Research Article

Existence of Resonance Stability of Triangular Equilibrium Points in Circular Case of the Planar Elliptical Restricted Three-Body Problem under the Oblate and Radiating Primaries around the Binary System

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This paper analyzes the existence of resonance stability of the triangular equilibrium points of the planar elliptical restricted three-body problem when both the primaries are oblate spheroid as well as the source of radiation under the particular case, when $e = 0$. We have derived Hamiltonian function describing the motion of infinitesimal mass in the neighborhood of the triangular equilibrium solutions taken as a convergent series. Hamiltonian function for the system has been derived and also expanded in powers of the generalized components of momenta. We have used canonical transformation to make the Hamiltonian function independent of true anomaly. The most interesting and distinguishable results of this study are establishing the relation for determining the range of stability at and near the resonance $\omega_2 = 1/2$ around the binary system.

1. Introduction

The elliptical restricted three-body problem describes the dynamical system more accurately on account of realistic assumptions of the motion of the primaries that are subjected to move along the elliptical orbit. If the primaries move along elliptical orbits, this can have significant effects because the orbits of the most of planets and stars are elliptical rather than circular, so this problem is turned as elliptical restricted three-body problem. The celestial bodies as we know are not perfectly spheres. They are either oblate spheroid or triaxial rigid body. The lack of sphericity of the heavenly bodies causes large perturbations from a two-body orbit. This motivates our studies of stability of triangular equilibrium points under the resonance, when both of the primaries are oblate spheroid and source of radiation. The stability of infinitesimal around the equilibrium points of the elliptical restricted three bodies has been studied [1–20] and also the authors have investigated the different aspects of the same problem. The existence

of the libration points and their stability in the radiational elliptical restricted three-body problem has been investigated [14]. The stability of the motion of infinitesimal around the triangular equilibrium points is depending upon μ and e . The different aspects of the same problem in details have been investigated by [16, 21–23]. The existence of libration points and their stability in the three-body photogravitational elliptical restricted problem have been studied [11]. The analytical investigation concerning the structure of asymptotic perturbative approximation for small amplitude motions of the third-point mass in the neighborhood of a Lagrangian equilateral libration positions in the planar and elliptical restricted three bodies have been investigated [24]. After a sequence of canonical transformations, they formulated the Hamiltonian function governing the motion of the negligible mass body using the eccentric anomaly of the primaries elliptical Keplerian orbit as the independent variable. They studied the liberalized system of differential equations of motion obtained from expanding the Hamiltonian around

FIGURE 1: Location of triangular points for Achird.

FIGURE 3: Location of triangular points for α Cen-AB.

FIGURE 2: Location of triangular points for Luyten.

FIGURE 4: Location of triangular points for Kruger-60.

a Lagrangian solution. The approximated integrations of the elliptical restricted three-body problem by means of perturbation technique based on Lie series and development, which led to an approximated solution of the differential system of canonical equations of motion derived from the chosen Hamiltonian function, have been discussed [25].

The present study aims to investigate the condition of existence of resonance stability of infinitesimal body around the triangular equilibrium points in the elliptical restricted three-body problem, when both of the primaries are oblate spheroid and also the source of radiation. We have adopted the method due to Markeev [21], in which the Hamiltonian function pertaining to the problem is made independent of time using several canonical transformations. The existence of resonance and the stability of infinitesimal near the resonance frequency has been analyzed.

The stability of the triangular equilibrium points in the circular restricted three-body problem considering the bigger primary as an oblate spheroid in linear case, has been studied and established that range of the mass parameter given rise to stable triangular solutions is lowered [12]. The values of critical mass ratio have been obtained for the various values of the oblateness parameters in this paper.

The nonlinear stability of the triangular equilibrium point L_4 in the restricted problem has been studied under

the condition that the potential between the bodies is considered as the potential forces. In the first case they discussed the nonlinear stability of the same problem by taking into account of various conditions such that the bigger primary is an oblate spheroid and in the second case both the primaries are oblate spheroid, whose axes of symmetry is perpendicular to the plane of relative motion of the primaries. They also discussed the nonlinear stability of triangular equilibrium points in circular restricted three bodies as the primaries are spherical in shape and the bigger primary is a source of radiation [26].

The nonlinear stability zones of the triangular Lagrangian points are computed numerically, where the bigger primary is an oblate spheroid in the planar circular restricted three-body problem. It is found that the oblateness has a noticeable effect and this is identified to be related to the resonance case and the associated curves in the mass parameters μ versus oblateness coefficient A_1 configuration space and represented this measure in the parameter space. The location of triangular equilibrium points around the binary system is represented by Figures 1, 2, 3, 4, and 5.

