Research Article

Latitudinal Dependence of Cosmic Rays Modulation at 1 AU and Interplanetary Magnetic Field Polar Correction

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The cosmic rays differential intensity inside the heliosphere, for energy below 30 GeV/nuc, depends on solar activity and interplanetary magnetic field polarity. This variation, termed solar modulation, is described using a 2D (radius and colatitude) Monte Carlo approach for solving the Parker transport equation that includes diffusion, convection, magnetic drift, and adiabatic energy loss. Since the whole transport is strongly related to the interplanetary magnetic field (IMF) structure, a better understanding of his description is needed in order to reproduce the cosmic rays intensity at the Earth, as well as outside the ecliptic plane. In this work an interplanetary magnetic field model including the standard description on ecliptic region and a polar correction is presented. This treatment of the IMF, implemented in the HelMod Monte Carlo code (version 2.0), was used to determine the effects on the differential intensity of Proton at 1 AU and allowed one to investigate how latitudinal gradients of proton intensities, observed in the inner heliosphere with the Ulysses spacecraft during 1995, can be affected by the modification of the IMF in the polar regions.

1. Introduction

The Solar Modulation, due to the solar activity, affects the Local Interstellar Spectrum (LIS) of Galactic Cosmic Rays (GCR) typically at energies lower than 30 GeV/nuc. This process, described by means of the Parker equation (e.g., see [1, 2] and Chapter 4 of [3]), is originated from the interaction of GCRs with the interplanetary magnetic field (IMF) and its irregularities. The IMF is the magnetic field that is carried outwards during the solar wind expansion. The interplanetary conditions vary as a function of the solar cycle which approximately lasts eleven years. In a solar cycle, when the maximum activity occurs, the IMF reverse his polarity. Thus, similar solar polarity conditions are found almost every 22 years [4]. In the HelMod Monte Carlo code version 1.5 (e.g., see [2]), the “classical” description of IMF, as proposed by Parker [5], was implemented together with the polar corrections of the solar magnetic field suggested subsequently in [6, 7]. This IMF was used inside the HelMod [2] code to investigate the solar modulation observed at Earth and to partially account for GCR latitudinal gradients, that is, those observed with the Ulysses spacecraft [8, 9]. In order to fully account for both the latitudinal gradients and latitudinal position of the proton-intensity minimum observed during the Ulysses fast scan in 1995, the HelMod Code was updated to the version 2.0 to include a new treatment of the parallel and diffusion coefficients following that one described in [10]. In the present formulation, the parallel component of the diffusion tensor depends only on the radial distance from the Sun, while it is independent of solar latitude.
2. The Interplanetary Magnetic Field

Nowadays, we know that there is a Solar Wind plasma (SW) that permeates the interplanetary space and constitutes the interplanetary medium. In IMF models the magnetic-field lines are supposed to be embedded in the nonrelativistic streaming particles of the SW, which carries the field with them into interplanetary space, producing the large scale structure of the IMF and the heliosphere. The “classical” description of the IMF was proposed originally by Parker (e.g., see [2, 5, 11–14] and Chapter 4 of [3]). He assumed (i) a constant solar rotation with angular velocity \(\omega\), (ii) a simple spherically symmetric emission of the SW, and (iii) a constant (or approaching an almost constant) SW speed \(V_{sw}\) at larger radial distances \(r\), for example, for \(r > r_b = 10R_\odot\) (where \(R_\odot\) is the Solar radius), since beyond \(r_b\) the wind speed varies slowly with the distance. The “classical” IMF can be analytically expressed as [15]

\[
B_{Par} = \frac{A}{r^2} \left( e_r - \Gamma e_\varphi \right) \left[ 1 - 2H \left( \theta - \theta' \right) \right],
\]

where \(A\) is a coefficient that determines the IMF polarity and allows \(B_{Par}\) to be equal to \(B_0\), that is, the value of the IMF at Earth’s orbit as extracted from NASA/GSFC’s OMNI data set through OMNIWeb [16, 17]; \(e_r\) and \(e_\varphi\) are unit vector components in the radial and azimuthal directions, respectively, \(\theta\) is the colatitude (polar angle); \(\theta'\) is the polar angle determining the position of the Heliospheric Current Sheet (HCS) [18]; \(H\) is the Heaviside function; thus, \([1 - 2H \left( \theta - \theta' \right)]\) allows \(B_{Par}\) to change sign in the two regions above and below the HCS [18] of the heliosphere; finally,

\[
\Gamma = \tan \Psi = \frac{\omega (r - r_b) \sin \theta}{V_{sw}},
\]

with \(\Psi\) the spiral angle. In the present model \(\omega\) is assumed to be independent of the heliographic latitude and equal to the sidereal rotation at the Sun’s equator. The magnitude of Parker field is thus

\[
B_{Par} = \frac{A}{r^2} \sqrt{1 + \Gamma^2}.
\]

