the photosphere, one finds a steep change into other layers: the chromosphere and the corona, from which only tiny amounts of energy escape, both outwardly into the solar wind and space, and others inwardly back into the Sun.

The photospheric magnetic field, in this author’s opinion, provides the Sun with the following: (1) a degree of granularity which affords the Sun with its ability to furnish changes on scales much faster than the granulation and supergranulation patterns seen and (2) when the field is concentrated into small structures, such as spots, pores, or intergranular lanes, it opens “channels” to form which permit cold downdrafts to form and descend from the photosphere into the solar interior towards the base of the convection zone. These aspects provided the impetus to develop our 2D algorithmic approach via cellular automata as distinct entities that provide a granularity or coarseness to magnetic field motions within the photosphere. For the Sun, itself, rather than our model, it is the magnetic field that imbues the photosphere with its widely varying
properties. As the great solar physicist, Robert Leighton, often said: “If the Sun did not have a magnetic field, it would be as uninteresting a star as most astronomers think it is.” In the absence of the Sun’s magnetic field, we observe virtually no semi-permanent large, spatially-varying structures that yield evident geometrical variations. We may clarify our use of granularity by pointing to some of its effects within the troposphere. If one lived within an atmosphere dominated by heat diffusion, one would find, for example, gradual temperature variations from hot to cold and vice-versa, rather than the more familiar weather patterns, where one finds a high degree of chaos (in the geophysical sense, namely both large-scale patterns and smaller scale stochastic variations). The larger-scale patterns have unique names, for example Hadley cell circulation, hurricanes/typhoons, pressure fronts, baroclinic instabilities, tornadoes, and so forth. Thus one finds some persistence in the atmospheric temperatures from day to day at any location, which we call coarseness. Namely, the patterns do not simply blend into molecular diffusion, but rather have motions and patterns on grander scales. Such behavior is typical of turbulence. Turbulence theory was originally developed for solar and stellar interior structures via mixing-length theory (MLT) and then evolved into general tropospheric usage by meteorologists. This theory has since been surpassed by more exact methods, which handle structures using computerized methodologies.

Returning to the Sun, in the absence of magnetic fields, upflows and downflows within the Sun’s convection zone do provide a tiny degree of granularity seen in its surface. These give the surface of the Sun a faint similarity to an orange peel, as seen in white light from the Earth. Yet, overall, the Sun’s surface, in the absence of magnetic fields, would resemble a totally desolate, unblemished desert that Leighton depicted rather than one with a more interesting terrain. The interesting terrain of the Sun on small scales, however, was captured by Robert Howard, when he described the Sun’s surface, seen at Mount Wilson, as “a can of worms.” This small-scale depiction accurately portrays the turbulent environment found within the Sun’s surface, owing to the roiling convection needed to resupply its surface with the energy it radiates freely into space. One might consider this tremendously wasteful except that we owe our existence to this grandiose generosity.

Thus the photosphere predominantly has its structurally varying properties associated with its embedded magnetic field. The solar magnetic field carries on throughout the entire photosphere and does not simply disappear at the boundaries of sunspots and faculae. Magnetic fields are non-zero everywhere, except at infinitesimally small boundaries. Thus it is the large-scale non-zero structures that are the predominant subject of this paper. How are these magnetic fields transported from their origins in the photosphere, as fresh new features, often displayed dramatically as sunspots or faculae, then travel to their demise, as remnant structures, barely discernable as they melt into the background noise of the turbulent photosphere?

Within the convection zone, mixing is so efficient that without the channels provided by magnetism, no significant non-radial, spatially-varying properties would develop. Because the gases beneath the photosphere are effectively in local thermodynamic equilibrium (LTE), except for tiny turbulent differences which serve to transport energy, the SAHA equation for LTE is used, along with the chemical composition, so that one can uniquely determine the conversion of pressure, temperature, density, and ionization level to ascertain the various percentage gases and ionization state within and below the Sun’s surface. Hence, it is as Leighton described, that without a magnetic field, the Sun would be a lot duller star than it is. A solar weather forecast, in the absence of magnetic field, would always consist of the following: hot today, hot again tomorrow; much the same subsequently.

In the absence of helioseismological observations, we predominantly would only see the Sun’s surface. So, it is there, and above the surface, in the chromosphere and corona, that interesting solar phenomena typically appear and are related to the Sun’s surface magnetic field. Despite MHD’s usefulness in the laboratory, where one does not need to consider energy transport on large scales, the meteorological equations do have applicability, as downflows undergo adiabatic heating, but owing to their superadiabatic surroundings, remain relatively cool, and upflows transport great amounts of energy in the solar case; thus such effects are placed within stellar energy transport equations. Because of the interest and importance of the Sun’s magnetic field throughout the Sun–Planetary system, this paper will concentrate on understanding the Sun’s large-scale magnetic field.

Because of the thinning of gases above the photosphere, many rapid, yet vast, changes occur above the thin photospheric layer. Temporal and spatial variations of >100% are seen in flares and coronal mass ejections. In addition to affecting the majority of the plasma, they also issue forth particles on the tail end of the distribution function; low energy solar cosmic radiation sweeps through the solar system while the magnetic field carried by the plasma particles sweeps out the inner heliosphere from incoming galactic cosmic radiation. Hence various “space weather” effects are felt throughout many Sun–Planetary systems. Thus the rest of this paper focuses predominantly on what happens to the magnetic field in that surface layer of the Sun, the photosphere. Near the surface of our Sun, physical conditions vary greatly. Complex interactions of magnetic field, plasma, and radiation happen, giving rise to many spectacular and complex phenomena, such as erupting prominences, sunspots, and many activity-related effects from flares to the solar wind and energetic particle emissions, difficult to understand, yet a wonder to behold. Astronomically, the photosphere is that region of the Sun from which light emanates. This happens approximately near an optical depth of \( r \sim 2/3 \). Thus the photosphere is both the deepest region that can be seen into, and above which visible radiation becomes transparent. Hence, it is predominantly within this unique layer where optical radiation leaves our
Nevertheless, to understand complex phenomena, it is based models. For these reasons many scientists are wary of treatments; thus it is hard for scientists to appreciate the result in any “beautiful equations,” as happens in analytic to be useful until the present epoch. Further, the results do processing made any agent-based models too cumbersome have been ubiquitous and the mainstay of the physics world “set of rules,” given to the agents. Di model is clearly dependent upon the correctness of the stated is. So, in many of the CA models, they are only as good or as old computer saying goes: GIGO: garbage in, garbage out. A number of problems often occur when one attempts to model highly nonlinear phenomena with differential equations. Effects such as chaos occur, and/or when the boundary conditions become too difficult to handle, as in turbulence, wherein effects happen on thin surface layers, which may affect the entire solution. One only has to witness a gentle stream turn into a raging river when a few inches of rain fall suddenly, to recognize the importance and omnipresent nature of nonlinear phenomena.

Cellular automata models, however, have been able to make progress in some areas of nonlinear phenomena from forest fires to magnetic spin states and the list of successful CA modeling of physical phenomena, using Netlogo alone, appears to be impressive and growing [2]. The main aspect of CA modeling, as opposed to traditional techniques, is that the phenomenon is expressed by a “set of rules,” which the automata can follow. Thus complex behaviors, which one would be hard pressed to write a differential equation for, such as bee motions in a hive, or birds flocking together as they fly, or ants finding sugar and laying down a well-trodden trail, so that the entire nest follows paths to the food source, have all been modeled with Netlogo, but not with differential equations. CA techniques, however, are not a panacea for modeling complex phenomena. In fact, as the old computer saying goes: GIGO: garbage in, garbage out. So, in many of the CA models, they are only as good or as bad, as the researcher or programmer who writes the codes is.

Consideration about the correctness of an agent-based model is clearly dependent upon the correctness of the stated “set of rules,” given to the agents. Differential equations have been ubiquitous and the mainstay of the physics world from the 17th to the 20th century. The power of computer processing made any agent-based models too cumbersome to be useful until the present epoch. Further, the results do not result in any “beautiful equations,” as happens in analytic treatments; thus it is hard for scientists to appreciate the bunch of numbers that are the primary output of agent-based models. For these reasons many scientists are wary of such numerical models compared with analytical treatments. Nevertheless, to understand complex phenomena, it is readily apparent that analytical treatments may be totally inadequate in certain areas. Thus we hope to show that for large-scale solar magnetic fields, an agent-based model may be better able to mimic behaviors more effectively than any analytic method to date. It is with this in mind, that we discuss the present model’s behaviors.

In our magnetic field mapping model, we take the photosphere, to be the domain in which we model the Sun’s magnetic fields. Non-zero surface magnetic fields are carried, in our model, by cellular automata. In their absence, the magnetic field is assumed to be near zero. In addition we simplify the computational problems enormously by choosing a large “graininess” of the field entities; the field is 0 where there are no cellular automata, and where there is a cellular automaton present, the field is either out of the Sun (shown by a blue cellular automaton, CA) or into the Sun (shown by a red CA) with a quantized amount of magnetic flux.

Let us state briefly how our model works, and later discuss a number of solar phenomena that are illustrated by this model. We are guided in choosing the following rules based upon “locality,” namely that field entities’ behaviors (birth, death, and motions) are affected solely through the influences of nearest neighbors upon them. There are but a few effects that agents have on other agents: (1) while at the poles, field agents are connected via the Babcock-Leighton (B-L) subsurface field to lower latitudes which allows them to undertake two duties there: (A) the B-L subsurface field spawns new photospheric magnetic flux (the processes are outside the photospheric domain, and thus are not the subject of this paper) and (B) the B-L subsurface field attracts lower-latitude fields via the long-range magnetic tension; (2) nearby agents affect each other’s motion by short-range interactions we describe; (3) when opposite flux agents get too close, we invoke a “death process,” that is, the fields and agents disappear from the photosphere. One may consider this as magnetic loops being pulled into the Sun (or out of the Sun), and hence disappearing from the photosphere, or any number of scenarios; they are again outside the domain of this model.

Hence what goes on above or below the Sun’s surface is not generally considered except in a minimal fashion, when, for example, we discuss the subsurface “Babcock-Leighton” toroidal magnetic field. A recent version of the model may be found on the Netlogo Community website [3]. It can either be run there, or downloaded with the netlogo 4.1.3 program, and run on one’s computer. The way the program works is that at its start, two buttons are pressed on the netlogo program interface: SETUP and GO/STOP. The SETUP button initiates a number of field agents at the poles. The GO/STOP button initiates the running of the program with three basic processes: (A) Birth, (B) Motion, and (C) Death, as follows. (A) Birth: the field entities are born in active regions with probabilities proportional to the polar field strength at the last polar field maximum value and have a geometry associated with Hale’s laws. (B) Motion: the agents move about on the solar surface by the flows and forces we describe briefly as follows. The motion of our field patterns are guided by three types of forces:
geometric, long-, and short-range forces. The geometric forces transform coordinates between the Sun's spherical surface and Netlogo's geometry. The Coriolis force is added to this general category, and the long- and short-range forces involve magnetic effects. The long-range force is associated with the magnetic tension from the “Babcock-Leighton” subsurface magnetic field, which winds its way from one pole to the opposite pole. It is assumed to pull on any field entity that emanates through the photosphere, since above this surface the magnetic field herniates, or splays outwards, into the corona thereby lessening its field strength, and thus the stress tensor pulls magnetic entities along the field-line path. Our model just considers the latitudinal motions, as differential rotation provides for the average E-W motions. The short-range forces are those between close neighbors; these guide the field entities in the following manner: they encourage field entities to move toward regions of weaker field strength, and, in addition, like entities (e.g., members of the bluebird breed) “flock together,” and unlike entities (opposite breed members) turn aside. (C) Death: when opposite field agents get too close (within the parameter kill-dist apart), they die off in opposite-sign pairs.

Field agents have a few properties; they possess a quantized amount of magnetic flux, $10^{23}$ Mx; the blue agents, sometimes called “bluebirds” have field directed radially outward from the photosphere, and the red agents have the similar field directed radially inward, and are sometimes called “cardinals”. The Netlogo interface has some quantities with b or c letters, denoting these species or breeds of birds, referring to the two types of field agents. Field agents keep track of their nearest neighbors, and interact with them locally, as we just described. The manner is very similar to bird flocking; however, one can see similar behavior in many nonliving environments. For example, many coherent behaviors give rise to the formation of semi-discreet objects in weather phenomena such as flakes of snow in a snow storm or ice in a hail-storm, cloud formations in the sky, the formation of river valleys on the Earth’s surface, volcanic outflows of lava or volcanic ash, the spin states related to magnetic domains in solids, and so forth. There are just innumerable living and inanimate phenomena, where “herd behaviors” prevail. Such linked behaviors do develop in both animate entities and inanimate ones. In physics, when this occurs, for example in magnetic domains in materials, it is usually referred to as “symmetry breaking.”

Let us now ask two questions and then discuss them. How can we model a highly restricted volume in space, namely the photosphere, and virtually ignore more than 99% of the Sun, volumetrically, or by mass, outside our chosen photospheric domain? Although this appears puzzling, and counter-intuitive, the answer is actually relatively simple. The laws of physics are obeyed locally, as well as globally, so one is always free to choose the volume under consideration. One does not need to consider what makes the “boundary values” what they are, but simply use them as the boundary values for the volume under consideration. Thus, for example, when an explosion occurs in a neighboring room, the wall is pressed away from the explosion, so the boundaries surrounding the chosen volume need to be handled appropriately. If we are in the neighboring room, we might be concerned about our future, when the walls of our room move suddenly after—towards us. It may be that our type of lower boundary condition is an overreach. We do not totally ignore the lower boundary effects: they are used to both supply the photosphere with field births and affect field transport, both dependent upon the Babcock-Leighton magnetic field. The success or failure of our model will reveal the quality of our assumptions.