The present paper described the effects of the oblateness and radiation of both the primaries on the existence of

FIGURE 5: Location of triangular points for Xi-Bootis.

resonance and the stability of the triangular equilibrium points of the planar elliptical restricted three-body problem in particular case when $e = 0$. We have adopted the method due to Markeev [21, 27] which made the Hamiltonian function independent of time by using several canonical transformations. The existence of resonance stability of the triangular equilibrium points near the resonance frequency has been analyzed using simulation technique.

2. Equation of Motion

The equation of motion of the planar elliptical restricted three-body problem under the photogravitational and oblateness of both the primaries in barycentric, pulsating, nondimensional coordinates is represented by [28]:

$$\begin{aligned} x'' - 2y' &= \phi \Omega_x; \\ y'' + 2x' &= \phi \Omega_y, \end{aligned} \quad (1)$$

where

$$\begin{aligned} \Omega &= \frac{x^2 + y^2}{2} + \frac{1}{1 + 3((A_1 + A_2)/2)} \\ &\times \left[\frac{(1 - \mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1 - \mu)q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} \right], \end{aligned} \quad (2)$$

where

$$r_1^2 = (x + \mu)^2 + y^2; \quad r_2^2 = (x - 1 + \mu)^2 + y^2 \quad (3)$$

and $\phi = 1/(1 + e \cos \nu)$.

Here, prime ($'$) denotes the differentiation with respect to the true anomaly and Ω_x and Ω_y denote the partial differentiation of Ω with respect to x and y , respectively, where A_1 and A_2 are the oblateness parameter of the primaries and q_1, q_2 are the source of radiation of both the primaries. There are two triangular equilibrium points in the plane of the finite bodies around the coordinates (x, y) and then the three bodies form nearly equilateral triangle. Since

the equilibrium points are symmetrical to each other, the nature of motion near the two triangular equilibrium points is the same

$$\begin{aligned} x_0 &= \frac{1}{2} - \mu + \frac{A_1}{2} - \frac{A_2}{2} + \frac{\epsilon^{(2)}}{3} - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} + \frac{1}{2}A_2\epsilon^{(2)}; \\ y_0 &= \pm \frac{\sqrt{3}}{2} \left[1 - \frac{A_1}{3} - \frac{A_2}{3} - \frac{2}{9}\epsilon^{(1)} \right. \\ &\quad \left. - \frac{2}{9}\epsilon^{(2)} - \frac{1}{3}A_1\epsilon^{(1)} - \frac{1}{3}A_2\epsilon^{(2)} \right]. \end{aligned} \quad (4)$$

Thus, the coordinate of the triangular equilibrium points has been obtained up to the first order terms in the parameters A_1, A_2, q_1 , and q_2 , which is represented by (4) referred from [22, 29].

The system (1) described the motion of dynamical system with Lagrangian which is represented as

$$\begin{aligned} L &= \frac{\dot{x}^2 + \dot{y}^2}{2} + (\dot{y}x - \dot{x}y) + \frac{1}{(1 + e \cos \nu)} \\ &\times \left[\frac{x^2 + y^2}{2} + \frac{1}{1 + 3((A_1 + A_2)/2)} \right. \\ &\quad \left. \times \left(\frac{(1 - \mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1 - \mu)q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} \right) \right] \end{aligned} \quad (5)$$

$$P_x = \frac{\partial L}{\partial \dot{x}} = \dot{x} - y; \quad P_y = \frac{\partial L}{\partial \dot{y}} = \dot{y} + x. \quad (6)$$

We constructed the expression for the Hamiltonian function of the problem using the formula

$$H = -L + \frac{\partial L}{\partial \dot{x}} \dot{x} + \frac{\partial L}{\partial \dot{y}} \dot{y}. \quad (7)$$

Hence, we have

$$H = -L + P_x \dot{x} + P_y \dot{y} = -L + (P_x^2 + P_y^2) + (P_x y - P_y x). \quad (8)$$

The perturbed Hamiltonian function of the given problem can be obtained using (5) and (8), which is reduced to the following form:

$$\begin{aligned} H &= \frac{(P_x^2 + P_y^2)}{2} + (P_x y - P_y x) + \frac{e \cos \nu}{(1 + e \cos \nu)} \left(\frac{x^2 + y^2}{2} \right) \\ &\quad - \frac{1}{(1 + e \cos \nu)(1 + 3(A_1 + A_2)/2)} \\ &\quad \times \left\{ \frac{(1 - \mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1 - \mu)q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3} \right\}. \end{aligned} \quad (9)$$