In 1989 [6], Jokipii and Kota have argued that the solar surface, where the feet of the field lines lie, is not a smooth surface, but a granular turbulent surface that keeps changing with time, especially in the polar regions. This turbulence may cause the footpoints of the polar field lines to wander randomly, creating transverse components in the field, thus causing temporal deviations from the smooth Parker geometry. The net effect of this is a highly irregular and compressed field line. In other words, the magnitude of the mean magnetic field at the poles is greater than in the case of the smooth magnetic field of a pure Parker spiral. Jokipii and Kota [6] have therefore suggested that the Parker spiral field may be generalized by the introduction of a perturbation parameter \([\delta(\theta)]\) which amplifies the field strength at large radial distances. With this modification the magnitude of IMF, (3), becomes [6]

\[
B_{Pol} = \frac{A}{r^2} \sqrt{1 + \Gamma^2 + \left( \frac{r}{r_b} \right)^2 \delta(\theta)^2}.
\]

The difference of the IMF obtained from (3) and (4) is less than \(-1\)% for colatitudes \(20^\circ \leq \theta \leq 160^\circ\) (e.g., see Figure 1(a)) and increases for colatitudes approaching the polar regions (e.g., see Figure 2 of [2]).

In the present treatment, the heliosphere is divided into polar regions and a equatorial region where different description of IMF are applied. In the equatorial region the Parker’s IMF, Equation (3) is used, while in the polar regions we used a modified IMF that allows a magnitude as in (4)

\[
B_{Pol} = \frac{A}{r^2} \left[ e_r + \frac{r}{r_b} \delta(\theta) e_\varphi \frac{\omega (r - r_b) \sin \theta}{V_{sw}} e_\varphi \right] \times [1 - 2H \left( \theta - \theta' \right)] \quad \text{Polar regions},
\]

\[
\cdot \frac{A}{r^2} \left[ e_r - \frac{\omega (r - r_b) \sin \theta}{V_{sw}} e_\varphi \right] \times [1 - 2H \left( \theta - \theta' \right)] \quad \text{Equatorial region},
\]

where equatorial regions are those with colatitude \(X' \leq \theta \leq (180^\circ - X')\). The symbol \(\theta_X\) indicates the corresponding polar regions.

In order to have a divergence-free magnetic-field we require that the perturbation factor \([\delta(\theta)]\) has to be

\[
\delta(\theta) = \frac{\delta_m}{\left[1 - 2H \left( \theta - \theta' \right) \right] \sin \theta},
\]

where \(\delta_m\) is the minimum perturbation factor of the field. The perturbation parameter is left to grow with decreasing of the colatitude. However, in their original work, Jokipi and Kota [6] estimated the value of the parameter \(\delta\) between \(10^{-3}\) and \(3 \times 10^{-3}\).

Since the polar field is only a perturbation of the Parker field, it is a reasonable assumption that stream lines of the magnetic field do not cross the equatorial plane, thus, remaining completely contained in the solar hemisphere of injection. This allows one to estimate an upper limit on the possible values of \(\delta_m\) (see, Figure 1(b)). Currently, we use \(\delta_m = 1 \times 10^{-5}\) by comparing simulations with observations at Earth orbit during Solar Cycle 23 (see, Section 5).