In the current model we address the boundaries around the photosphere as follows: (1) we add active regions proportional to the Babcock-Leighton field, the North-South toroidal field they describe in their papers [4, 5], and (2) we consider the magnetic field tension from these magnetic fields to create long-range magnetic forces, which pull field entities towards the appropriate pole. Thus by choosing a small volume (a thin spherical shell) around the photosphere to examine, the model contains elements of strengths and weaknesses. The strengths relate to the model’s staying away from the areas of solar physics which have the least understanding and the fewest observations of, namely, the deeper regions of the Sun below the photosphere, and the thinner outermost regions (the chromosphere and corona) where temporal and spatial variations exceed those in the photosphere. Thus the photospheric properties are the best observed within the Sun. Our model also capitalizes on a number of the mainstays of solar physics; for example, new field entities are based upon Hales’ laws of sunspot polarities, and other features of active region fields such as Joy’s law, the Babcock-Leighton field, and so forth. There are a number of weaknesses of the model as well. Perhaps foremost in our mind is that the model really has little to say about what goes on outside of its domain—the thin spherical shell surrounding the Sun’s photosphere. Our model should remain silent about physical processes outside of its photospheric domain. We wish it could do more, both to test it, and to learn from it. Still, we are content with its current development, and hope it will become a useful tool to study the Sun’s magnetic behaviors.

We now provide some sample runs with our model, thereby illustrating how our model mimics the following behaviors, as seen in the photosphere: Solar Cycle (∼11-year) Oscillations, the Waldmeier effect (large cycles are shorter than small cycles), Unipolar Magnetic Regions/Sectors/Coronal Holes (large-scale field patterns), Maunder Minimum (extended periods of time where low field levels exist in the photosphere), the March/Rush to the Poles (details of polar field reversal at high latitudes), and how the Sun sometimes appears as a magnetic monopole (wherein the Sun sometimes appears to violate Maxwell’s divergence B requirement), which of course requires no alteration to Maxwell’s equations.

2. Solar Field Mapping Model

To understand how the present model works, and what it does, we begin by inspecting Figure 1. This is the Field Mapping interface of this model written in Netlogo 4.1.3.
Figure 1: Shown is the Field Mapping version 1p07 interface. This directs cellular automata to follow our algorithms written in the Netlogo 4.1.3 language. The blue slider bars on the left and below allow various parameters to be set, and the display in the center progresses through time, allowing large-scale solar fields to be “mapped.” The time history of the fields is displayed in the two plots to the right of the interface. The white monitors also provide instantaneous values of various parameters to be displayed. There are two graphs on the right; the lower one labeled Plot A, displays the polar field variations. In it are three colored curves: blue, red, and black. The blue curve displays the amount and sign of the magnetic flux in the northern magnetic pole, namely, the mean field above 60 degrees latitude. The red curve, similarly counts the mean field in the Southern polar region. The black curve marks the Absolute Value Sum of the red and blue curves. So, this curve in black displays the amount of magnetic flux in the polar regions. It has been artificially placed 60 units downwards, for increased clarity, so 0 lies at the bottom of the graph. It essentially reaches a peak, during a solar cycle minimum. The very astute observer will observe a fourth black curve at the bottom of the graph. This last curve at the bottom of Plot A shows when the total polar field values reach a “peak.” This is taken as the start of a new cycle and is shown as a uniform tiny “blip” in this bottom curve. Thus one can count solar cycles with these blips. The second plot, towards the right of the display, is Plot B. It shows the mean latitude of all active regions born during a tick unit of time. In this manner one may observe the solar cycle “butterfly graph,” although only roughly, as the graph program does not average, and so forth.

The model may be found at [3], wherein the Netlogo software package may be found together with latest code. In Figure 1, the model is run for the first two solar cycles. Movie 1 of Supplementary Material available online at doi:10.1155/2012/923578 shows the model’s display running with Apple’s Quicktime player. The run is started with the SETUP button pressed, and then the GO/STOP button. Of particular importance is the Random-Seed slider bar, which can be varied to choose a different run with each setting. Various blue slider bars on the left and below allow various parameters to be changed and modified. The top central display is a synoptic map of the Sun (in a Mercator projection format, rather than an equal area format, owing to the geometry of Netlogo). Fields in the model are “born” in the photosphere, based upon the strength of the polar field (based on the Babcock-Leighton model [4, 5]), and used by Schatten and others who use “polar field precursor” methods to predict solar activity [6–11]. The geomagnetic precursor indices appear not to have fared as well as the solar precursor method based directly upon the Sun’s polar field [9, 11]. In our field mapping program, one may change the rate at which the model calculates field motions by the slider in the program above the interface. By increasing the speed, however, one cannot observe the individual flux element motions as the model runs. The model still calculates field motions precisely; however, the display does not do justice to the calculations.

Returning to the interface seen in Figure 1, one observes two plots in the lower right; the lower one is labeled Plot A, displaying the polar field variations versus time. In each Plot A, from any figure, are shown three colored curves: blue, red, and black. The blue curve displays the amount and sign of net magnetic flux in the northern regions, namely, the mean outward field above 60 degrees latitude. The red curve, similarly counts the net magnetic flux near the South Pole. The black curve marks the Absolute Value Sum of the red and blue curves, so the black curve displays the absolute value of the magnetic flux located in the polar regions. This black curve has been artificially lowered 60 units downwards, for increased clarity, so the three curves do not run together. The black curve, representing total polar flux, essentially reaches a peak, during a solar cycle minimum. The astute observer will note a fourth black curve at the very bottom of each Plot A graph. This bottom curve in Plot A shows when the previously discussed, total polar field (the black curve) values reach a “peak.” Thus one can count solar cycles from the number of these blips. Each subsequent Plot A graph will have these three main colored graphs, plus the bottom blip graph, showing polar field maxima.

The second plot, above Plot A, is Plot B. Plot B shows the mean latitude of all active regions born during a specific period of time, called a “tick unit” in Netlogo language. In this manner one may observe the solar cycle “butterfly graph.” Unfortunately the Netlogo plotting software requires
one value for each tick unit of time. Hence it does not allow multiple or 0 values for the ordinate. Hence one cannot graph butterfly diagrams in the conventional way (many or no values for a particular time). So the default value is 0, when no new active regions are born. Hence one observes “false marks” along the abscissa, which should be ignored. Thus the curve in Plot B is an imitation of the butterfly diagram, with the limited graphing method allowed in Netlogo.

Figure 2 illustrates polar field variations over a few solar cycles (top), and a much longer period of time (4000), time units at the bottom. In this latter graph, one can observe firstly that there is an anticorrelation of cycle length (shown by the tick marks at the bottom of Plot A) with the size of the solar cycle, and additionally, the modeled Sun sometimes goes through lengthy periods of time, reminiscent of our Sun’s Maunder Minima, near about 3000 tick units. The motions of field lines at various times are interesting to watch; however, any simple graphs we draw will not do full justice to the modeled fields motions, just as a snapshot does not render the time history of a movie well. One important aspect of the present model is that the actual solar dynamo implements roughly a thousand active region field lines throughout an entire solar cycle, for a single field line emanating near the Sun’s poles during solar activity minimum, when the polar field maximizes between solar cycles (see Wang et al. [12, 13]). Our view is that the Sun can simply afford to do this as most field lines are cancelled in the neighborhood of active regions. Thus few field lines succeed in their progression to the poles, but, having reached there, are able to regenerate much equatorial magnetic flux. Thus just as mosquitoes lay many, many eggs, in the hope that a few might survive, the Sun also has a relatively weak polar magnetic flux relative to its equatorial active region flux. We support the Babcock-Leighton viewpoints that the high latitude fields have a key role in the solar dynamo by serving as seeds for the toroidal field of the next solar cycle. Their work has served us and others well [9] by allowing predictions of solar activity to be based upon the Sun’s polar field observations near the time of solar minimum. The method was extended from the original work [10] by considering how much field is embedded within the Sun, based upon a Solar Dynamo Amplitude index (SODA), where one adds information related to the Sun’s toroidal field along with the polar field. Although this method seems like a natural outgrowth of the original idea by allowing the Sun’s fields to be continuously monitored, it is not 100% clear how much this method adds to the original polar-field-based method, owing to the statistics being not good enough to compare the two methods.

From the quality of predictions based upon the polar field method [9] and from the observational and theoretical aspects of the Babcock-Leighton dynamo, the polar field does indeed seem to play an important role in the rise of the next cycle’s toroidal field. In the present model, the polar field itself simply arises from just the remnants of the current solar cycle’s magnetism. These magnetic fields wander almost aimlessly across the Sun to finally arrive at their safe haven at the poles. This long traversal of magnetic elements across the photosphere, from one cycle to the next, leads to a certain amount of Brownian type random walking, which may explain why the solar cycle prediction methods based solely upon Fourier type analyses of sunspot number and mathematical methods of their ilk lead only to the conclusion that sunspot number itself has not been shown to serve as a good index to predict solar activity upon its own. We move forward now to try and better understand how the elements of solar magnetism move across the solar disk.

We will see, from the field maps in this paper that the cyclical behavior of the solar activity cycle, as seen in the Sun’s polar field lines, may be regarded as the flotsam and jetsam of the solar cycle. These terms are nautical terms referring to the leftovvers when a ship broke apart due to the ravages of the sea; the jetsam are those goods jettisoned overboard by the crew in their attempts to save them from a watery grave, whereas flotsam are those that floated away on their own, having been insecurely stowed. Their long-lost triplet, lagan, is the goods going to the bottom of the ocean. For the case of solar magnetic fields, we may regard the disappeared flux as lagan, and not distinguish between flotsam and jetsam, but simply apply these terms to fields that survive to collect in UMRs and the polar regions which then reform into the next cycle’s toroidal field and future solar activity. It does not matter not to the current model, since this is a photospheric model, whether this reformation happens by field being transported by shallow flows just below the photosphere, or at great depth, near the bottom of the Sun’s convection zone. Such events, outside the domain of this model are simply outside this model’s purview. As discussed earlier, this model tries to consider only those elements that happen in/or near the photosphere, and thus treats new flux that enters the photosphere as “updated boundary conditions.” We look forward to this model being updated to allow observed sunspot fields to serve as actual boundary conditions, as was done with Leighton’s model [14], and then seeing how accurately this model portrays the large-scale field, as was done with Leighton’s model.

Another question which may arise is how are we able to model the photospheric field with a model which is incomplete, by not considering all the workings of the entire Sun? We do this as follows. The model is designed to sweep under the photospheric rug, metaphorically, the nature of how magnetic fields arrive in the photosphere. One may be surprised that one can construct a model this way. But yes, one can. One is always free to choose any volume of interest, apply the laws of physics to this volume, and consider the surface surrounding the volume as a boundary condition. This is how our choice of the photosphere as the domain of this model is designed; nothing more, nothing less. By restricting our domain, of course, we limit what we may learn or say about phenomena outside this volume. We are fundamentally trying to understand the photosphere, and what role the magnetic field and fluid motions therein play in affecting the large-scale photospheric magnetic field.

Let us continue our discussion of the magnetic flux calculations, as many have taken this as an important aspect of “flux transport models.” Our program may be regarded as
We next examine and calculate the earlier, only 1 line per 1000 field lines arrives at the poles). et al. [12, 13]:

Examinations of the lengths between field minima, shown by the tick marks in the figure, readily display an inverse correlation between field minima lengths and periods of high activity. The extended periods of low activity (e.g. the one near 3000 time units) may be similar to Maunder Minima type phenomena. Examinations of the lengths between field minima, shown by the tick marks in the figure, readily display an inverse correlation with polar field magnitudes. Namely, when the fields are larger, the periods are shorter. This effect was discovered by Waldmeier.

![Field mapping polar field variations](a)

![Field mapping polar field variations](b)

**Figure 2:** Displayed are two graphs of polar field variations versus time for the first 8 solar cycles (a), and for 4000 tick time units (b). Each of the graphs is of the Plot A type using the Field Mapping model 1p07 run with the Netlogo 4.1.2 code version. Shown most prominently are three colored curves: blue, red, and black. The blue curve displays the amount and sign of the magnetic flux in the northern magnetic pole, namely, the mean field above 60 degrees latitude. The red curve similarly counts the mean field in the Southern polar region. The two polar fields anti-correlate. The black curve marks the Absolute Value Sum of the red and blue curves, so it corresponds to the mean polar flux. It has been displaced downward, so the −60 value corresponds with 0 flux, for increased clarity. The black curve essentially reaches a peak, during a solar cycle minimum, when the polar fields reach their maximum values. Again, a fourth black curve is drawn, with little blips, and placed at the bottom of each plot showing when the actual peak in the polar field occurs. These provide a good indicator of the start of a solar cycle, so one can count solar cycles with these blips. The lower graph shows an extended period of low solar activity interspersed within typical periods of high activity. The extended periods of low activity (e.g. the one near 3000 time units) may be similar to Maunder Minima type phenomena. Examinations of the lengths between field minima, shown by the tick marks in the figure, readily display an inverse correlation with polar field magnitudes. Namely, when the fields are larger, the periods are shorter. This effect was discovered by Waldmeier.