Since, the two triangular equilibrium points are symmetrical, the nature of the oscillations of infinitesimal near the two

points will be the same. We investigated the motion of infinitesimal near the equilibrium point. In order to discuss the motion of infinitesimal near the equilibrium point L_4 , shift the origin at L_4 by using the change of variables given as follows:

$$\begin{aligned} x &= \xi + q_1; & y &= \eta + q_2; \\ p_x &= p_\xi + p_1; & p_y &= p_\eta + p_2, \end{aligned} \quad (10)$$

where, the displacement of infinitesimal at and near the equilibrium point L_4 is represented as follows:

$$\begin{aligned} \xi &= \left[\frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \\ &\quad \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right]; \\ \eta &= \pm \frac{\sqrt{3}}{2} \left[1 - \frac{A_1}{3} - \frac{A_2}{3} - \frac{2}{9}\epsilon^{(1)} \right. \\ &\quad \left. - \frac{2}{9}\epsilon^{(2)} - \frac{1}{3}A_1\epsilon^{(1)} - \frac{1}{3}A_2\epsilon^{(2)} \right]; \\ p_\xi &= -\frac{\sqrt{3}}{2} \left[1 - \frac{A_1}{3} - \frac{A_2}{3} - \frac{2}{9}\epsilon^{(1)} \right. \\ &\quad \left. - \frac{2}{9}\epsilon^{(2)} - \frac{1}{3}A_1\epsilon^{(1)} - \frac{1}{3}A_2\epsilon^{(2)} \right]; \\ p_\eta &= \left[-\frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu \right. \\ &\quad \left. + \frac{\epsilon^{(2)}}{3} - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right]. \end{aligned} \quad (11)$$

The solution (11) in the new variables is given by the equilibrium position:

$$q_1 = q_2 = p_1 = p_2 = 0. \quad (12)$$

Expanding the Hamiltonian function (9) in the powers of p_1 and q_1 , we obtained

$$H = \sum_{K=0}^{\infty} H_K = H_0 + H_1 + H_2 + H_3 + H_4 + H_5 + \dots, \quad (13)$$

where $H_0 = H(\xi, \eta, p_\xi, p_\eta) = \text{constant}$ and $H_1 = 0$.

We evaluated $H_2, H_3, \dots$, using (10), and the terms of the Hamiltonian (9) are expanded one by one, the terms are

not depending upon p_i and q_i , and those of order one are neglected. Thus, we obtain

$$(i) \quad \frac{p_x^2 + p_y^2}{2} = \frac{1}{2} \left\{ (p_\xi + p_1)^2 + (p_\eta + p_2)^2 \right\} \quad (14)$$

$$= \frac{1}{2} (p_1^2 + p_2^2),$$

$$(ii) \quad (p_x y - p_y x) = (p_\xi + p_1)(\eta + q_2) - (p_\eta + p_2)(\xi + q_1) \quad (15)$$

$$= (p_1 q_2 - p_2 q_1),$$

$$(iii) \quad \frac{e \cos \nu}{2(1 + e \cos \nu)} (x^2 + y^2) = \frac{e \cos \nu}{2(1 + e \cos \nu)} \left\{ (\xi + q_1)^2 + (\eta + q_2)^2 \right\} \quad (16)$$

$$= \frac{e \cos \nu}{2(1 + e \cos \nu)} (q_1^2 + q_2^2),$$

$$(iv) \quad \frac{1}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}} \cdot \left\{ \frac{(1 - \mu)(1 - \epsilon^{(1)})}{r_1} + \frac{\mu(1 - \epsilon^{(2)})}{r_2} \right. \\ \left. + \frac{(1 - \mu)(1 - \epsilon^{(1)})A_1}{2r_1^3} + \frac{\mu(1 - \epsilon^{(2)})A_2}{2r_2^3} \right\}, \quad (17)$$

where $r_1^2 = (x + \mu)^2 + y^2 = (\xi + q_1 + \mu)^2 + (\eta + q_2)^2$,

$$\begin{aligned} r_1^2 &= \left(q_1 + \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu \right. \\ &\quad \left. + \frac{\epsilon^{(2)}}{3} - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 \\ &\quad + \left\{ q_2 + \frac{\sqrt{3}}{2} \left(1 - \frac{A_1}{3} - \frac{A_2}{3} - \frac{2}{9}\epsilon^{(1)} - \frac{2}{9}\epsilon^{(2)} \right. \right. \\ &\quad \left. \left. - \frac{1}{3}A_1\epsilon^{(1)} - \frac{1}{3}A_2\epsilon^{(2)} \right) \right\}^2, \end{aligned}$$