3. The Propagation Model

Parker in 1965 [1] (see also, [2], Chapter 4 of [3] and references therein) treated the propagation of GCRs through the interplanetary space. He accounted for the so-called adiabatic energy losses, outward convection due to the SW and drift
effects. In the heliocentric system the Parker equation is then expressed (e.g., see [2, 19]):

\[
\frac{\partial U}{\partial t} = \frac{\partial}{\partial x_i} \left( K^S_{ij} \frac{\partial U}{\partial x_j} \right) - \frac{\partial}{\partial x_i} \left[ (V_{sw,i} + v_{d,i}) U \right] + \frac{1}{3} \frac{\partial V_{sw,i}}{\partial x_i} \frac{\partial}{\partial t} (\alpha_{rel} T U),
\]

(7)

where \(U\) is the number density of particles per unit of particle kinetic energy \(T\), at the time \(t\). \(V_{sw,i}\) is the solar wind velocity along the axis \(x_i\) \([20]\). \(K^S\) is the symmetric part of diffusion tensor \([1]\), \(v_{d,i}\) is the drift velocity that takes into account the drift of the particles due to the large scale structure of the magnetic field \([21–23]\), and, finally,

\[
\alpha_{rel} = \frac{T + 2m_e c^2}{T + m_e c^2},
\]

(8)

where \(m_e\) is the rest mass of the GCR particle. The last term of (7) accounts for adiabatic energy losses \([1, 24]\). The number density \(U\) is related to the differential intensity \(J\) as \([2, 25]\), Chapter 4 of [3] and references therein:

\[
J = \frac{\nu U}{4\pi}
\]

(9)

where \(\nu\) is the speed of the GCR particle.

Equation (7) was solved using the HelMod code (see the discussion in [2]). This treatment (i) follows that introduced in \([10, 26–30]\) and (ii) determines the differential intensity of GCRs using a set of approximated stochastic differential equations (SDEs) which provides a solution equivalent to that from (7). The equivalence between the Parker equation, that is a Fokker-Planck type equation, and the SDEs is demonstrated in \([28, 31]\). In the present work, we use a 2D (radius and colatitude) approximation for the particle transport. The model includes the effects of solar activity during the propagation from the effective boundary of the heliosphere down to Earth's position.

The set of SDEs for the 2D approximation of (7) in heliocentric spherical coordinates is

\[
\Delta r = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 K^S_{rr} \right) \Delta t - \frac{1}{r} \frac{\partial}{\partial \mu} \left( K^S_{\mu r} \sqrt{1 - \mu^2 (\theta)} \right) \Delta t
\]

\[
+ (V_{sw} + v_{d,r}) \Delta t + \left( 2K^S_{rr} \right)^{1/2} \omega_r \sqrt{\Delta t},
\]

(10a)

\[
\Delta \mu (\theta) = -\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r K^S_{\mu r} \sqrt{1 - \mu^2 (\theta)} \right] \Delta t
\]

\[
+ \frac{\partial}{\partial \mu} \left( K^S_{\mu r} \frac{1 - \mu^2 (\theta)}{r^2} \right) \Delta t
\]

\[
- \frac{1}{r} \frac{\partial}{\partial \mu} \left( 1 - \mu^2 (\theta) \right) \Delta t
\]

\[
- \frac{2K^S_{rr}}{r} \left[ 1 - \mu^2 (\theta) \right]^{1/2} \omega_r \sqrt{\Delta t}
\]

\[
+ \frac{1}{r} \left[ 1 - \mu^2 (\theta) \right] \left( K^S_{r r} K^S_{\mu r} \right) \left( \frac{K^S_{rr}}{0.5K^S_{rr}} \right)^{1/2} \omega_\mu \sqrt{\Delta t},
\]

(10b)

\[
\Delta T = -\frac{\alpha_{rel} T}{3r^2} \frac{\partial V_{sw}^2}{\partial r} \Delta t,
\]

(10c)
where $\mu(\theta) = \cos \theta$ and $\omega_i$ is a random number following a Gaussian distribution with a mean of zero and a standard deviation of one. The procedure for determining the SDEs can be found in [2].

In a coordinate system with one axis parallel to the average magnetic field and the other two perpendicular to this the symmetric part of the diffusion tensor $K_{ij}^{S}$ is (see e.g., [32]):

$$K_{ij}^{S} = \begin{bmatrix} K_{||} & 0 & 0 \\ 0 & K_{L,\perp} & 0 \\ 0 & 0 & K_{L,\beta} \end{bmatrix},$$

with $K_{||}$ the diffusion coefficient describing the diffusion parallel to the average magnetic field and $K_{L,\perp}$ and $K_{L,\beta}$ are the diffusion coefficients describing the diffusion perpendicular to the average magnetic field in the radial and polar directions, respectively. In this work $K_{||}$ is that one proposed by Strauss and collaborators in [10] (see also [33–35]):

$$K_{||} = \frac{\beta K_0}{3} \frac{P}{1 \text{GV}} \left(1 + \frac{r}{1 \text{AU}}\right),$$

where $K_0$ is the diffusion parameter—described in Section 2.1 of [2]—which depends on solar activity and polarity, $\beta$ is the particle speed in unit of speed of light, $P = pc/[Z]e$ and $r$ is the radial distance from the Sun.
is the particle rigidity expressed in GV, and, finally, $r$ is the heliocentric distance from the Sun in AU.