We define an efficiency formula defined similar to Wang et al. [12, 13]:

\[
\text{Field transport efficiency} = \frac{\text{polar field transport rate per cycle}}{\text{single species birth rate}}.
\]

We may use the acronym for field transport efficiency (FTE). We define the “single species birth rate” to be the average number of field entities of one sign, per cycle. We define the “polar field transport rate” as the amount of single sign magnetic field entities brought to the pole. With this in mind, we use specifics for the 322 tick data shown on the interface, seen in Figure 1. There are four solar cycles; during which time the absolute polar field peaks (North + south) are \( \sim 50 \) (\( \sim 25 \) for each field sign). The monitors in the upper right show the 4 solar cycles, the 273 total numbers of active regions, and the 10,960 number of field lines (backfield count), most of which (10,784) died. The polar field transport rate per cycle is obtained using the peak-to-peak amplitude of about 40, (\( \sim 40/1370 \)) with 20 field entities in a peak pole but requiring 40 to be transported there (20 to reverse the field, and 20 to reestablish the reverse polarity). Thus, the single species birth rate is 10,960/2 = 5,480 per 4 cycles = 1,370 field entities per cycle. The field transport efficiency then becomes FTE = 40/1370 \( \sim 2.9\% \) or 1 : 34 in the earlier notation, using these numbers just discussed. This “efficiency,” 2.9% or 1 : 34, compares to the much lower earlier value for the observed solar fields (0.1% or 1 : 1000) for the Sun.

Lowering the Sun’s ratio to our model’s, of 1 : 34, rather than 1 : 1,000, is a ratio of \( \sim 30 \). Our model’s greater efficiency has one primary advantage. It allows us feeble human beings to see the displays without the full field wastage that the Sun partakes in. The change of the Sun’s field wastage ratio of 1 : 1000 to our model’s wastage ratio of 1 : 34 implies a field wastage of 97%. This wastage, we suggest, contains predominantly “chaos,” removed by the present model. To this author’s reasoning, the field wastage (for the Sun of 99.9%, and for this model \( \sim 97\% \)) represents fields in active regions being born there through chaotic upbringings [15], involving field percolation (a churning of the field) in the superadiabatic neighborhood of growing active regions, and this leads to their wastage fields’ premature chaotic deaths in the very neighborhood where these fields were born. Such is the short life of most sunspot fields. It adds nothing to
our understanding if our model was to capture the wastage, so long as we remember, that it exists and this model is not modeling it, nor capturing it, when statistics are compared with the Sun’s, but that the extra wastage fields do exist on the Sun; however they are a complexity that we haven’t dealt with, just as the molecular motions on the atomic level surpass our ability to distinguish. All our sensory organs can feel its pressure, the average force against our skin, but not the individual force of each molecule that strikes our skin.

Lest the reader be not as inclined by our reasoning, as we are; let us briefly add a bit more. One should not be at all troubled by our removing the wastage fields by considering the actual reality of “counting items,” as in the following. Imagine a straight field line just below the photosphere. Now we twist it, so one loop pops up above, so there are two new field sources (a+ and a−) in a small region. One can easily twist and pop two loops up. What happens to these little loops can be quite different. One little loop may simply be pulled back down; the other loop spread apart. There really is no gold standard in counting magnetic loops through a surface. The kinds of “counting processes” that we undertake of field lines through a mathematical surface (such as an idealized photosphere) simply provides us with a rough gauge of magnetic flux. This is simply what we do: we can count, but counting is nothing physical that the Sun can respond to. Counting is not a physical process. It is something that humans do, as we try to quantify physical attributes.

There are a number of parameters leading to interesting behavioral aspects of this model, which may be investigated. We shall be showing these in the next few sections. Let us first mention one of the ingredients in this model compared with earlier field models, like Leighton’s [4]. Although this model may appear to behave dispersively, this model is not diffusive, instead there is a parameter, which controls the velocity of the field motions and combined with the kill-dist parameter, one can obtain a rough diffusion coefficient (cm²/sec) for the approximate rate at which fields cancel. In the current model, the magnetic field is advected by the various flows, and the field disperses by field cancellation, primarily at the boundaries of large-scale field unipolar regions. The actual process just involves the removal of field agents when they get within a kill-dist (fixed distance) apart. Most importantly, this is a deterministic model. Random elements are associated with the manner in which our sunspot fields are chosen.

For any who are concerned about the ratio of our model’s reduced ratio of random fields to ordered fields compared with the Sun’s, let us mention one final analogy to understand this important point. Let us consider solar field lines to come in 100 flavors, say no. 1–no. 100, in each of two colors, red and blue. Only the red and blue lines labeled with a particular number, say no. 1, are counted; the rest are discarded. If we were to show the extra 99 field lines of higher flavors, that fail to go to the poles, it would be even more confusing to the patterns shown than it already is. This mathematical transformation would remove the numerical deficiency of the model we display, through a simple mathematical trick. This is why, in our mind, not fitting all the solar dynamo numbers (the Sun having a wastage of 99.9% and this model, 97%) is not of any significance. Our simplified “toy model” that we show here allows these field displays to be perceived by humans without the added complexity of having a significantly higher average field cancellation that the Sun provides.

Hence this simulation enhances the ratio of polar field lines to equatorial field, so that the most significant field lines may, hopefully, be adequately seen. Another way, of saying this is that the Sun’s active region fields are relatively strong compared with polar fields, and the inefficiency of the Sun’s dynamo is not adequately represented by the current field mapping model. If we were to make a direct one-to-one portrayal of field lines, we would need many more field lines in the active regions, and then have the model simulation spending a lot of time canceling many more, and running for longer periods to show the overall magnetic structure. We have compromised this aspect for an improved display.

2.1. Model Usage and Behaviors. The most obvious behavior of our model is that it portrays the oscillatory 11-year solar cycle, (seen in Plot A). In addition to seeing oscillatory behavior, one sees the irregularity that has become legendary to those of us who marvel at the erratic behaviors that the Sun displays with little rhyme and reason. This has led many theories to fall into the waste-bin of theoretical ideas that many solar cycle predictors have had to trash, as they attempt to understand the Sun’s enigmatic behaviors, as they try to support the agencies whose enterprises are affected by solar activity. NASA, as well as other national and international agencies, has numerous technological achievements in space that may be affected by excessive solar activity. With increasing demands and dependency on these satellites, and so forth, knowledge of long-term solar behavior is becoming an increasingly important subject of practical as well as theoretical study. The long-term method by which such predictions are made has not yet reached a sufficiently agreed upon methodology [11] and, as with much of solar physics, remains an individualistic, pioneering endeavor, rather a fully established field with sufficient clarity that teachers outside of the specialists within the field feel comfortable in teaching to the next generation. This is unfortunate, as the Sun is the only star that we can see with sufficient clarity, so that explicit detailed models may be tested. Just as in a kiln, impurities are separated out to create iron, in the observational crucible; faulty theories are removed, hopefully leaving behind some semblance of truth. It is primarily for this reason, that we have constructed the current model, so that it may focus on the observable Sun, most of which comes from the photosphere and above, leaving out the cleverly designed helioseismological observations that provide glimpses into the solar interior.

2.2. The Waldmeier Effect. Following our viewing of the most apparent solar cycle behavior, let us now consider a less well known effect. In addition to the variances in solar cycle amplitude we discussed, there is another effect, which Waldmeier [16, 17] found. This is that high cycles reach
their peak in a shorter time interval than low cycles do. This behavior has not been well explained; however it has become so well known to solar physicists, that is, becomes as ubiquitous as the air we breathe.

For simplicity, consider just one type of species, although the results apply to the two equal species that we are dealing with. Consider also an approximately cyclical pattern, where we take the rate of polar field births, \( n_p \), proportional to the number Sun’s polar fields, \( n_p \), and inversely proportional to the period, \( \tau \), to obtain the approximate number of births per unit time:

\[
\dot{n}_p = \frac{2\pi n_p}{\tau}
\]

(2)

and we use an identical relationship between the periods of surface field entities. Now the rate of field deaths is related to the integrated effects of field deaths, being locally proportional to the product of blue and red magnetic agent numbers:

\[
\dot{n}_D = k_2 \sum_{\text{Sun}} (n_B n_R),
\]

(3)

where the number and death rates of surface fields are equivalent to the number and rates of blue and red species: \( n_D = n_B = n_R \) and \( n_D = n_B = n_R \). Given that our model utilizes a birth rate of surface field entities in active regions proportional to the polar field numbers, \( n_p \), with a fixed number of field entities per active region, \( \text{population}_{bc} \), and a rough equality between the integrated field births and deaths balance, on average, our model:

\[
\dot{n}_D \approx \dot{n}_p.
\]

(4)

In this model there is a balance between field birth and field death, so

\[
\dot{n}_D = k_2 \sum_{\text{Sun}} (n_B n_R) \approx \dot{n}_p = \frac{2\pi n_p}{\tau}.
\]

(5)

If we examine this numerically, with our nominal field values, we obtain the surface death rate of \( \sim 0.054 \) active regions per tick timestep. For the first solar cycle, this equates to \( \sim 1 \) field entity per tick timestep, since this cycle has a period of \( \sim 70 \) ticks, providing a death rate of \( \sim 4 \) entities. The births for the first solar cycle, including initial polar field entities, was 5280 field entities and the number of field deaths was \( \sim 4800 \) (with half the number of each color). Using (5), with the above numbers, for our screen size display of \( 43 \times 85 \) grid points, yields 3655 cells, having an average density of 1.3 entities per tick. This yields a value for \( n_D \) of 1.3 entities per 70 ticks or 0.018 deaths per unit per tick, yielding \( \sim 5000 \) deaths per cycle for the entire grid, which is necessitated by the above equation, on average. These numbers display a rough self-consistency, namely, the cycle lasts approximately as long as the time it takes to kill roughly as many entities as are born during that cycle.

We may consider the above aspects to gain some further understanding of the interplay between these the life and death of field entities. For simplicity (to allow understanding without the detailed tracking of both entities, since our model does that), we consider a 1D model, where we have polar and surface, red and blue agents; however since the number of each color is identical, we just consider the total number of red and blue surface and polar fields, \( n_S \) and \( n_P \), respectively.

Now consider some of the elements of the Babcock-Leighton dynamo: the surface fields arise from a magnification process that amplifies the poloidal fields and converts them to toroidal field where they erupt into surface fields, hence we choose

\[
n'_S = m_{B-L} n_P,
\]

(6)

where \( n_p \) is the number of polar field lines, of both sign poles, \( n_S \) is the number of surface fields of both signs on the surface of the Sun, not including polar fields, and \( m_{B-L} \) is a rate of magnification and eruption by the Babcock-Leighton dynamo process that amplifies magnetic fields into surface fields. We are ignoring all the real detailed spherical geometry, time history, and so forth. Now, the polar field arises from the following portions of the erupted (sunspot fields) surface fields, which are led back to the pole (in our model by the magnetic attraction of the polar field, but in the Babcock-Leighton model by geometry and diffusion of the following field in active regions). These fields are of opposite sign to the original polar field and so serve to erase that polar field and build one of opposite sign. Hence we express that by a simplified differential equation:

\[
\dot{n}'_p = -v_d n_S,
\]

(7)

where \( v_d \) represents a drift velocity (1/s), in terms of the time it takes for the solar surface field to drift poleward, destroy the polar field of that sign, and reverse the polar field. As they stand, these simplified equations are famous as leading to one frequency in a Fourier series. If we start our time series off with a specific poloidal field, \( n_P(0) = B_P(0) \), and a surface field (e.g., related to the toroidal field in this simplified picture of 0), this yields an oscillation of

\[
n_S = m_{B-L} B_P(0) \sin \frac{2\pi t}{\tau},
\]

(8)

\[
n_P = B_P(0) \cos \frac{2\pi t}{\tau},
\]

where \( \tau \) is given by \( 2\pi/\sqrt{m_{B-L} v_d} \).

If we now move on from these equations, we can consider the case for functional time dependence, rather than the uniformly oscillating situation we began with. Consider first a more general form of the above equations that allows the components to interact nonlinearly. One well-studied type of first order differential equations takes the Lotka-Volterra form:

\[
x' = ax - \beta xy,
\]

(9)

\[
y' = \gamma y - \delta xy,
\]

where \( x \) and \( y \) functions interact through a predator-prey type relationship. Analytic solutions to these equations are
We have included a functional form, $F(t)$, and have chosen it to be $F(t) = 1/(t + \delta)$, with $\delta$ being a small constant, 0.1, so that the form does not go out of bounds at 0. The purpose of the second term on the right hand side of the above equations, $en^2$, is allowing for decay of these fields through “collisions” with opposite field signs and using the assumption of equal numbers of both signed fields (species). The terms between blue and red fields would be proportional to a collision frequency (related to the number density of opposite color entities, $en_5$), but a second factor of $n_5$ occurs when one integrates over the solar surface to obtain the total death rate of each species, and this provide for the integrated sum of all collisions. The $-n_5/\tau_D$ terms provides for some decay, with an arbitrary time constant, which can be modified to consider various decay time constants.