$$r_1^{-1} = \left[\left(q_1 + \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \right. \\ \left. \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 + \left(q_2 + \frac{\sqrt{3}}{2} \right)^2 \right]^{-1/2},$$

$$r_1^{-1} = f(q_1, q_2),$$

$$r_2^2 = (x - 1 + \mu)^2 + y^2 = [(\xi + q_1 - 1 + \mu)^2 + (\eta + q_2)^2],$$

$$\begin{aligned}
r_2^2 &= \left(q_1 - \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \\
&\quad \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 + \left(q_2 + \frac{\Lambda}{2} \right)^2, \\
r_2^{-1} &= \left[\left(q_1 - \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \right. \\
&\quad \left. \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 + \left(q_2 + \frac{\Lambda}{2} \right)^2 \right]^{-1/2}, \\
r_2^{-1} &= g(q_1, q_2), \\
r_1^{-3} &= \left[\left(q_1 + \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \right. \\
&\quad \left. \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 + \left\{ q_2 + \frac{\Lambda}{2} \right\}^2 \right]^{-3/2}, \\
r_1^{-3} &= \alpha(q_1, q_2), \\
r_2^{-3} &= \left[\left(q_1 - \frac{1}{2} + \frac{A_1}{2} - \frac{A_2}{2} - \mu + \frac{\epsilon^{(2)}}{3} \right. \right. \\
&\quad \left. \left. - \frac{\epsilon^{(1)}}{3} - \frac{1}{2}A_1\epsilon^{(1)} - \frac{1}{2}A_2\epsilon^{(2)} \right)^2 + \left(q_2 + \frac{\Lambda}{2} \right)^2 \right]^{-3/2}, \\
r_2^{-3} &= h(q_1, q_2),
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\frac{\Lambda}{2} &= \frac{\sqrt{3}}{2} \left[1 - \frac{A_1}{3} - \frac{A_2}{3} - \frac{2}{9}\epsilon^{(1)} \right. \\
&\quad \left. - \frac{2}{9}\epsilon^{(2)} - \frac{1}{3}A_1\epsilon^{(1)} - \frac{1}{3}A_2\epsilon^{(2)} \right].
\end{aligned} \tag{19}$$

3. Stability of Triangular Equilibrium Points of the Problem in Circular Case

We discussed the stability of infinitesimal under elliptical restricted three-body problem, when both of the primaries are oblate spheroid and the source of radiation under the particular case, when $e = 0$. We have constructed out a suitable Hamiltonian to test the stability of the perturbed system by substituting $e = 0$ in the expression (A.9) (see the appendix); we thus obtained the Hamiltonian for the circular problem up to the second order terms (see the appendix), which is represented as follows:

$$\begin{aligned}
H_2 &= \frac{p_1^2 + p_2^2}{2} + (P_1q_2 - P_2q_1) + e \cos \nu \frac{(q_1^2 + q_2^2)}{2(1 + e \cos \nu)} \\
&\quad - \frac{1}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}}
\end{aligned}$$

$$\begin{aligned}
&\times \left[\left(-\frac{1}{8} + \frac{9}{8}A_1 - \frac{9}{16}A_2 - \frac{73}{24}A_1\epsilon^{(1)} + \frac{11}{48}A_2\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. - \frac{27}{16}\mu A_1 + \frac{27}{16}\mu A_2 + \frac{17}{8}\mu\epsilon^{(1)} - \frac{7}{9}\mu\epsilon^{(2)} + \frac{1}{2}\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. - \frac{13}{8}\epsilon^{(1)} \right) q_1^2 \right. \\
&\quad \left. + \sqrt{3} \left(\frac{3}{4} + \frac{17}{4}A_1 + \frac{7}{8}A_2 + \frac{139}{24}A_1\epsilon^{(1)} + \frac{15}{8}A_2\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. - \frac{41}{8}\mu A_1 - \frac{41}{8}\mu A_2 - \frac{1}{6}\mu\epsilon^{(1)} + \mu\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. - \frac{3\sqrt{3}}{2}\mu - \frac{1}{3}\epsilon^{(2)} - \frac{7}{12}\epsilon^{(1)} \right) q_1q_2 \right. \\
&\quad \left. + \left(\frac{5}{8} + \frac{27}{8}A_1 + \frac{21}{16}A_2 + \frac{107}{16}A_1\epsilon^{(1)} + \frac{11}{16}A_2\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. - \frac{33}{16}\mu A_1 + \frac{33}{16}\mu A_2 - \frac{7}{8}\mu\epsilon^{(1)} + \frac{11}{8}\mu\epsilon^{(2)} - \frac{1}{2}\epsilon^{(2)} \right. \right. \\
&\quad \left. \left. + \frac{7}{8}\epsilon^{(1)} \right) q_2^2 \right].
\end{aligned} \tag{20}$$