In the current treatment, $K_\|_\theta$ has a radial dependence proportional to $r$, but no latitudinal dependence. Mc Donald and collaborators (see, [36]) remarked that (i) a spatial dependence of $K_\|_\theta$—like the one proposed here—can affect the latitudinal gradients at high latitude and (ii) it is consistent with that originally suggested in [6]. Furthermore, the perpendicular diffusion coefficient is taken to be proportional to $K_\|_\theta$ with a ratio $K_\perp/K_\|=0.13$ for both $r$ and $\theta$-coordinates. The latter value is discussed in Section 5.

In addition, the practical relationship between $K_\theta$ and monthly Smoothed Sunspot Numbers (SSN) [37] values—discussed in Section 2.1 of [2]—is currently updated using the most recent data from [38]. As in [2], the $K_\theta$ data are subdivided into four sets, that is, ascending and descending
Table 1: Definition of ascending and descending periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>Years</th>
</tr>
</thead>
</table>

phases for both negative and positive solar magnetic-field polarities (Table 1). It has to be remarked that after each maximum the sign of the magnetic field (i.e., the A parameter in (5)) is reversed. The updated practical relationships between $K_0$ in AU$^2$ GV$^{-1}$ s$^{-1}$ and SSN values for $1.4 \leq$ SSN $\leq 165$ for the four periods (from I up to IV, listed in Table 1) are

(I) $K_0 = 0.000297 - 2.9 \cdot 10^{-6}$SSN + $8.1 \cdot 10^{-9}$SSN$^2$

+ $1.46 \cdot 10^{-10}$SSN$^3 - 8.4 \cdot 10^{-13}$SSN$^4$,

(13a)

(II) $K_0 = \begin{cases} 0.000304846 & \text{if SSN} \leq 20, \\ 0.00195 & \text{if SSN} > 20, \\ -5.8 \cdot 10^{-6}$SSN, \\ -2.3 \cdot 10^{-10}$SSN$^2 + 9.1 \cdot 10^{-5}, \end{cases}$

(13b)

(III) $K_0 = 0.0002391 - 8.453 \cdot 10^{-7}$SSN

(13c)

(IV) $K_0 = 0.000247 - 1.175 \cdot 10^{-6}$SSN.

(13d)

The rms (root mean square) values of the percentage difference between values obtained with (13a), (13b), (13c), and (13d) from those determined using the procedure discussed in Section 2.1 of [2] applied to the data from [38] were found to be 6.0%, 10.1%, 7.0%, and 13.2% for the period I (ascending phase with $A < 0$), II (descending phase with $A < 0$), III (ascending phase with $A > 0$), and IV (descending phase with $A > 0$), respectively.

4. The Magnetic Field in the Polar Regions

Section 2 describes an IMF following the Parker Field with a small region around the poles in which such a field is modified. As already mentioned (see e.g., [6, 39–43]), the correction is needed to better reproduce the complexity of the magnetic field in those regions.

Moreover, Ulysses spacecraft (see e.g., [44–46]) explored the heliosphere outside the ecliptic plane up to ±80° of solar latitude at a solar distance from ~1 up to ~5 AU. Using these observations, the presence of latitudinal gradient in the proton intensity could be determined (e.g., see Figure 2 of [9] and Figure 5 of [8]). The data collected during the latitudinal fast scan (from September 1994 up to August 1995) show (a) a nearly symmetric latitudinal gradient with the minimum near ecliptic plane, (b) a southward shift of the minimum, and (c) an intensity in the North polar region at 80° exceeding the South polar intensity. In [9] a latitudinal gradient of ~0.3%/degree for proton with kinetic energy $>0.1$ GeV was estimated. While in [8] the analysis to higher energy was extended estimating a gradient of ~0.22%/degree for proton with kinetic energy $>2$ GeV. The minimum in the charged particle intensity separating the two hemispheres of the heliosphere occurs ~10° South of the heliographic equator [9]. In addition, an independent analysis that takes into account the latitudinal motion of the Earth and IMP8 confirms a significant (~8° ± 2°) southward offset of the intensity minimum [9] for $T > 100$ MeV proton. Furthermore in [8], a southward offset of about $\approx 7°$ is evaluated; this offset of the intensity minimum results to be independent of the particle energy up to 2 GeV. Finally, in [9], the intensity in the North polar region at 80 is observed to exceed the South polar intensity of ~6% for protons with $T > 100$ MeV.