For numerical simplicity, we choose $mb-L = \nu_d = 5$, $\epsilon = 0.1$, and $\tau = 5$, with $\epsilon = 0.1$. The solutions to these equations are shown in Figure 3. One can observe an inverse relationship between cycle amplitude and cycle duration, similar to that observed by Waldmeier. The polar field precedes the surface field, and when the amplitude decreases, the cycle period lengthens. We may consider this inverse relationship happening with our model, for the following reason. When solar activity increases, the dissipation of field regions grows disproportionately, owing to the second order term. As a consequence, the cycle duration shortens. The inverse is also true. The present field mapping model displays these properties and one can see them by examining the lengths and sizes of cycles, in the figures.

2.2.1. Unipolar Magnetic Regions (UMRs), Sectors, Coronal Holes, and Poleward Field Surges. In addition to the most evident features of the solar cycle on the face of the Sun, in white light, sunspots and faculae, on the global scale, difficult to observe features, however, are signs of the Sun’s polar fields. Observed by polar faculae [18], and polar plumes seen at the times of eclipses, the Sun’s polar fields have been used as “solar precursors” to predict the size of the next cycle. The large unipolar field regions of the Sun seem to reside at other latitudes as well, although their structure is sometimes not well recognized when they are broken up by smaller-scale active regions. These regions were called Unipolar Magnetic Regions (UMRs) [19] and had the shape of backwards “C-” shaped field patterns, discovered on the Sun. It is likely that they are identical with the sector patterns found shortly thereafter. These are “north-south” aligned magnetic features on the Sun, described first as sectors of a divided orange, seen both on the Sun and in interplanetary space [20]. The slight difference in the description of these two patterns is not as significant as the discovery by these two groups of the large-scale solar patterns, intermediate between polar fields and sunspots. This is just like discovering Mt. Everest; it is not important to quibble about whether its shape is more like the beak of a bird or a triangular pyramid.

We now often refer to large-scale unipolar regions on the Sun, whether at the poles or at lower latitudes universally, as “coronal holes”, owing to the manner in which unipolar field regions on the Sun allow the coronal plasma to vacate these field lines rather quickly. We now know that the patterns seldom, if ever, form mathematical idealized geometric structures. Nevertheless, early on, coronal holes had these earlier names. We now understand the phenomenon of the highest closed arches as forming when the Alfvén velocity allows the magnetic field to travel downwards at the same speed that the coronal plasma is advected outwards. So a balance exists between the outward speed of the plasma and the inward tension pulling the field lines inward. It is an intricate race that allows the highest closed arches to lie about a solar radius above the Sun. In this manner, the “source surface” is that general height for these highest closed arches on average. Above these lines, the solar wind would extend field structures into interplanetary space, and inside of the height the field lines can reconnect to the Sun.

When running the current Solar Field Mapping program, the pattern of field on the solar display appears to show a variety of patterns and general motions of field quite similar to the backwards C-shaped UMRs and sectors discussed by the previous authors, when not too encumbered with new active regions. One often can see various numbers of the backwards C-shaped field structures emanating from active regions as the fields break up and drift towards opposite poles. One may consider that they are driven by the magnetic forces, and swept back by differential rotation.
These may be some of the main driving influences, rather than diffusive properties. One cannot dismiss totally some of the other likely smaller influences, such as meridional flows and some amount of "diffusion" from turbulent mixing. To this author, it appears that magnetism from the Babcock-Leighton subsurface field provides a meridional (North-South) motion that may be regarded as the long-range magnetic force driving the 11-year oscillation, and the differential rotation draws the field out into a "streakline", the technical fluid dynamical term that describes the patterns generated when a time-dependent wind blows over a "tracer" source leaving a "trail" of emissions, as in the case of a smokestack or in the case of the Sun, its magnetic fields, and fluid flows.

This viewpoint is similar to that previously espoused [21], although the effects of the subsurface Babcock-Leighton magnetic field may not have been previously considered. These authors examined the amount of "open magnetic flux" by looking at the connections of solar field lines to the corona. Babcock and Leighton did not have these models in mind, but Babcock and Leighton certainly understood solar physics and Maxwell's equation, so likely this was at the heart of their models. In the potential field source surface (PFSS) model, Wang and Sheeley examined antisymmetric pairs, and they tend to add generally, to the "open flux," and consequently these tend to augment the dynamo, by adding preferential North-South field.

If the fields were symmetric (in their or our model), such fields might result in open flux, but not in the N-S plane, as they would not have a North-South dipole moment, and so, in the viewpoint of this paper, would not affect the N-S solar dipole, and hence would not tend to add to the Babcock-Leighton polar field. Of course, for the actual Sun, open flux can easily result in a large-scale magnetic torque on field lines from its interaction with the global field, and it is difficult to say what the effect would be of such a torque on the field regions below. Nevertheless, from the viewpoint in the present paper, what happens, in these perfectly symmetric cases, is that on the way to the opposite pole, the fields "interfere" with reverse sign fields, so cancellation would occur. If, of course, at a particular longitude, the North and South patterns interfere "constructively," rather than canceling, the two patterns become symmetrical in the North-South direction, and a complete backwards-C results. Total constructive interference leads to backwards C-shaped structures similar to Babcock and Howard's patterns. In addition to these patterns, Wang et al. at NRL have shown the patterns of field from magnetism diffusing and flowing from active regions in a variety of cases.

Let us now examine some of the random patterns that the present model generates. One can see a couple of patterns "sectors" in our Figure 4. The polar field in this display is near a polar reversal with roughly equal sign fluxes at both poles. Although the number of such sectors is often depicted to be 2 or 4, per rotation (the equatorial circumference of the Sun), when the number of new regions erupts in many places nearly at the same time on the Sun, the Sun's surface may have more sectors per rotation as our simulations show. This goes against any simple rules (of only low order harmonics on the Sun) that are generally thought true. Thus the viewpoint of Wilcox and Ness [20] of sectors is viewed more as a simplifying generalization than any exact representation.

The "source surface" model [22] first utilized Green's functions in the corona to attenuate the high spatial frequency harmonics found on the Sun, converting the complex photospheric field structures to the simpler sector structures seen in space [20]. Utilizing improved photospheric boundary conditions, rather than surface monopoles to fit the photospheric boundary conditions, Altschuler and Newkirk [23] improved the PFSS model through their improved use of spherical harmonics. Pneumann and Kopp [24] developed an MHD model for the specific case of a perfect dipole field on the Sun and subsequently Schatten obtained a current sheet model [25] stressing the importance of thin current sheets in the formation of interplanetary sectors, with the model being able to calculate the field for a force-free corona, from any arbitrary photospheric field configuration, with zero volumetric currents. Currents in this model were forced to reside only in locations where \( B = 0 \), as in surfaces where \( B = 0 \). The quantity \( J \times B \) would always be 0 as when \( B \neq 0 \), \( J = 0 \), and when \( J \neq 0 \), \( B = 0 \), so the magnetic force, \( J \times B \), is always 0 in this force-free current sheet model. A mathematical "trick" allows this to be accomplished and, at the same time, balances the Maxwell stresses across the "source surface." One simply reverses any inward fields on the source surface, so they cannot reconnect with the outward fields, and then, after the field lines are calculated, reverses the inward directed field lines to the correct orientation. This, of course, creates an artificial, fictitious monopole for the purposes (in the calculation) of handling the coronal field, but this is nothing beyond a mathematical manipulation. It is this model that Wang and Sheeley [21] use when they describe the PFSS model they now utilize. It is a more difficult calculation than the original PFSS model, so often, regretfully, mostly the 1969 is used.

In any case, the question remains: to what extent do the spherical harmonics, \( Y_{lm} \), describe the Sun's magnetic fields versus the coronal and interplanetary "smoothing" of the solar field? Do the photospheric fields inherently consist of high order harmonics or low order harmonics?

A deeper understanding of the manner in which active regions add field to the larger scale structures was greatly furthered by Wang et al. [26]. These authors clarified many of the puzzles in the field patterns. Firstly, the "polar surges" of magnetic field that these authors found have a profound effect upon the Sun's large-scale field structures. Additionally they asked the puzzling question: How could rigid rotation of coronal fields exist in the presence of the photospheric differential rotation? It seemed counter-intuitive. These authors explained this, as the difference between patterns (the coronal rigid rotation), and the rotation of actual fluid (the differential rotation). Such phenomena as the "spokes" in Saturn's rings are another dramatic example of a similar effect. Although puzzling, the phenomena are quite common. Anyone at the beach sees patterns, rolls of ocean waves, arriving at the shore. Yet the motion of the
wave pattern is quite different from the motion of the water. The water does not travel at the tops of the waves directly from hundreds of miles away! So, the phase velocity and fluid velocity differ significantly. The group velocity of a wave is the velocity which the overall pattern or shape of the wave amplitudes propagates through space. The fluid elements move in a cylindrical motion (in the deep ocean), so they never travel far from their initial position, owing to the wave patterns that they partake in. The wave patterns are transported horizontally to the shore, where the waves break in the shallow water. In the present model, one can observe surges of field towards the poles, similar to the previous authors’ patterns. Fisk, Zurbuchen, and Schwadron [27] offered a pictorial display elucidating the field geometry and reconnection effects occurring in the solar corona.

More recently a hybrid heliospheric modeling system was developed to forecast interplanetary properties directly from solar fields, in a multidecadal advance useful for solar-terrestrial research [26, 28], beyond what anyone could have imagined in the early days of Babcock and Leighton.

2.2.2. Maunder Minima. Figure 5 shows the end stage of the Field Mapping interface, of a long run of 2000 timesteps, which culminates from the same starting parameters as in Figure 1. We wanted to illustrate a quiet interval near 3,000 timesteps of low solar activity. One could obtain this, more readily, by reducing the AR-rate; however, we just ran the standard run for a sufficiently long interval and were fortunate to obtain an interval with low activity events. Lowering the AR-rate enhances the number of weak region effects, and although one still gets many oscillations, they are of lengthier periods and have weaker activity levels; the field eventually recovers. In Plot A, one sees the long time history, which goes through the aforementioned weak field interval; naturally, we and likely others would refer to this as a Maunder Minimum type event.

Let us now examine the composite history of this model run for 2000 tick timesteps. First we note that as the activity lessens, the oscillation periods lengthen, as studied in the earlier section with regard to the Waldmeier effect. Figure 5 additionally, shows some of the weak field patterns during the quiet interval. We have shown in the lower half of Figure 5, some of the display field maps. One occasionally observes, particularly in the lowest two maps, rivers of field traveling in circumpolar jets (occasionally eastward or westward directed relative to the fluid). The displays show a relatively simple field. Thus in these patterns the fields are so very weak, that it is difficult to discern the detailed patterns, without viewing the actual evolution.

Let us now examine the composite history of this model run for 2000 tick timesteps. We focus first on the overall behavior and then some interesting aspects related to the quietest second period shown in Figure 5 (top), Plot A. First, for the overall behavior, note that, as before, the largest peaks have the shortest periods. Now, we examine the efficiency, as before, using (1), the ratio of the “single species birth rate”, per cycle, to the polar field transport rate, the amount of single sign magnetic field entities brought to the pole in a whole cycle. With this in mind, we use specifics for the 2000 tick data shown on the interface, seen in the lower half of Figure 1. During this whole time period 24 solar cycles have come and gone. Commensurate with this, 1,821 active regions developed for a total of 72,880 field lines have been born.
The polar field transport rate per cycle is obtained using the peak-to-peak amplitude of about \(46, (\sim \pm 23)\), with \(\sim 23\) field entities in a peak pole, requiring \(46\) to be transported there (23 to reverse the field, and 23 to reestablish the reverse polarity). The monitors in the upper right of the interface, display, respectively, the 24 solar cycles, the 72,880 total numbers of field entities (or 36,440 single species entities). The single species birth rate is thus 1,518 entities per cycle. The field transport efficiency then is the ratio of these numbers: \(\text{FTE} = \frac{46}{1518} \approx 3\% \approx 1:33\), virtually identical to our earlier estimate of \(2.9\% = 1:34\), showing a remarkable uniformity at converting equatorial field to polar field transport. We are somewhat surprised at this uniformity, but that is likely due to our calculating long run
averages, as opposed to cycle-to-cycle statistics. Again this number (3%) compares to the remarkable inefficiency of the Sun, for these calculations (1 : 1000, or 0.1%).

With the significantly higher efficiency of this model, compared with the Sun (3% versus 0.1%), we need to point out again, that the field lines calculated and displayed are not the tightly confined sunspot and active region fields, most often discussed by solar physicists interested in dynamic solar events such as flares. The field lines we study best in this model are not the most visible magnetic features, sunspots, but rather their weaker remnants, such as faculae and network fields, both rarely seen, but ever present. These may be of greater interest to some, since one finds they generally have a longer lasting global impact and help drive the Sun’s future activity behavior. Another interesting aspect that can be seen if one watches the model run long enough is that there is often a degree of asymmetry in the Sun’s polar field, usually some oppositely oriented field embedded within the polar field. This often disappears as the cycle progresses; however, we believe it to be a real effect, and that this asymmetry appears to us to be a tilting of the Sun’s dipole much like the Earth’s dipole is not centered on the Earth’s rotational axis, but currently some 11 degrees inclined to the axis; here we relate the tilt to Cowling’s [29] theorem against having an axisymmetric dynamo.