The variational equation for the circular problem can be written as

$$\dot{p}_i = -\frac{\partial H_2}{\partial q_i}, \quad \dot{q}_i = -\frac{\partial H_2}{\partial p_i}, \quad i = 1, 2, \tag{21}$$

where the Hamiltonian of second order H_2 is represented by (20). Thus, the variational equations for the circular problem are given as

$$\begin{aligned}
\ddot{q}_1 - 2\dot{q}_2 &= A^*q_1 + Bq_2; \\
\ddot{q}_2 - 2\dot{q}_1 &= B^*q_1 + Cq_2,
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
A^* &= \left(\frac{3}{4} + \frac{9}{4}A_1 - \frac{9}{8}A_2 - \frac{73}{12}A_1\epsilon^{(1)} + \frac{11}{24}A_2\epsilon^{(2)} - \frac{27}{8}\mu A_1 \right. \\
&\quad \left. + \frac{27}{8}\mu A_2 + \frac{17}{4}\mu\epsilon^{(1)} - \frac{7}{4}\mu\epsilon^{(2)} + \epsilon^{(2)} - \frac{13}{4}\epsilon^{(1)} \right), \\
B &= k + \sqrt{3} \left(\frac{17}{4}A_1 + \frac{7}{8}A_2 + \frac{139}{24}A_1\epsilon^{(1)} + \frac{15}{8}A_2\epsilon^{(2)} \right. \\
&\quad \left. - \frac{41}{8}\mu A_1 - \frac{41}{8}\mu A_2 - \frac{1}{6}\mu\epsilon^{(1)} + \mu\epsilon^{(2)} \right. \\
&\quad \left. - \frac{1}{3}\epsilon^{(2)} + \frac{7}{12}\epsilon^{(1)} \right),
\end{aligned}$$

$$C = \left(\frac{9}{4} - \frac{9}{4}A_1 + \frac{9}{8}A_2 + \frac{73}{12}A_1\epsilon^{(1)} - \frac{11}{24}A_2\epsilon^{(2)} + \frac{27}{8}\mu A_1 - \frac{27}{8}\mu A_2 - \frac{17}{4}\mu\epsilon^{(1)} + \frac{7}{4}\mu\epsilon^{(2)} - \epsilon^{(2)} + \frac{13}{4}\epsilon^{(1)} \right), \quad (23)$$

where $k = 3\sqrt{3}(1 - 2\mu)/4$.

Now, the characteristic equation based on the problem can be obtained on putting

$$\begin{aligned} q_1 &= Le^{\lambda t}, & q_2 &= Me^{\lambda t}, \\ \dot{q}_1 &= L\lambda e^{\lambda t}, & \dot{q}_2 &= M\lambda e^{\lambda t}, \\ \ddot{q}_1 &= L\lambda^2 e^{\lambda t}, & \ddot{q}_2 &= M\lambda^2 e^{\lambda t}. \end{aligned} \quad (24)$$

The characteristic equation is obtained by substituting the results obtained from (24) in (22), and we get

$$\begin{vmatrix} \lambda^2 - A^* & -2\lambda + B \\ 2\lambda - B & \lambda^2 - C \end{vmatrix}. \quad (25)$$

Solving the characteristic equation, we get

$$\lambda^4 - \lambda^2(A^* + C - 4)\lambda^2 + (A^*C - B^2) = 0, \quad (26)$$

where $A^* + C - 4 = -1$ and

$$(A^*C - B^2) = \left\{ \frac{27\mu(1-\mu)}{4} \right\} \left\{ 1 - \frac{9A_1}{64} + \frac{2A_2}{3} - \frac{\epsilon}{2} \right\}. \quad (27)$$

The characteristic equation (26) has been reduced to the following form:

$$\begin{aligned} \lambda^4 + \lambda^2 + \left\{ \frac{27\mu(1-\mu)}{4} \right\} \\ \times \left\{ 1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right\} = 0. \end{aligned} \quad (28)$$

We observed that $A_1 = 0$ and $A_2 = 0$, the characteristic equation (27) is representing the classical circular restricted three-body problem:

Taking $\lambda_1 = i\omega_1$ and $\lambda_2 = i\omega_2$, we obtained from (28)

$$\begin{aligned} \omega^4 - \omega^2 + \left\{ \frac{27\mu(1-\mu)}{4} \right\} \\ \times \left\{ 1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right\} = 0. \end{aligned} \quad (29)$$