Using the present HelMod code (version 2.0), we could investigate (i) the latitudinal gradient of GCR intensities resulting from solar modulation and (ii) how the magnetic-field structure of the polar regions, as defined in Section 2, is able to influence the GCR spectra on the ecliptic plane. As previously defined, we denote with $\theta_{X}$ a polar region of amplitude $X$ from polar axis, that is, $0 < \theta < X$ and $0 > 180° - X$. Three regions with $X = 20°, 25°$, and $30°$ were investigated. Outside any of these regions, the ratio between $B_{pol}$ and $B_{par}$ in (5) is less than ~1% (see Figure 1) and, thus, it ensures a smooth transition between polar and equatorial regions. For the purpose of this study we consider an energy binning closer to those presented in [8]. The KET instrument [47] collects proton data in three “channels” one with energies ranging from 0.038 GeV up to 2.0 GeV and two for particle with kinetic energy $T > 0.1$ GeV and $T > 2$ GeV, respectively. A successive reanalysis of the collected data allowed the authors to subdivide the 0.25–2 GeV “channel” in three “subchannels” of intermediate energies. Since the Present Model is optimized—as discussed in [2]—for particles with rigidity greater than 1 GV (i.e., $\approx 0.44$ GeV), the present results are compared only with the corresponding “channel” or “subchannels” suited for the corresponding energy range (see Table 2).

At 1 AU and as a function of the solar colatitude, the GCR intensities for protons are shown in Figure 2. For a comparison with Ulysses observations, the modulated intensities of protons—resulting from HelMod code—were investigated from 80° (North) and down to ~80° (South). They were
obtained using the HelMod code and selected using the energy bins reported in Table 2. In Figure 2, the latitudinal intensity distribution is normalized to the corresponding South Pole intensity. The quoted errors include statistical and systematic errors. The distributions were interpolated using a parabolic function expressed as

$$I(\theta_{\text{lat}}) = a + c(\theta_{\text{lat}} + d)^2,$$  \hspace{1cm} (14)

where $I(\theta_{\text{lat}})$ is the normalized intensity, $\theta_{\text{lat}}$ is the latitudinal angle (the latitudinal angle is $\theta_{\text{lat}} = 90^\circ - \theta$), and $a$, $b$, and $c$ are parameters determined from the fitting procedure. The so obtained fitted curves are shown as continuous lines in Figure 2. Furthermore, the latitudinal positions of minimum intensity ($\theta_{\text{lat, min}}$), percentages of North-South asymmetry of intensities ($\Delta_{N,S}$), and differences in percentage between the maximum and minimum intensities ($\Delta_{\text{max}}$) were also determined from the fitting procedure and are listed in Tables 3, 4, and 5, respectively. The quoted errors—following a procedure discussed in [2]—are derived varying the fitted parameters two times larger than the one resulting from the best fit ($I_{\text{best}}$). $\eta_{\text{rms, lat}}$ is defined as

$$\eta_{\text{rms, lat}} = \sqrt[2]{\frac{\sum_i (\eta_{i,\text{lat}}/\sigma_{\eta_i,\text{lat}})^2}{\sum_i 1/\sigma_{\eta_i,\text{lat}}^2}},$$  \hspace{1cm} (15)

with

$$\eta_{i,\text{lat}} = \frac{I(\theta_i) - I_{\text{best}}(\theta_i)}{I_{\text{best}}(\theta_i)},$$  \hspace{1cm} (16)

where $\theta_i$ is the central value of the $i$th latitudinal bin of the differential intensity distribution and $\sigma_{\eta_i,\text{lat}}$ are the errors including the experimental and Monte Carlo uncertainties. For $\theta > 30^\circ$, that is, assuming a modified polar magnetic-field for $\theta < 30^\circ$ and $\theta > 150^\circ$, we found a general agreement with Ulysses observations. The position of $\theta_{\text{lat, min}}$ is compatible within the errors with one observed in [8], as well as the values of $\Delta_{N,S}$ and $\Delta_{\text{max}}$.