With the Sun using only ~0.1% of its active region fields to regenerate its polar fields, this clearly makes the actual solar cycle a very inefficient process. Nevertheless, it is the photosphere’s job to transport the Sun’s energy outwards not to make magnetic fields. So, what may be described as “wasting 99.9% of its energy,” an inefficiency (to us, humans who are interested in the solar cycle), may be, to some higher vantage point, a marvelously efficient radiator. We really cannot judge such things; they just are what they are. In other words, we can simply agree that the Sun allows 99.9% of its active region magnetic flux to cancel before the remaining 0.1% makes its way to the opposite poles. The strength of active region fields compared with the background field may be seen in many field plots [30–32]. Included are individual images of the Sun’s fields as well as synoptic maps of the Sun’s full surface. Active region fields show magnetism covering the disk as well as the enormous field variations present on the Sun’s surface, with weak polar fields barely visible in the earlier displays, when the Sun’s field was very active in the last half of the 20th century. Let us return to our field modeling displays and observe that it too gives rise to Maunder Minimum type behavior.

2.2.3. March/Rush to the Poles, or Rivers of Magnetism. Another event corresponds with the following solar behavior. On the Sun, there are many temporal relationships between events. One that is often referred to is a “march” or “rush” to the poles of polar crown filaments. These motions often occur just before the time of polar field reversal during that phase of the solar cycle (solar maximum). These polar crown filaments, when seen above the Sun’s limbs, are referred to, most commonly, as prominences.

In times past, before the space age, they were most “prominent,” hence their name, at times of solar eclipses, when they displayed their rich, ruby-red light to terrestrial observers as they peaked above the moon’s limbs on the day of a solar eclipse. This color contrasted beautifully with the steel blue electron scattering of the photospheric light by the inner corona. The bluish-white corona displays the feeling of shivery-cold moonlight reflecting off icebergs above a freezing ocean, since eclipses often result in a temperature drop of many degrees. Such a feeling belies the corona’s heat of millions of degrees, and the prominences possess a ruby-red color, belying their cool, chromospheric temperatures, thousands of degrees lower than the dazzling white photosphere. Nature thus gives us fair warning not to judge it with our frail human senses. Instead, we must apply our equally frail mental faculties to interpret its mysteries. All we can do is pause to think and reflect how little we know.

Thus prominences appear in the ruby-red Hydrogen alpha line, at a wavelength of 656.281 nm. They are dark when contrasted against the solar disk (hence called filaments), because they absorb the light from the bright photospheric disk beyond. At times of eclipses, however, they are seen most prominently in ages past with their bright red appearance. Most commonly, although they are quiescent and scarcely, a tenth of solar radius above the photosphere, just lying peacefully like some kind of solar cow, nestled under the protective base of a helmet streamer, showing the inner corona’s base. These coronal streamers, of course, got their namesake, owing to the WW I shaped helmets warn by the Prussian soldiers to protect them from barrages; however, their shapely pointed tops probably made the soldiers more visible, rather than provided any real protection. Most commonly, in this modern era, however, the prominences only gain real fame, when they are ejected rapidly into the corona or solar wind, via coronal mass ejections (CMEs). These occur often with (or without) solar flares, marking their brightened appearance in the chromosphere, or infrequently with a white-light flare, in the photosphere.

The particular aspect of these features under discussion is their famous “march” or “rush” to the poles. Much as soldiers marched or rushed into battle, streamers all around the Sun gather en masse and march to the poles on queue, as if all called by a distant drummer on the Sun. We cannot hear the sound above the loud din emanating from the Sun’s surface, since the granules have material speeds approaching the sonic velocity. If we could hear sounds, it would be the loudest possible, essentially white-hot sound. We would go deaf instantly, but the march of all the magnetic features towards the Sun’s poles simultaneously would be a rather spectacular sight, and one that is not totally understood.

Gopalswamy et al. [33] provide reasonable explanations for this behavior. They report on the relationship between the disappearance of high latitude coronal filaments (PCFs), with coronal mass ejections (CMEs), and the overall extended solar cycle. The march to the poles is discussed in their paper as a “rush to the poles,” and so the terminology varies a bit from decade to decade. As we know, the filaments lie at the base of closed field structures, and before one can create a new polar field at the poles, the old fields first require removal. Consequently, the old field lines march to
the poles to reverse the polar fields of the old cycle. This seems eminently reasonable, and hence the march or rush to the poles associated with the polarity field reversal seems like a natural and necessary step in the solar cycle.

For the purposes of this paper, we see such fields parading to the poles at high latitudes, as rivers of field gradually moving poleward, just prior to field reversal. Figure 6 shows these field patterns with our model: top, the time history of the polar fields, and bottom, the field patterns at those times listed, just prior to field reversal shown. Usually, there is thin filament of old polar field, which gets pushed to the poles, and at reversal, it totally disappears, as new flux pushes up against the old flux till it finally gives way. This is basically the viewpoint that Gopalswamy, and references therein, report [33]. Hence we find it pleasing that this interesting aspect of the solar cycle is mimicked too by our model. Prior to recognizing this phenomenon as belonging to a preexisting name: “rush to the poles of crown filaments,” we were calling this “rivers of magnetism,” for the following reason. As one sees in a number of panels of Figure 6, the high latitude fields, but not the extreme polar fields, seem to run almost contiguously from the East to the West, meaning these fields are circumpolar rivers of magnetism, like the solar equivalent of a jet stream, but involving magnetism, not wind flow direction. They may be a high latitude extension of Cowling [29] backwards C shaped structures; however, now we simply equate it with the march/rush to the pole phenomenon.

At the pole, these field patterns just become increasingly stretched by differential rotation. Thus they may be a pattern drifting to the poles while at the same time rotating differentially, thus stretching increasingly in an East-West direction, and thus gradually decreasing the tilt angle of the magnetic striping of the field line. Thus it might resemble a thin sheet on the surface of the Sun, similar in geometry to the terrestrial aurora, which is circumpolar too, however circumpolar about the magnetic axis, not the axis of rotation. To us, the magnetic fields looked like rivers, with one polarity floating towards one pole, while the reverse polarity floated to the opposite pole. The pre-existing term “crown filament rush to the poles” seems adequate to describe this interesting behavior.

To obtain them from a model run, one must reset the value tick-end to either 0 to run continuously or to past the time one wants to examine for a longer run, for example, 300. In most examples the two polar fields do not reverse simultaneously, and there is even a slight difference at the poles, with magnetism at some longitudes disappearing after others. Such is the nature of the real solar cycle too. Such real phenomena are never as clear cut as our simple views suggest.

2.2.4. The Sun Appearing as a Monopole. Perhaps because this subject is the most controversial, we are leaving it to the end. Wilcox [34] stated that the Sun does sometimes “appear as a (magnetic) monopole.”

We are raising this matter, even though we are not positively disposed to revamping basic laws of physics based upon solar observations. Yet, we do recognize that Helium was first discovered on the Sun, and that neutrino oscillations too revolved around the Sun. Lastly, of course, we take pride in Leighton’s wonderful comment that if the Sun had no magnetic field, it would be as boring a star as most astronomers think it is. Hence, we do not simply toss out puzzling observations of solar magnetic fields. Instead, we ask: What can we learn from them? Further, does the current model shed any light on the origin of these puzzling observations? Let us first review the findings.

Wilcox [34] found that early in 1965, from February 3 to April 24, “For three or four consecutive months the observations (of the Sun) show that the solar field was predominantly directed out of the Sun at all solar latitudes.” Wilcox's observations from Mt. Wilson were supported, he states, by Crimean observations at high solar latitudes, although the polarity of their sign convention seems to have been opposite to the sign polarity used throughout the US, making it possible for there to have been a possible misunderstanding. The observations are backed by the interplanetary field as well, reports Wilcox. The field near the end of year, however, showed a reversed behavior, when several rotations displayed all but a few days of outward field, from June 1965 through October 1965. So, this adds more puzzling behavior of the Sun’s magnetism since Maxwell’s “Divergence B equation”

$$\nabla \cdot \mathbf{B} = 0$$

(11)

requires that the field have zero divergence at every location and for all times. The equation is not a bank that one can borrow against, if one agrees to pay the bank interest; the law applies equally at all times. We shall be using, predominantly, not the differential form, but rather the integral form of this law, based upon Stoke’s theorem, so that

$$\oint \partial \mathbf{V} \cdot d\mathbf{A} = 0,$$

(12)

where the integral is summed over the volume of a closed surface, \(\partial \mathbf{V}\), and \(d\mathbf{A}\) is an element of area normal to the surface. Nevertheless for ease in understanding we shall be referring to it as Maxwell’s Divergence B equation.

Before discussing the relevance of our model to these observations, let us first voice concerns we have about the solar observations that Wilcox reports on. We shall enumerate these reservations for clarity.

It appears that the detailed synoptic maps, made at Mt. Wilson at the time (from which Wilcox obtained his solar data), only show fields equatorward of 40 degrees latitude, owing to the difficulty in registering significant Zeeman splitting to obtain sufficient signal at that time. This allows the maps to have significant regions of the Sun from which positive or negative flux might emanate.

The regions of the Sun’s polar field were not examined in Wilcox’s study, although he quotes support from Severny; however, there is no “hard data” in Wilcox’s report on this. Going back to Severny and colleague’s reports [35–37] does not clarify the answer to this question, as the polar fields seem to have been fairly weak during this period. The discussion between Wilcox and Severny seems to be just a conversation between these individuals and neither is alive now. The Sun’s
polar fields may be an important aspect of this situation. If the Sun’s polar fields were “dipolar”, as they are often described, they would not affect the global field accounting, the key issue that needs to be studied to resolve this problem. Certainly, regardless of the sign convention, there seems to be less doubt that the polar fields were, at some portion of this time, of the same sign, so that may be important. The direction of magnetism has been a standard since the days of Maxwell [38]; many of the units have been revised, but the definitions are inherent in Maxwell’s fundamental equations.

Concerning the fields in the Sun’s polar regions, even though they did not always appear on synoptic maps, we have been fortunate to have Robert Howard, an experienced and dedicated observer serving many years at Mt. Wilson, to describe the Sun’s polar fields quite adequately from 1960–1971, as well as during other years. So, let us mention at least what happened during that interesting time, 1965, and not focus on the remainder of the period. The interested reader will want to follow the entire history of the Sun’s polar fields at Mt. Wilson, and Howard [39] provides an excellent starting place. During 1965, it appears that the Sun as seen from Mt. Wilson had the following behavior: for Jan. to ∼1/2 April, the South polar field was predominantly of the same sign as North (both + outward, but for Feb. the N polar field was very weak, and only the two polar fields had ∼identical fluxes. to within 1/2 Gauss, so not really much magnetic flux diverged from the Sun’s poles during this time period). In May, there seemed to be no observations. From June till half

**Figure 6**: Again the top panel shows the polar field variations during the run for the first few “field reversals,” shown with timesteps at the bottom. Thus again at the top is a Plot A graph with the three colored curves: blue, red, and black. The blue curve displays the amount and sign of the magnetic flux in the northern magnetic pole; the red curve similarly counts the mean field in the Southern polar region, and the black curve marks the Absolute Value Sum of the red and blue curves, placed 60 units downwards, for increased clarity. It essentially reaches a peak, during a solar cycle minimum. The lower panels are the field maps at these times, just prior to the complete reversal. One sees the new polar field rising from below, and the old polar field, on the verge of total annihilation. As mentioned in the text, the new fields are gradually marching to the poles. If one were to view the Sun’s limbs at these times, the border between old and new polar field would have a dark filament at high latitude, circling the poles, with the demarcation line being the old and new field boundaries.
of Oct., the field of South Pole was $+$, and of opposite sign to the North Pole; the South 0-1 G, and the North 1-2 G; for the remainder of the year, the fields fluctuated around somewhat, but the South was continually (as it had been mostly the whole year) emanating more outward magnetic flux. Thus, overall, there was not a significant net magnetic flux emanating through the Sun's poles during 1965, as this author interprets the graphs of Howard [39]. The reason is as follows. Whenever the fields were of the same sign (early in the year and late in the year), they were weak, and whenever (middle of the year), they were larger, they had opposite sign. Hence there appears to be little net magnetic flux, at least as we interpret Howard's report.

Let us now briefly discuss the Sun's polar fields. Interestingly, Severny reviewed Babcock's work and stated that H. D. Babcock used the Mt. Wilson magnetograph at seven points in the Sun's polar regions, and found that in 1957-8, the North polar field of the Sun reversed so suddenly, that both poles of the Sun had the same polarity for almost a full year. Severny also states that in 1959 the solar polar field was parallel to the Earth's. Thus Severny was well aware of the sign convention used by solar physicists to measure solar magnetism. As mentioned earlier, the sign convention is not just an American solar magnetism standard; it is the universal standard for magnetic field signs in physics [38]; Maxwell's equations have not changed sign since they were invented. That the Sun's polar field retains the same sign magnetism is relevant to our model, as we shall see shortly.