Hence, we have

$$\begin{aligned} \omega_{1,2}^2 = \frac{1}{2} \left[1 \pm \left\{ 1 - 27\mu(1-\mu) \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \right\}^{1/2} \right]; \end{aligned} \quad (30)$$

that is,

$$\begin{aligned} \omega_1 &= \frac{1}{\sqrt{2}} \left[1 + \left\{ 1 - 27\mu(1-\mu) \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \right\}^{1/2} \right], \\ \omega_2 &= \frac{1}{\sqrt{2}} \left[1 - \left\{ 1 - 27\mu(1-\mu) \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \right\}^{1/2} \right]. \end{aligned} \quad (31)$$

The equilibrium position is stable, if $\omega_{1,2}$ are purely imaginary. In order $\omega_{1,2}$ to be purely imaginary, the quantity $\omega_{1,2}^2$ must be negative and hence the discriminant of (29) is represented as follows:

$$1 - 27\mu(1-\mu) \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \geq 0. \quad (32)$$

If the equality relation holds in (31), we get

$$1 - 27\mu(1-\mu) \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) = 0; \quad (33)$$

that is,

$$\begin{aligned} 27 \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \mu^2 \\ - 27 \left(1 - \frac{125}{6}A_1 + \frac{40}{3}A_2 + \frac{5}{6}\epsilon^{(1)} - \frac{35}{18}\epsilon^{(2)} \right) \mu + 1 = 0; \end{aligned} \quad (34)$$

that is,

$$\begin{aligned} \mu = 9 \pm \sqrt{69} \left(1 - \left(\frac{125}{69} \right) A_1 + \left(\frac{80}{69} \right) A_2 + \left(\frac{5}{69} \right) \epsilon^{(1)} - \left(\frac{35}{207} \right) \epsilon^{(2)} \right) (18)^{-1}. \end{aligned} \quad (35)$$

Since $\mu \leq 1/2$, the positive sign is inadmissible. Hence, the region of stability in first approximation can be written as

$$\begin{aligned} 0 < \mu < \left\{ 9 - \sqrt{69} \left(1 - \left(\frac{125}{69} \right) A_1 + \left(\frac{80}{69} \right) A_2 + \left(\frac{5}{69} \right) \epsilon^{(1)} - \left(\frac{35}{207} \right) \epsilon^{(2)} \right) \right\} (18)^{-1}. \end{aligned} \quad (36)$$

Thus, the value of μ responsible for stable equilibrium points is given by

$$\begin{aligned} \mu_{\text{critical}} = 0.0385208965 \dots + 0.83601A_1 + 0.535048A_2 \\ - 0.0334405\epsilon^{(1)} + 0.0780279\epsilon^{(2)}. \end{aligned} \quad (37)$$

Also, we observed that in the case when $A_1 = A_2 = \varepsilon = 0$

$$\omega_1(\mu_c) = \omega_2(\mu_c) = \frac{1}{\sqrt{2}}, \quad \omega_1(0) = 1, \quad \omega_2(0) = 0. \quad (38)$$

The existence of resonance is possible in the neighborhood of the value of μ for which the frequencies ω_1 and ω_2 given by (31) satisfy at least one of the following relations:

$$\omega_1 = \frac{N}{2}, \quad \omega_2 = \frac{N}{2}, \quad \omega_1 - \omega_2 = N, \quad (39)$$

where N is the natural number.

The dependence of ω_1 and ω_2 on μ can be examined by using the simulation technique by drawing the curves by finding out correlation between μ and $\omega_{1,2}$. It is clear that in the region (32), only the resonance $\omega_2 = 1/2$ is possible.

The corresponding value of μ for $\omega_2 = 1/2$ is given by

$$\begin{aligned} \mu_0 = & 0.02859547921 \cdots + 11.04854A_1 \\ & - 2.35702260A_2 - 127.2792\varepsilon^{(1)} + 296.9848481\varepsilon^{(2)}. \end{aligned} \quad (40)$$

4. Conclusion

We discussed the stability of infinitesimal around the triangular equilibrium points under the elliptical restricted three-body problem, when both of the primaries are oblate spheroid and source of radiation for $e = 0$, in circular case near the resonance frequency. We have constructed a suitable normalized convergent Hamiltonian function and investigated the stability of infinitesimal around the triangular equilibrium points and the perturbed system analytically and numerically due to oblateness and radiation of primaries in circular case up to the second order terms. The region of stability and instability of the linear problem for the value of eccentricity " e " = 0 has been analyzed using simulation technique of the problem. The region of stability of the linear problem in μ and $\omega_{1,2}$ in the plane has been clearly marked. In the region outside the boundary, the triangular points are unstable while, in the region inside the boundary curve, the necessary condition of linear stability is satisfied in μ and $\omega_{1,2}$ plane. We concluded that the effect of the oblateness and radiation of primaries affects the location and resonance stability of triangular equilibrium points of the elliptical restricted three-body problem in the particular case when $e = 0$ at and near the resonance frequency $\omega_2 = 1/2$, which is analyzed from the graphical behavior of the triangular equilibrium points around the binary system.