The HelMod code allows one to investigate the relevance of the treatment of the polar region magnetic-field with respect to the resulting modulation of GCR. Thus, the latitudinal normalized intensities were obtained excluding a few (small) regions nearby the poles. This was determined from reducing the latitudinal spatial phase-space admissible for pseudo-particles (see [2]), that is, the latitudinal extension of GCR particles taken into account—to $1^\circ < \theta < 179^\circ$ ($\theta_{R,1}$), $2^\circ < \theta < 178^\circ$ ($\theta_{R,2}$) and $10^\circ < \theta < 170^\circ$ ($\theta_{R,10}$). So obtained latitudinal gradients are compared with the full latitudinal extension, $\theta_{R,0}$ ($0^\circ < \theta < 180^\circ$), in Figure 3. By an inspection of Figure 3, one may lead to the conclusion that the GCR diffusion nearby the polar axis has a large impact on the latitudinal gradients in the inner heliosphere. As a consequence, the IMF description in the polar regions is relevant in order to reproduce the observed modulated GCR spectra.

### Table 3: Latitudinal positions of minimum proton intensity ($\theta_{\text{lat, min}}$) (in degrees) as a function of the kinetic energy, using three values for the extension of the polar regions.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>$\theta_{20^\circ}$</th>
<th>$\theta_{50^\circ}$</th>
<th>$\theta_{10^\circ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35–0.98 GeV</td>
<td>$-5.01$</td>
<td>$-4.94$</td>
<td>$-4.93$</td>
</tr>
<tr>
<td>0.76–2.09 GeV</td>
<td>$-5.16$</td>
<td>$-5.05$</td>
<td>$-10.04$</td>
</tr>
<tr>
<td>2.09–200 GeV</td>
<td>$-3.67$</td>
<td>$-7.74$</td>
<td>$-9.6$</td>
</tr>
</tbody>
</table>

### Table 4: Percentages of North-South asymmetry of proton intensities ($\Delta_{N,S}$) as a function of the kinetic energy, using three values for the extension of the polar regions.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>$\theta_{20^\circ}$</th>
<th>$\theta_{50^\circ}$</th>
<th>$\theta_{10^\circ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35–0.98 GeV</td>
<td>$-11.3%$</td>
<td>$-13.4%$</td>
<td>$-13.4%$</td>
</tr>
<tr>
<td>0.76–2.09 GeV</td>
<td>$-6.5%$</td>
<td>$-9.3%$</td>
<td>$-9.3%$</td>
</tr>
<tr>
<td>2.09–200 GeV</td>
<td>$-3.3%$</td>
<td>$-4.2%$</td>
<td>$-4.2%$</td>
</tr>
</tbody>
</table>

### Table 5: Differences in percentage between the maximum and minimum proton intensities ($\Delta_{\text{max}}$) as a function of the kinetic energy, using three values for the extension of the polar regions.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>$\theta_{20^\circ}$</th>
<th>$\theta_{50^\circ}$</th>
<th>$\theta_{10^\circ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35–0.98 GeV</td>
<td>$-35.4%$</td>
<td>$-36.4%$</td>
<td>$-36.4%$</td>
</tr>
<tr>
<td>0.76–2.09 GeV</td>
<td>$-23.5%$</td>
<td>$-25.3%$</td>
<td>$-25.3%$</td>
</tr>
<tr>
<td>2.09–200 GeV</td>
<td>$-9.3%$</td>
<td>$-10.5%$</td>
<td>$-10.5%$</td>
</tr>
</tbody>
</table>

### 5. Comparison with Observations during Solar Cycle 23

The agreement of HelMod simulated spectra with observations during solar cycle 23 is investigated via quantitative comparisons using (17) and (18). However, since the structure of the heliosphere is different in high and low solar activity the two periods are separately analyzed.