The Sun's polar fields occupy those regions of the Sun poleward of $\approx 60^\circ$. These are the locations where the Sun's main global dipole fields typically reside. Despite often hearing the phrase "the solar dipole", the Sun's polar fields are not strictly dipolar. Namely, they do not have equal and opposite poles. Further the Sun does not possess the same latitudinal dependence that a strict dipole has. The Sun's fields are NOT similar in virtually any way that makes sense to a dipole magnet. Instead, they consist of numerous tiny regions from which magnetic field of both signs emanate, covering the high latitude polar regions (greater than $\approx 60^\circ$) of the Sun. Additionally, the Sun has many, many bipoles all over its surface [40]. There usually is a predominance of one polarity, at either pole. Most often, except when the Sun changes polarity (near solar maximum, but each pole reverses polarity at different times, as much as a year apart), one sign field predominates the polarity at each pole, and most often the polar fields are reversed from each other and are not closely equal and opposite from each other. Our knowledge of the Sun's polar fields is minimal.

We may even call the polar regions of the Sun, the "ignorolatitudes" (modifying the term "ignorosphere," used by atmospheric and space physicists to describe the mesosphere—the middle atmosphere at 50–85 km above the earth, which is not accessible by aircraft, too high, and not accessible by spacecraft, too low). The polar latitudes are the locations on the solar surface least studied by solar physicists, because there does not seem to be a lot going on there: no flares, no new activity centers, and so forth. Nevertheless, these regions may, in some ways be equally important to the balance of solar magnetism, as the active latitudes. The active latitudes are a great source of solar magnetism; however, the polar latitudes may be the graveyard of the many solar fields. Thus they deserve, perhaps, an equal amount of attention. We do not know what really happens there, except these regions are the source of the steadiest high speed solar winds, a key to understanding the solar dynamo, and useful as a predictor of future sunspot activity. As a consequence of this last point, they beg examination. How is it that these fields are a key component to understanding future solar activity, and yet more than 99% of the solar physicists do not study them?

Beyond the high latitude gaps in data, Wilcox shows three solar rotations of detailed Mt. Wilson data, and there is a period of about 10 days in Carrington rotation 1802, where the entire period seems to be devoid of data, thus missing about 1/3 of the data during this interval. Let us add to this the fact that the magnetograph can see at best just the visible hemisphere of the Sun (and most often only a fraction of that for greatest clarity), so the synoptic maps seen of the Sun's field show an accumulated data display made from many, many different times (roughly 10 or more), not a snapshot over the Sun's surface made at one time, from satellites peering at the Sun from all angles. Knowing the workings of the observatory, and that one was restricted to make observations as different longitudes of the Sun face the Earth, the geometry is clarified.

Additionally, and this is typical of the magnetograph contours back in the 1960s, even when data exists, the majority of the solar disk, even when well observed, still has insufficient signal to allow non-zero contour lines to show over a broad portion of the Sun's surface. In other words, at low latitudes where this data situated, most areas show zero data—not really because there is exactly a zero field present (although the Sun's field is thought to exist in patches of high field and larger areas of weak, or near zero field), but all we can really say, is that the line of sight splitting is below the level of detection. This is just the state of observing the Sun's longitudinal field at that time from Mt. Wilson. Mathematical zeros are not likely in the physics of fields; however, levels below detectability are.

“Observing window effect.” Lastly, we should be remiss if we did not mention the following aspect, which we call the “observing window effect.” Solar observations only see light that is emitted from the Sun's photosphere, typically, and goes into the telescope observer window where it is processed. Hence, if regions on the Sun's surface possess a magnetic field pointing into the Sun (at a particular time) but these regions happen to lie on rather darkened lanes (and there are lanes on the Sun that are relatively dark; there is a mottling of the Sun's surface due to lowered temperatures (e.g., intergranular lanes, owing to the convective patterns)), then it is likely that this field sense (the toward the Sun field) would be undercounted, since the magnetograph integrates the line splitting (into the observing window). Hence any solar observation is subject to “observer window” effects and irregularities in the Sun's surface brightness, and other irregularities in its surface from an ideal sphere.
Table 1: Wilcox [34] suggested that the interplanetary field (IMF) showed a monopole-like behavior, when the IMF, or its proxy, showed time intervals when it was predominantly pointed towards or away from the Sun for a good fraction of a solar rotation. Svalgaard [41] shows the following time periods with unipolar patterns.

<table>
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In the next few paragraphs, let us first put to rest the problems associated with the interplanetary fields from near Earth (not observations of the Sun's surface). According to Wilcox, the Sun appeared virtually unipolar of one sign for an entire or even more than one solar rotation as observed from Earth using the Sun's extended interplanetary field (IMF) polarity interpreted from geomagnetic observations. These are based on a highly accurate method, spearheaded single-handedly by Svalgaard [41]. His observations are shown in Table 1. Same sense magnetic fields are displayed with rotations showing predominantly towards the Sun field from May 1967 through October 1967. The reverse situation next occurs near solar minimum in 1987, when one rotation, all away from the Sun field, manifests itself at Earth. The rotations before and after the February 14th rotation (Bartels Rotation no. 2098: these are Bartels rotations, an Earth-based geomagnetic numbering system, different from the Carrington solar-based numbering system used at Mt. Wilson) show predominantly the same polarity.

Let us add some cold water to dampen our enthusiasm for this discordance with Maxwell's Divergence $B$ equation. The naive interpretation (ours, not Svalgaard's) of this outward or inward directed field is that the Sun has a non-zero divergent field. We are even more out on a limb here, because what really is being observed by Svalgaard is the field in the ecliptic plane at Earth. So, there is less integration over the surface of a volume than the incomplete observations of the Sun from Mt. Wilson. Here, we have accumulated observations from the Earth, as the Sun rotates in space and the solar wind carries out the Sun's field, but it still is basically
the observations of one latitude and longitude at a time; there is hardly any way to integrate over a closed surface that the integral form of Maxwell’s equation demands. The final nail in the coffin of using the interplanetary magnetic field near Earth, being a good location to examine interplanetary magnetic fields from the Sun for a divergence test, comes from the report by Echer and Svalgaard [42], who report that the interplanetary sector structure at Earth is closely associated with the Rosenberg-Coleman effect (a latitude effect) wherein the field at the Earth shows a high degree of polarity changes associated with the heliolatitude of the Earth. These authors also report an asymmetry in the effect, which may be due to any of a number of causes; they suggest an “internal solar variation.” The Rosenberg-Coleman effect was observed in the late 1960s. The current sheet model with currents circling the Sun was illustrated in the early 70s [25], with the current sheet shown in projection, as well as in the fields modeled; however, it was never referred to as the now “ballerina skirt” model, an unmathematical but evocative picture is most often cited. The magnetic fields in interplanetary space seem eminently consistent with no solar monopole; one just takes the leading term; when there is no monopole, the leading term is a solar dipole, with the higher harmonics falling off in the inner corona progressively with powers of inverse radius, until the magnetic fields are “frozen in” to, in accord with Alfvén’s theory, the solar wind which carries the extended solar magnetic field, out into interplanetary space, where they are observed by spacecraft or Earth. So, now all we have left are the direct solar observations.

Putting aside our observational shortcomings, let us now consider some of the model’s behaviors that may shed light on the Sun’s strange field wanderings. Has the Sun gone on a bender, or off the deep end of a large swimming pool? Let us consider this seeming violation of one of Maxwell’s equations. Well, one might argue that Maxwell had 4 famous equations, and 3 out of 4 are a superb batting average. Of course, as physicists we cannot think that way. Instead, we are trained to acknowledge the many successes of these 4 equations, and that they help form the basis of special relativity, and quantum electrodynamics, the best tested set of equations in Physics. So, we must acknowledge this, and that something must be going on with the Sun that is indeed puzzling, but our classical bedrocks remain the inviolability of each and every Maxwell equation, particularly \( \nabla \cdot B = 0 \), unless magnetic monopoles themselves are directly found, say in cosmic radiation or perhaps in space, where the moon was searched for magnetic monopoles [43, 44] and no monopoles were found there to a finer extent than any search on Earth! So, we must pay homage to the Maxwell’s Divergence B equation in its commensurate integrated form, (12), and attempt to understand what might have gone wrong with the Sun, or the interplanetary magnetic field, to make them so unreliable.

So, we consider (12) is obeyed; however, the Sun may still appear as a monopole from Earth if the following strange behavior occurred on the Sun. Namely, if the solar fields move about in such a manner that the Sun has a large quadrupole moment, so that although \( \nabla \cdot B = 0 \), the divergence of the Sun’s field as seen from the earth only provides the illusion of \( \nabla \cdot B \neq 0 \) ! Thus we shall be considering not that Maxwell’s Divergence B equation is disobeyed, but the Sun’s behavior gives us the illusion that Maxwell’s Divergence B equation is disobeyed! This may allow us to find out something interesting about our Sun’s behavior that we do not know, but only if we can understand what tricks stars can evoke.

Rather than talk about the field of the Sun, in terms of the field on its surface at each latitude, and so forth, it is simplest to describe its spherical harmonic expansion, in terms of Legendre polynomials and their associated spherical harmonics, \( Y_{lm} \). Along with these are their \( g_{lm} \) coefficients [38], and sometimes Schmidt normalization is used in solar and geophysics, in which case one uses \( g_{lm} \) and \( h_{lm} \). The Divergence B equation, \( \nabla \cdot B = 0 \), implies that the \( g_{00} \) term is 0, when analyzed over a closed surface. Thus we turn to trying to understand how the solar field might accumulate error, which might give the appearance of Maxwell’s Divergence B equation being off.

The simplest way for the Sun to give this appearance is that a large solar quadrupole moment exists (a strong \( g_{20} \) term), which yields a spherical harmonic form of the \( Y_{20} \) type (or another even multipole moment of \( Y_{N0} \) type, \( N \) even could suffice; however the \( Y_{20} \) term has the broadest equatorially symmetric form, apart from \( Y_{00} \)). Figure 7 shows the geometric appearance of the spherical harmonic functions, \( Y_{lm} \). These are best understood by recognizing that the Legendre polynomials possess \( \ell - |m| \) vertical nodes or zeros, whereas in the longitudinal direction, owing to

\[
\nabla \cdot B = 0, \\
\text{Figure 7: Visual representations of the first few spherical harmonics. Red portions represent regions where the function is positive, and green, negative, respectively. The right hand side of the figure shows } +M \text{ numbers, and the left side, negative } M \text{ numbers, so the pattern solutions shown distinguish the symmetric from the antisymmetric solutions. As usual, the } L \text{ number denotes the total number of nodes, with } M \text{ being the number of vertical nodes, so } M = 0 \text{ denotes axisymmetric solutions, vertically downward from the top. The } L = M \text{ mode denotes sectoral nodes, having no vertical nodes, as a vertically sliced orange would. These sectoral nodes are shown diagonally downward on the right hand side, and the antisymmetric form } M = -L \text{ diagonally downward on the left. The original solar sector pattern had an } L = 2, M = \pm 2 \text{ form, shown in the third row downward on the right or left. The very top spherically symmetric node is the monopole term, positive everywhere, or alternatively negative everywhere, and the second one vertically downward is the quadrupole term } (l = 2, m = 0), \text{ negative in the equatorial plane and positive at both poles, or vice-versa. The } g \text{ coefficient can multiply the form to be negative of the actual spherical harmonic.}
\]
the appearance of the sine and cosine functions, the forms possess $2|m|$ zeros, which result in horizontal nodes, as shown in Figure 7. When the spherical harmonic order $m$ is zero (the central vertical column in the figure), the spherical harmonic functions do not depend upon longitude and are referred to as azimuthally symmetric or zonal. When $\ell = |m|$ (the right and left diagonals in the figure), there are no zero crossings in latitude, and the functions are referred to as sectoral. For the other cases, the functions checker the sphere, and they are referred to as tesseral, and the $l$ number governs the total number of checkered patterns. These functions occur in quantum mechanics as well as electromagnetism and are simply orthonormal representations of geometric forms. Thus they may be described as a spherical form of Fourier analysis.

It is clear that the $Y_{20}$ term appears when one is preferentially sampling the equatorial solar field. Both $Y_{20}$ and $Y_{00}$ have a single sign at all longitudes at low latitudes (below ~45 degrees) hence when preferentially sampling the low latitudes (missing the high latitude fields completely), one can easily mistake a quadrupole form for the $Y_{00}$ form. The forms are chosen so $Y_{20}$ is negative at the equator, whereas $Y_{00}$ is positive, but the coefficient, $g$ number, adjusts so that either can represent a positive or negative value. The monopole form is totally outward, or inward, uniformly over all latitudes, whereas the quadrupole form changes sign at high latitude. Hence $Y_{00}$ is special; it is uniformly of one sign, whereas all other terms have 0 net divergence. So, how can one mistake a quadrupole form for a monopole form?

The Sun may fool us as follows. When we view the Sun from Earth, we are at low heliographic latitudes. There are two ways we underestimate the Sun’s polar fields: (1) the high latitudes show only a much smaller projected angle (the cosine angle of normal of the surface to the observer), and (2) the Zeeman splitting is roughly proportional to the projected field along the line of sight, providing another cosine factor. Since the Sun is tilted by 7.25 degrees from the ecliptic, this angle gets at most only towards the Earth at an angle of ~86.75 degrees directed towards Earth, and the cosine of this angle is 0.057. The value squared is 0.0032, about 0.3%, illustrating how weakly the polar fields are observed, and this is the best case, during the time one pole is preferentially oriented towards Earth (in the spring and fall). When one observes the Sun and then attempts to run a best “fit” algorithm (even, at best, when such an algorithm is undertaken), they allow a non-zero fit to the monopole term. Additionally, typically, error analyses are neglected, and one just hears of the best fit “value.” In this manner, the Sun fools us, or we fool ourselves into thinking a solar monopole field may exist, without a true recognition of the error sources.