Hence, we arrived at the conclusion that the shifting of the locations and the resonance stability of triangular equilibrium points in the particular case when $e = 0$ around the well-known binary system would be possible by changing the oblateness and radiation parameters which are observed from Figures 6, 7, 8, 10, 11, 12, 14, 15, and 16 and the combined Figures 9, 13, 17, 18, 19, 20, and 21. Nonlinear behavior of infinitesimal around the triangular equilibrium points under the above mentioned perturbing forces is yet to be investigated.

FIGURE 6: Correlation between μ and $\omega_{1,2}$.

FIGURE 7: Correlation between μ and $\omega_{1,2}$.

Appendix

In order to expand H in powers of q_1 and q_2 , we are required to obtain the expansion of $f(q_1, q_2)$, $g(q_1, q_2)$, $\alpha(q_1, q_2)$, and $h(q_1, q_2)$ with the help of Taylor's series, and we get

$$\begin{aligned} f(q_1, q_2) = & f(0, 0) + [q_1 f_1(0, 0) + q_2 f_2(0, 0)] \\ & + \frac{1}{2} [q_1^2 f_{11}(0, 0) + 2q_1 q_2 f_{12}(0, 0) + q_2^2 f_{22}(0, 0)] \\ & + \frac{1}{6} [q_1^3 f_{111}(0, 0) + 3q_1^2 q_2 f_{112}(0, 0) \\ & \quad + 3q_1 q_2^2 f_{122}(0, 0) + q_2^3 f_{222}(0, 0)] \\ & + \frac{1}{24} [q_1^4 f_{1111}(0, 0) + 4q_1^3 q_2 f_{1112}(0, 0) \\ & \quad + 6q_1^2 q_2^2 f_{1122}(0, 0) + 4q_1 q_2^3 f_{1222}(0, 0) \\ & \quad + q_2^4 f_{2222}(0, 0)] + \cdots, \end{aligned} \quad (A.1)$$

FIGURE 8: Correlation between μ and $\omega_{1,2}$. FIGURE 10: Correlation between μ and $\omega_{1,2}$.

FIGURE 9: Combined graph for Figures 1, 2, and 3.

FIGURE 11: Correlation between μ and $\omega_{1,2}$.

$$\begin{aligned}
 g(q_1, q_2) &= g(0, 0) + [q_1 g_1(0, 0) + q_2 g_2(0, 0)] \\
 &+ \frac{1}{2} [q_1^2 g_{11}(0, 0) + 2q_1 q_2 g_{12}(0, 0) + q_2^2 g_{22}(0, 0)] \\
 &+ \frac{1}{6} [q_1^3 g_{111}(0, 0) + 3q_1^2 q_2 g_{112}(0, 0) \\
 &+ 3q_1 q_2^2 g_{122}(0, 0) + q_2^3 g_{222}(0, 0)] \\
 &+ \frac{1}{24} [q_1^4 g_{1111}(0, 0) + 4q_1^3 q_2 g_{1112}(0, 0) \\
 &+ 6q_1^2 q_2^2 g_{1122}(0, 0) + 4q_1 q_2^3 g_{1222}(0, 0) \\
 &+ q_2^4 g_{2222}(0, 0)] + \dots,
 \end{aligned} \tag{A.2}$$

$$\begin{aligned}
 \alpha(q_1, q_2) &= \alpha(0, 0) + [q_1 \alpha_1(0, 0) + q_2 \alpha_2(0, 0)] \\
 &+ \frac{1}{2} [q_1^2 \alpha_{11}(0, 0) + 2q_1 q_2 \alpha_{12}(0, 0) + q_2^2 \alpha_{22}(0, 0)] \\
 &+ \frac{1}{6} [q_1^3 \alpha_{111}(0, 0) + 3q_1^2 q_2 \alpha_{112}(0, 0) \\
 &+ 3q_1 q_2^2 \alpha_{122}(0, 0) + q_2^3 \alpha_{222}(0, 0)]
 \end{aligned}$$