The HelMod Code [2] (version 2.0) allowed us to investigate how the modulated (simulated) differential intensities are affected by the (1) particle drift effect, (2) polar enhancement of the diffusion tensor along the polar direction ($K_{\parallel, \theta}$, e.g., see [48]), and, finally, (3) values of the tilt angle ($\alpha_r$) calculated following the approach of the “R” and “L” models [49]. This analysis also allows us to estimate the values of IMF parameters that better describe the modulation along the entire solar cycle. The effects related to particle drift were investigated via the suppression of the drift velocity (No Drift approximation), this accounts for the hypothesis that magnetic drift convection is almost completely suppressed during solar maxima. The differential intensities were calculated for $K_{\perp, \mu} = \rho E K_{\perp, \mu}$ with values of $\rho E$ of 1, 8, and 10, that is, no enhancement, that suggested in [50] and that suggested in [2, 48] (and reference there in), respectively. Furthermore, the modulated proton spectra were derived from a LIS whose normalization constant depends on the experimental set of data and were already discussed in [2]. In addition, the differential intensities were calculated accounting for...
particles inside heliospheric regions where solar latitudes are lower than $|5.7^\circ|$.

During the period of high solar activity for the solar cycle 23, the BESS collaboration took data in the years 1999, 2000, and 2002 (see sets of data in [51]). For period not dominated by high solar activity in solar cycle 23, BESS, AMS, and PAMELA collaborations took data, that is, BESS-1997 [51], AMS-1998 [52], and PAMELA-2006/08 [53].

Following the procedure described in [2], the observation data were compared with those obtained from HelMod code using the error-weighted root mean square ($\eta_{\text{rms}}$) of the relative difference ($\eta$) between experimental data ($f_{\text{exp}}$) and those resulting from simulated differential intensities ($f_{\text{sim}}$). For each set of experimental data and with the approximations and/or models described above, we determined the quantity

$$\eta_{\text{rms}} = \sqrt{\frac{\sum (\eta_i/\sigma_{\eta,i})^2}{\sum 1/\sigma_{\eta,i}^2}},$$  

where $T_i$ is the average energy of the $i$th energy bin of the differential intensity distribution and $\sigma_{\eta,i}$ are the errors including the experimental and Monte Carlo uncertainties; the latter account for the Poisson error of each energy bin. The simulated differential intensities are interpolated with a cubic spline function. The modulation results are studied varying the parameters $\delta_m$—from 0, that is, the nonmodified Parker IMF, up to $3 \times 10^{-5}$, see Figure 1(b)—, $K_{\perp}/K_{\parallel} = \rho_k$ (from 0.10 up to 0.14) and $K_{\perp}/K_{\perp} = \rho_k$ (from 1 up to 10) seeking a set of parameters set that minimize $\eta_{\text{rms}}$. In Table 6, the average values of $\eta_{\text{rms}}$ (in percentage, %) for low solar activity periods are listed. They were obtained in the energy range (Above 30 GeV, the differential intensity is marginally (if at all) affected by modulation.) from 444 MeV up to 30 GeV using the “L” and “R” models for the tilt angle $\alpha_t$ and for the No Drift approximation and without any enhancement of the diffusion tensor along the polar direction ($K_{\perp}$). The results derived with the enhancement of the diffusion tensor along the polar direction indicate that for $\rho_k = 8$ and 10 one obtains a value of $\eta_{\text{rms}}$ that is from 1.5 up to 3 times larger with respect the case without enhancement (For a comparison, the scalar approximation presented in [2], that is, assuming that the diffusion propagation is independent of magnetic structure, leads to and average $\eta_{\text{rms}}$ of $\sim 15\%$). From inspection of Table 6, one can remark that the drift mechanism leads to a better agreement with experimental data. Furthermore the “R” and “L” models for tilt angles are comparable within the precision of the method (discussed in [2]). The minimum difference with respect to the experimental data occurs when $\rho_k = 0.11–0.13$ and $\delta_m = 1.0 \times 10^{-5}$ for both “R” and “L” models, with the “L” model slightly preferred to “R”.

In Figures 4, 5, and 6, the differential intensities determined with the HelMod code are shown and compared to the experimental data of BESS-1997, AMS-1998, and PAMELA-2006/08, respectively; in these figures, the dashed lines are the LIS as discussed in [2]. These modulated intensities are the ones calculated for a heliospheric region where solar latitudes
are lower than \(5.7^\circ\), using \(\rho_k = 0.13\), \(\delta_m = 1.0 \times 10^{-5}\) and \(\rho_E = 1\) with the “L” model. Finally, one can remark that the present code, combining diffusion and drift mechanisms, is also suited to describe the modulation effect in periods when the solar activity is no longer at the maximum.