More explicitly, what happens is that the quadrupole term $(2,0)$ (see Figure 7) appears significant from the Earth at low latitudes, and we miss or underrepresent the opposite sign high latitude fields that offset the low latitude contributions, since the Earth samples the Sun preferentially at low latitudes. As a consequence, the Sun sometimes “looks like a monopole.” Could such forms be strong enough that they might be mistaken for a solar monopole? Well, we cannot go back and measure the polar fields of the Sun in 1965 or the other periods wherein these strange behaviors arose. We may wait for them, but we must understand how we might be better prepared than in 1965. For now, we can examine our model, and see whether this model has anything to say about a solar quadrupole moment. Well, when we started running Quad-Blaster set to 200 to remove the annoying quadrupole moments it seemed like a neat way to remove the pesky fields. Even with this high a value, it does not totally remove the $g_{20}$ term; however, it greatly reduces it.

Nevertheless, although it stilled monopole moments, perhaps it did this more than the Sun does, and thus perhaps we were “throwing the baby out with the bath water.” Namely, perhaps a more realistic solar model would be obtained by including some of these field drifts, which induce a quadrupole moment. We now consider this by running our model with a significantly reduced quadrupole moment. Figure 8 shows our model run with Quad-Blaster set to 0, and a number of quadrupole fields of the $Y_{20}$ type emerge (as seen from the panels, where the polar fields are of the same sign, and the equatorial field is predominantly the reverse sign). We note that there are significant quadrupole terms, as seen from the bottom panel, showing the polar field, gradually drifting upwards. At the end of this run, the polar fields are both positive and roughly the largest of any throughout the entire run. It is hard to know what the best value might be for this quadrupole removal term, or whether it should be there at all, in this form. Clearly, if one wishes to have the quadrupole term removed for improving the solar field maps, we need to know how much the Sun’s polar fields vary. Thus, use of this term and this model may represent a useful application to compare the Sun’s polar field variations with, in order to better understand the Sun’s strange wandering polar field patterns! Since the Sun’s polar fields are important, not just for better understanding the origins of the solar dynamo, but also as entry points to the terrestrial environment of the local galactic cosmic ray flux, first observed by Forbush (his 20-year wave). Both subjects appear worthy of future scientific study, both from the Earth and from space.

Thus, to reinforce this aspect, we need to know what the polar field of the Sun actually does, before deciding what the best factor is for this parameter. Nevertheless, this lack of definition just serves to illustrate the point we tried to make earlier, that the polar fields are amongst the least well observed magnetic fields in the Sun’s photosphere.

In running our model, we see, in addition to the solar cycle related out-of-phase oscillations of the Sun’s polar fields (related to the solar cycle), occasional in-phase wanderings of both polar fields towards either positive or negative territory. Some are rarely seen and others seemingly new phenomena, appear in this large-scale global model. Of interest are the infrequent events wherein both polar regions have predominantly the same polarity, with a reverse polarity predominantly near the Sun’s equator. This effectively describes the Sun having a large quadrupole moment, the $Y_{20}$ term, with a degree of symmetry about its polar axis. One can tell that this is beginning to happen by examining the polar field graph (Plot A) when both polar fields drift either up or down: positively or negatively, namely, not symmetrical...
Figure 8: The Sun appearing as a monopole. This model is obtained by using a 0 value for the “quad-blaster” setting. The lower graph shows the polar fields, again a Plot A type graph of the polar field variation seen in many of the preceding figures. The Plot A graph has the three colored curves: blue, red, and black. The blue curve displays the amount and sign of the magnetic flux in the northern magnetic pole; the red curve similarly counts the mean field in the Southern polar region, and the black curve marks the Absolute Value Sum of the red and blue curves, placed 60 units downwards, for increased clarity. The polar fields overall, are observed drifting well into the positive arena (thus with both polar regions positive, with the assumption of Maxwell’s divergence B condition, one may assume that the equatorial field of the Sun balances the magnetic flux), indicative of a quadrupole moment, and these three field maps are shown by the three graphs above indicated by the times. These are all forays of one sign, but either sign is equally likely. This upward drift indicates increasingly positive or blue polar fields (at the poles) and negative or red equatorial fields. The solar fields calculated by the model are not monopolar since blue and red entities always have identical numbers, but rather quadrupolar. This leads to the false impression at Earth, of a predominant non-zero solar magnetic polarity at low latitudes to those who choose this explanation.

Figure 8: The Sun appearing as a monopole. This model is obtained by using a 0 value for the “quad-blaster” setting. The lower graph shows the polar fields, again a Plot A type graph of the polar field variation seen in many of the preceding figures. The Plot A graph has the three colored curves: blue, red, and black. The blue curve displays the amount and sign of the magnetic flux in the northern magnetic pole; the red curve similarly counts the mean field in the Southern polar region, and the black curve marks the Absolute Value Sum of the red and blue curves, placed 60 units downwards, for increased clarity. The polar fields overall, are observed drifting well into the positive arena (thus with both polar regions positive, with the assumption of Maxwell’s divergence B condition, one may assume that the equatorial field of the Sun balances the magnetic flux), indicative of a quadrupole moment, and these three field maps are shown by the three graphs above indicated by the times. These are all forays of one sign, but either sign is equally likely. This upward drift indicates increasingly positive or blue polar fields (at the poles) and negative or red equatorial fields. The solar fields calculated by the model are not monopolar since blue and red entities always have identical numbers, but rather quadrupolar. This leads to the false impression at Earth, of a predominant non-zero solar magnetic polarity at low latitudes to those who choose this explanation.

about zero. This has been seen infrequently on the Sun, predominantly because the polar fields themselves have only been observed infrequently, and with poor confidence, and only recently. Many times, the polar fields are not seen at all (e.g., when one solar pole is tilted away from the Earth). It would of course benefit such observations to have a satellite measuring the Sun’s polar fields from a vantage point near the opposite side of the Sun, but situated so that communications with Earth are possible.

These events support the model’s behavior of occasional “apparent monopole-like behavior,” as observed from the ecliptic viewpoint of Earth, which is preferentially situated near the Sun’s equator. We have incorporated a term in the model, called quad-blaster, which is just a term tending to reduce the quadrupole moment of the Sun, but the parameter only works moderately well, and the quadrupole moment still wanders around owing to the random walk nature of the surface fields in our model. If quad-blaster is greater than 0, and the nominal value is 100, then the term tends to reduce the quadrupole moment and tends to remove any false monopole term. If quad-blaster is less than 0, then the term can magnify an apparent monopole term, and if quad-blaster equals 0, then there is no effect on the monopole term. We should like to mention one more recent bit of news on this subject supporting the quadrupole interpretation. Yang et al. [45] observed a coronal halo event; in this, the Sun undergoes a “quadrupole dimming” effect, namely, its quadrupole moment changes, consistent with pairs of
oppositely oriented dipoles (e.g., two dipoles of opposite sign disappearing, as would happen in a complex active region, or when fields reconnect, as in Babcock’s picture of field lines in the corona, close to the source surface area, reconnecting), so that whole loops are emitted into space, via a global scale field reconnection. This suggests that perhaps having quad-blaster equals 0, might be a preferable model. To really have the “best model” requires a better understanding of the Sun’s global fields, not just simple pictures. Nevertheless, it is up to the theoreticians to supply the solar observers with a “format” that they are able to “compress” their data into. Namely, should we be using \( g_m \) and \( h_{nm} \), or other convective eigenmodes that express the Sun’s harmonics better? The traditional spherical harmonics are orthonormal expansions into a “complete” series, like Fourier’s sines and cosines, except that the spherical harmonics are 3D, not 1D. Nevertheless, angular forms which take into account the Sun’s differential rotation and the manner in which sources from small magnetic features evolve may yield some information overlooked by the brute force methods that spherical harmonics were never fine-tuned for. This is simply a suggestion that may open the door for further research.

It is this author’s viewpoint, that the best eigenmodes are the UMRs of Bumba and Howard [19], coupled with the North-South meridional transport, similar to the patterns that Wang and Sheeley suggested. These might allow the Sun’s field (plus its changes) to be incorporated into a global model, that would be for the Sun, like the terrestrial weather’s global patterns (i.e., Hadley circulation, El Nino Southern Oscillation (ENSO), Arctic Oscillation (AO), planetary waves, and so forth that meteorologists have used to organize the complex terrestrial weather patterns into). For the Sun, we are lucky; the chemical constituency (the equation of state) is governed predominantly by the temperature and pressure via the SAHA equation [46], and one does not need to account for (except in the atmosphere above the photosphere, predominantly) the changing microscopic chemical properties as one does in the Earth’s atmosphere, because the mixing on the Sun is so great. One just needs the local state properties \( (T, P, B, \rho, \text{etc.}) \), plus the boundary conditions.

The boundary conditions for the Sun are more problematic, since the global aspects of the Sun are not so well understood. These are governed by global studies: the inflow (turbulent torques and pressures into the photosphere; however stellar models can provide these), and outflow equations: effluent angular momentum (governed by the outflow of solar wind versus latitude on open field lines, etc.). The global Sun needs to be examined, not just the microscopic aspects. The turbulence in a river is controlled by how much energy is available in the stretch of a river governed by the amount of water moving towards lowered gravitational potential, so that the recent rainfall affects the turbulent state, via the Reynolds number. In a similar manner, the turbulence in the solar atmosphere may be sensitive to the local and global state of its magnetism. Field inhibition [47] of Biermann’s may be coupled with flow dynamics [48, 49] in the following fashion. Initially, the magnetic field of an active region (AR) may inhibit the flow; however, inhibiting the outward flow of solar energy this way is similar to trying to stop the heat escape a boiling pot by placing a little lid on it; it usually is insufficient, and effect is temporary; energy simply escapes around the closed lid, in the following fashion. For the pot example, the energy into the pot keeps increasing until it eventually releases in a less gentle fashion, for example, with an outpouring of energy in a less gentle fashion, perhaps with hot steam leaking outwards. For the Sun’s case, the energy too leaks out via less gentle mechanisms, that is, in the visible, superheated gases flow around the blockage and the fluid erupts so that hotter interior gases rise above the normal photosphere, releasing energy in the UV and EUV, and so forth, and thus radiating the energy in shorter wavelengths, and so forth. Energy in active regions cannot be contained easily by local magnetic fields. The flow is just readily diverted to an alternate pathway. Any puny attempts predominantly serve to displace the flow towards less direct outlets that are simply more turbulent and serve to bring up more of the heat below owing to the increased turbulence patterns created. Hence the global aspects of a problem involving turbulence cannot be swept under the rug, much as one might wish to.

While discussing unusual solar behaviors, another aspect of this model that we think might occur or have occurred on the Sun is that sometimes the polar (dipole) fields do not reverse. They can head towards zero, seemingly about to reverse, but then goes right back to their former sign or they may have just a short, small reversal before heading back. Such behavior does not seem like our solar dynamo, but we only have about a century of observations of the Sun’s fields, begun by George E. Hale. It is hoped that some of these odd behaviors will be better understood, and if unrealistic, the model will be improved.

3. Future Investigations Using This Model

This section considers two broad aspects: how the solar field mapping model relates to (1) broader aspects of geophysics, namely, those areas affected by solar activity: the solar wind, through the magnetosphere, ionosphere, and the various layers in the Earth’s atmosphere, and (2) physical models and improvements.

The reasoning behind using the model for “broader aspects of geophysics” is that knowing the relationships between the Sun as a source and as a driver of various solar-terrestrial indices may be useful in the subdisciplines mentioned. The relationships between indices are at the heart of this, and using this model to improve that connection can only strengthen our understanding and usage of these interactions. In this section, we shall mention aspects of this model that one might study, through which the model can be further investigated, and also many more areas where it could well stand improvement in. The author shall be delighted to comment on, and help where possible, others undertaking such endeavors, not just with this model we refer to as Solar Field Mapping Model 1.07a, but other advances. Working with this model is as much fun as it is work, since Netlogo is such an easy language to program in.
Let us first discuss the use of indices as a future direction, global and daily indices. We may take the daily indices to indicate the daily count of a global index; however, there is not generally any easy transformation from a “global index” to a daily one, because each index has its own unique transformation qualities, rather than the same exact mathematical structure. To be more explicit, daily indices often require some complex weighting function of, or over, physical solar parameters and this process represents a some type of conformal mapping, for example, a radiative transfer function (or another type of) integration over solar atmospheric parameters. Such parametric functions may be photospheric, chromospheric, or coronal, but each index refers to a unique and unknown “mapping.” Often such an “inversion” represents trying to find a solution to an “inverse” problem in the sciences. Usually is not a unique answer. Consider a simple example: find the density within the Earth, knowing $g$ on its surface. This cannot be done uniquely. For if one uses any model, and tries to do one’s best, there are other models that can do equally good. If one chooses a constant density, sure that is a solution, but there also are others equally “good” where the density varies around this solution, but no knowledge about which is right (unless one is “given” that the solution must have constant density). Even solutions with most of the mass density near the center of the Earth are not easily ruled out by only knowledge of $g$ near the surface. Information that pertains to the internal density structure is required to ascertain properties of where the mass is, such as using gravity waves which travel through the interior.