In Table 7 we present the averages \(\eta_{\text{rms}}\) (in percentage, %) during the periods dominated by high solar activity. The simulated differential intensities were obtained for a heliospheric region where solar latitudes are lower than \(5.7^\circ\) without any enhancement of the diffusion tensor along the polar direction \((K_{\perp,\mu})\). The simulations with \(\rho_E = 8\) and 10 lead to \(\eta_{\text{rms}}\) comparable with those presented in Table 7. However, using \(\rho_E = 1\) provides a better agreement to experimental data at lower energy. From inspection of Table 7, one can note that “R” and “L” models for tilt angles yield comparable results within the precision of the method. Furthermore the minimum difference with the experimental data occurs when \(\rho_k = 0.10\) and \(\delta_m = (2.0 - 3.0) \times 10^{-5}\) with the “L” model slightly preferred to “R”. The HelMod parameter configuration, which minimizes the difference to the experimental data, are reported in Table 8: one may remark that the No Drift approximation is (almost) comparable to a drift treatment for both BESS-2000 and BESS-2002 with data collected during and after the maximum of the solar activity. Apparently, the drift treatment is needed in order to describe BESS-1999 with data taken during a period approaching the solar maximum.

In Figures 7, 8, and 9, the differential intensities determined with the HelMod code are shown and compared with the experimental data of BESS-1999, BESS-2000, and BESS-2002, respectively; in these figures, the dashed lines are the LIS as discussed in [2]. These modulated intensities are the ones calculated for a heliospheric region where solar latitudes are lower than \(5.7^\circ\), using \(K_{\perp,\mu} = K_{\perp,r}\) independently of the

![Figure 6: Proton differential intensity determined with the HelMod code (continuous line) compared to the experimental data of PAMELA 2006/08; the dashed line is the LIS (see the text).](image)

![Figure 7: Proton differential intensity determined with the HelMod code (continuous line) compared to the experimental data of BESS-1999; the dashed line is the LIS (see the text).](image)

### Table 6: Average values (last three columns) of \(\eta_{\text{rms}}\) (in percentage, %) as a function of \(\delta_m (\times 10^{-5})\) and \(\rho_k\) for BESS-1997, AMS-1998, PAMELA-2006/08, obtained from (17) without enhancement of the diffusion tensor along the polar direction \((\rho_E = 1)\), using “R” and “L” models for the tilt angle and No Drift approximation.

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<th>(\delta_m (\times 10^{-5}))</th>
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<th>“L” model</th>
<th>No Drift</th>
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Table 7: Average values (last three columns) of $\eta_{\rm rms}$ (in percentage, %) as a function of $\delta_m \times 10^{-5}$ and $\rho_k$, for BESS-1999, BESS-2000, BESS-2002 obtained from (17) without enhancement of the diffusion tensor along the polar direction ($\rho_E = 1$), using “R” and “L” models for the tilt angle and No Drift approximation. The differential intensities were calculated accounting for particles inside the heliospheric regions for which solar latitudes are lower than $|5.7^\circ|$.

<table>
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<th>$\delta_m (\times 10^{-5})$</th>
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is suited to describe the modulation effect in periods with high solar activity [2, 50, 54].

6. Conclusion

In this work an IMF, which combines the Parker Field and its polar modification, is presented. In the polar regions, the Parker IMF was modified with an additional latitudinal components according to those proposed by Jokipii and Kota in [6]. We found the maximum perturbed value with this component yielding, as a physical result, streaming lines completely confined in the solar hemisphere of injection.

The proposed IMF is, then, used within the HelMod Monte Carlo code to determine the effects on the differential intensity of protons at 1 AU as a function of the extension of polar region, in which the modified magnetic-field is employed. We found that a polar region contained within $30^\circ$ of colatitude is that one ensuring a very smooth transition to the equatorial region and allows to reproduce qualitatively
and quantitatively the latitudinal profile of the GCR intensity, and the latitudinal dip shift with respect to the ecliptic plane. Finally we determined how the polar region diffusion is mostly responsible of the proton intensity latitudinal gradient observed in the inner heliosphere with the Ulysses spacecraft during 1995.

Acknowledgments

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References

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