In the same vein, the solar field mapping model does not calculate sunspot number, really just field entities, and then performs simple statistics on these, to enable observables, like the polar field (abs-count $= n$-pole to be calculated which might relate to polar faculae), or the “Active Region Count”, (AR-count) in the upper right monitors. There are a plethora of solar-terrestrial/space weather parameters/indices that solar- and geophysicists have tracked in their attempts to monitor the Sun. The oldest is, of course, sunspot number, plus the plage and facular indices, the Calcium K index, and the facular count. After this, there are various geomagnetic indices, such as $Kp$, and its linear complements: $Ap$ and $aa$ indices. Additionally, one might be interested in modern observables, more directly related to the Sun's output, many commonly employed by space researchers, with direct connections to the Sun at short wavelengths (e.g., solar EUV, UV, etc.) and various chromospheric parameters, Mg II, Calcium K2, or F10.7, used often as a proxy for solar EUV, and useful for space orbital drag calculations, for example, or total solar irradiance (TSI). This list is not complete. Modern updates to these involving specific spectral observations (for the Sun), or ionospheric and atmospheric layers (for the Earth), and magnetospheric and interplanetary areas (for space) may be possible broad ways to consider the ranges of this research.

Now I shall point out weaknesses in the current model, namely, ways that it might stand improvement, or at least further investigation. I shall enumerate these and give brief identifying names so as to improve clarity, in any later discussions.

Momentum conservation: the model has a number of simplifying assumptions; one is the innocent choice that field entities have constant speed. This appears to conflict with momentum conservation since if field agents turn relative to one another and still retain the same speed, there is no guarantee that they conserve momentum. This sounds troubling, but this problem is really minor when we consider that the lower ends of any magnetic feature in the photosphere interacts with much denser material below the photosphere. Thus we may invoke that the lower density material absorbs the magnetic field tensions so as to conserve momentum. Still this is an area that deserves further investigation.

Coordinate system: the coordinate system could well be transformed to an equal area synoptic chart configuration. This would be an improvement to this model; however, Netlogo does not at present employ this geometry.

Field agent parameters: how the model chooses the birth of field could be modified, and there are various ways they might be improved or modify this model’s choice. Some might require major renovation others minor adjustments, depending upon the type of change considered. Our choice affects how field agents spread away from the active regions in which they were spawned. Their motions are consequently dependent upon the random velocity (heading) of their initial motion. This clearly introduces elements of (apparent noise) into our deterministic system. This, of course, means that the noise is not stochastic but rather is sensitive to initial conditions, meaning it is “chaos” noise (like the kind that Lorenz first investigated in weather system modeling—namely high sensitivity to initial conditions). Although this is more attractive, it still means that the flows, which ideally we wish to study, are encumbered with a degree of noise aspects, and that some parametric choice would be ideal for the Sun. It is unclear how much quantization noise is “right” for the Sun.

Hale boundary active regions: one might consider rather than adding sunspot fields with random longitudes, adding them at preferential longitudes associated with the appropriate Hale boundary in the source surface or current sheet boundaries mapped down to the Sun. This stems from the work of Svalgaard and Wilcox [50], and supported again by Svalgaard et al. [51] with interesting statistical results showing that active regions do not fall randomly in photospheric longitude, although their latitude bands are well defined by the Butterfly diagram into broad bands, but they also preferentially appear when the source surface or sector boundaries line up with a Hale polarity (that polarity matching the sunspot regions).

Observational feedback to model: here we envision putting in active region information, and generating field entities that are put into the model. As these move about on the solar surface, one imagines some comparison of the agents changed positions to observational data. For example, this could be using the agents to obtain spherical harmonic coefficients, $g_{lm}$ and $h_{lm}$, as commonly used in the Schmidt normalization of spherical harmonics for...
geophysical phenomena; testing and improving the dynamics of field motions so that the computed motions are tied to observed field motions, as opposed to flow patterns that this model yields. These disparate fits might lead to improvements in this model, so that the model more closely matches the observations. It is possible that the initial birds’ speeds will have to be reduced and/or some other initialization method to add active regions to this model. For example, real active regions do not have flux balance, yet this model only adds field entities in active regions which do have flux balance, so this will obviously require some “handling.” This may lead to removing glaring errors or findings of field motions that are hitherto unknown. So, it is a ripe field ready for harvesting.

Another aspect of this is to use actual new active region fields, as opposed to computer generated ones, and then ascertain how large-scale fields emanate from these regions. This surely will result in significant findings. We may also learn better how to predict changes in solar activity, better than we do at present.

Coronal field and solar wind implications: it would be of considerable interest to ascertain how these results would reflect in coronal field patterns and motions. One may see how closed and open fields change forms, and whether these variations are consistent with what is observed and lead towards improved modeling aspects, or are the results too disparate, and so point to some key failings in the model, from which it cannot recover, without being totally gutted.

### 4. Summary and Discussion

We have developed an algorithm that allows a calculation of photospheric field motions, given the placement of field in the photosphere. The photospheric placement is done by the present program using a pseudo-random placement in accord with the various “rules” developed by Hale, and others related to the solar cycle, with parameters such as the polar field strength, solar cycle phase, and so forth. Despite uncertainties in our model’s algorithms, we have accomplished this by making certain assumptions that allow us to find specific solutions to the motions of the Sun’s surface fields. Clearly, this is a daring task, since the fields appear to move on the photospheric surface in rather chaotic ways. It remains to be seen then, to what extent the solutions bear any resemblance to observed solar field behaviors. When the strengths and weaknesses of this model are evaluated, then subsequently, one may advance the methodology towards improved understandings and/or solutions. It may, of course, be that the methodology is a dismal failure, but then perhaps it will shed light on dismissing the ideas contained herein that are intractable.

To summarize the model, a photospheric field mapping program 1.07 was developed using Netlogo algorithms involving cellular automata entities. These entities were of two types: bird species or breeds: either bluebirds or cardinals, for the two signed magnetic flux directions, inward or outward. The model tracks photospheric field motions, once given the initial pseudo-random placement of photospheric fields, based upon solar cycle parameters: polar field amplitude, phase of the solar cycle, and so forth. The model is online at the Netlogo community website and may be run either there and/or further developed by the interested user. The author shall try to be available to help the interested user.

The model is able to mimic the following: Solar Cycle Oscillations, the Waldmeier effect, Unipolar Magnetic Regions/Sectors/Coronal Holes, Maunder Minima, March/Rush to the Poles, or Rivers of Magnetism prior to Polar Field Reversal, and that the Sun sometimes appears as a monopole, but most likely has a magnetic quadrupole form giving rise to preferentially one sign at the solar equator. We then discuss various changes/improvements to the program that we might hope to see.

We are surprised that as simple a model as this is, that numerous photospheric phenomena are able to be displayed without specific knowledge of flows hidden deep within the solar interior; namely, the model does not have access to the dynamic structure of the Sun’s interior (state variables such as density, pressure, temperature, composition, ionization, magnetic field distributions, global flows, etc. versus height and their temporal variations). We do not know what this implies but are nonetheless happy for any serendipitous behaviors. There are at least two possibilities concerning the interior information.

1. It may be that the interior information is making its presence known predominantly through the arrival of sunspots in the photosphere, which we place randomly, but the Sun does this through processes not fully understood, and that these sources of new magnetic flux are, in fact, the “solar interior information that allows the dynamo to happen.”

2. It may be that the surface motions are able to “sweep aside” much of the interior behavior, except for the amount and location of new magnetic flux eruptions, as well as the Babcock-Leighton magnetic tension force. Thus our model sweeps other factors aside much as the Earth’s oceans reveal little of the undersea terrain and what lies beneath for at least a few oceanic wavelengths.

These are questions our model brushes aside. Since this is a discussion, however, we consider this second possibility further, not because it is more likely, but because it is less likely, and therefore less popular, that somehow the surface “sweeps aside” much of the interior information.

How can a physical entity sweep aside its roots? It somehow leaves the photosphere behaving as if the internal structure mattered little. Aside from the ocean analogy, there are more behaviors of this kind: black holes, it is thought, with the “no hair theorem” states, that above the event horizon of a black hole, only a few elementary properties are felt “outside” owing to what exists “inside,” namely, their gross mass, electric charge, and angular momentum. The internal aspects inside the black hole are brushed aside, and only the most elemental properties are allowed to “escape.” The sins of the black hole are buried inside the event horizon.
Our personal viewpoint of the successfulness of the current model is the following: that any model may consider any volume and fit reasonable boundary conditions to the surfaces surrounding that volume, and if one handles the boundary conditions correctly and the physics within that volume correctly, then one need not consider the physics outside that chosen volume. Hence one can brush aside the dynamics of the fields and flows inside the Sun as long as one includes how they affect the boundary conditions near the chosen volume! For the purposes of this model, all that the photosphere needs to be supplied with are the properties of erupting magnetic field, and the photosphere then carries fields along its thin layer as is observed. It may be that my present understanding of this model is incorrect, and perhaps a different viewpoint explains why the current model works. I shall be happy to listen to other interpretations.

To briefly summarize our current viewpoint, which appears similar to some of the earlier work [12, 13, 19] of solar field patterns. Ensembles of entities of “field,” when arranged on a sphere, undergo flows of the type we prescribe and are thus able to travel in patterns [19], which sometimes are referred to as “convective eigenmodes.” Thus self-organized behaviors may result in surface patterns that are the natural consequences of convection. For the Sun, magnetic fields may allow the field to serve as one of the few convenient “tracers” of the larger-scale convective patterns that are available. The surface magnetic field then can be viewed as a solar equivalent to the terrestrial oceanic term “jetsam,” discussed in our earlier analogy of sailors jettisoning material from sinking ships, that they wished could be saved. In a similar vein, for the gaseous planets, the chemicals found in cloud patterns (red and white spots, colored bands, etc.) serve as convenient tracers for the upper- and downflow patterns, that we know water vapor and its various aerosols provide for our terrestrial atmosphere.

A whole field of mathematical science has begun around phenomena which occur in new groupings, which otherwise are inexplicable. This is the field of “self-organized criticality” (SOC). One of the earliest papers in this field is that of Bak et al. [52] who showed that 1/f noise can develop from SOC behavior. This form of noise is known to occur in many diverse subjects: to those studying turbulence, cosmic rays, and many other areas important in astrophysics. It is possible that in certain circumstances the surface of an object and its development may be somewhat independent from the interior supporting it. Fey et al. [53] find, for example, in doing theoretical work on percolation patterns involving avalanches in sand piles, that some general properties develop as to the distribution forms.

On the other hand, it may be that the photosphere stands as a unique place in the solar atmosphere. It is a driver for much that occurs throughout the convection zone. We know that the interior of a star cannot be calculated without the “outer boundary condition.” Modeled stellar surface conditions provide key parameters for stellar models of their deep interiors. Without knowledge of the radiative aspects of the photosphere, the deep interior cannot be constructed; the upper boundary conditions are vital to the whole solution. Even for the Earth’s atmosphere, the outer boundary conditions are vital. This is important for stars as it is for the Earth’s atmosphere.

With regard to the Sun, Brandenburg et al. [54] discuss theoretical aspects of shallow dynamos, in particular, how turbulent pressure can drive magnetic field to become concentrated near the Sun’s surface. If this is so, then since the field is a major player on the Sun’s surface for driving inhomogeneities, then the surface may play a role in providing a source for variations from average behavior deeper inside the Sun. Our model, however, does not distinguish depth, and it appears to shed no light on fascinating questions, such as “What happens, and how does field behave, below the photosphere?” For the present model, the interior of the Sun is a black box.

Alternatively, this model appears to shed light concerning large-scale photospheric field motions. We recognize that the surface of the Sun is more complicated than counting apples. So let us end by saying that we hope that the current simplified model may be improved, and that at present, we are encouraged with the model’s preliminary behaviors. We have not finely tuned the model to agree closely with the Sun, and surely this could improve its workings; however, as we stated, we do not yet know totally how to equate terrestrial time with model “tick” time, and thus this would be a great starting place.

We see our Solar Field Mapping Model as an opportunity to allow solar knowledge to grow, and try to make a “testable” model, as opposed to one separated from reality. Our early work in Source Surface theory [22], where we later published the predictions [55–57] of our model prior to the appearance of the corona at the times of solar eclipses, was to “test” the model with subsequent observations. Thus we must strive to make our theories “testable”, not just provide bald theories as if they lived on a higher plane, above comparisons with reality. If the scientific fields were to take this point of view, we would still be living in the geocentric Universe, and the Sun would be a perfect sphere in the sky. So, it is only against the hard realities of observations that we can test our understanding against reality. Of course, we see cellular automata as a way of handling the complex nonlinear behaviors that the Sun displays, and hope that these methods may make a foothold and serve to improve more detailed solar models.

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References


http://gong.nso.edu/info/press/GONG_magfield pri/.


hale sector boundaries,” Astrophysical Journal Letters, vol. 733,
no. 1, article no. 49, 2011.

an explanation of 1/f noise,” Physical Review Letters, vol. 59,

[53] A. Fey, R. Meester, and F. Redig, “Stabilizability and per-
culation in the infinite volume sandpile model,” Annals of

[54] A. Brandenburg, D. Sokoloff, and K. Subramanian, “Current
status of turbulent dynamo theory: From large-scale to small-

[55] K. H. Schatten, “Prediction of the coronal structure for the

[56] K. H. Schatten, “Prediction of the coronal structure for the
1970.

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