$$\begin{aligned}
 &+ \frac{1}{24} [q_1^4 \alpha_{1111}(0, 0) + 4q_1^3 q_2 \alpha_{1112}(0, 0) \\
 &+ 6q_1^2 q_2^2 \alpha_{1122}(0, 0) + 4q_1 q_2^3 \alpha_{1222}(0, 0) \\
 &+ q_2^4 \alpha_{2222}(0, 0)] + \dots,
 \end{aligned} \tag{A.3}$$

$$\begin{aligned}
 h(q_1, q_2) &= h(0, 0) + [q_1 h_1(0, 0) + q_2 h_2(0, 0)] \\
 &+ \frac{1}{2} [q_1^2 h_{11}(0, 0) + 2q_1 q_2 h_{12}(0, 0) + q_2^2 h_{22}(0, 0)] \\
 &+ \frac{1}{6} [q_1^3 h_{111}(0, 0) + 3q_1^2 q_2 h_{112}(0, 0) \\
 &+ 3q_1 q_2^2 h_{122}(0, 0) + q_2^3 h_{222}(0, 0)] \\
 &+ \frac{1}{24} [q_1^4 h_{1111}(0, 0) + 4q_1^3 q_2 h_{1112}(0, 0) \\
 &+ 6q_1^2 q_2^2 h_{1122}(0, 0) + 4q_1 q_2^3 h_{1222}(0, 0) \\
 &+ q_2^4 h_{2222}(0, 0)] + \dots,
 \end{aligned} \tag{A.4}$$

FIGURE 12: Correlation between μ and $\omega_{1,2}$. FIGURE 14: Correlation between μ and $\omega_{1,2}$ for binary system Achird-I.

FIGURE 13: Combined graph for Figures 5, 6, and 7.

FIGURE 15: Correlation between μ and $\omega_{1,2}$ for binary system Achird-II.

$$r_1^{-1} = f(q_1, q_2)$$

$$\begin{aligned}
 &= \left(1 + \frac{1}{2}A_2 + \frac{11}{18}A_1\epsilon^{(1)} - \frac{2}{9}A_1\epsilon^{(2)} - \frac{2}{9}A_2\epsilon^{(1)} \right. \\
 &\quad \left. + \frac{11}{18}A_2\epsilon^{(2)} + \frac{1}{3}\epsilon^{(1)} \right) \\
 &+ \left[q_1 \left(-\frac{1}{2} - \frac{A_1}{2} - \frac{A_2}{4} - \frac{11}{12}A_1\epsilon^{(1)} - \frac{1}{6}A_1\epsilon^{(2)} \right. \right. \\
 &\quad \left. \left. + \frac{1}{3}A_2\epsilon^{(1)} - \frac{3}{4}A_2\epsilon^{(2)} - \frac{1}{6}\epsilon^{(1)} - \frac{1}{3}\epsilon^{(2)} \right) \right] \\
 &+ \left[q_2 \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6}A_1 - \frac{7\sqrt{3}}{12}A_2 - \frac{7\sqrt{3}}{12}A_1\epsilon^{(1)} \right. \right. \\
 &\quad \left. \left. - \frac{7\sqrt{3}}{6}A_2\epsilon^{(2)} - \frac{7\sqrt{3}}{18}\epsilon^{(1)} + \frac{\sqrt{3}}{9}\epsilon^{(2)} \right) \right] \\
 &+ \frac{1}{2} \left[q_1^2 \left(-\frac{1}{4} + \frac{3}{2}A_1 - \frac{9}{8}A_2 - \frac{31}{12}A_1\epsilon^{(1)} \right. \right. \\
 &\quad \left. \left. + \frac{11}{24}A_2\epsilon^{(2)} - \frac{13}{4}\epsilon^{(1)} + \epsilon^{(2)} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 &+ 2q_1q_2 \left(\frac{3\sqrt{3}}{4} + \frac{2\sqrt{3}}{4}A_1 + \frac{7\sqrt{3}}{8}A_2 \right. \\
 &\quad \left. + \frac{51\sqrt{3}}{24}A_1\epsilon^{(1)} + \frac{15\sqrt{3}}{8}A_2\epsilon^{(2)} \right. \\
 &\quad \left. + \frac{7\sqrt{3}}{12}\epsilon^{(1)} + \frac{1}{\sqrt{3}}\epsilon^{(2)} \right) \\
 &+ q_2^2 \left(\frac{5}{4} - \frac{3}{2}A_1 + \frac{21}{8}A_2 + \frac{31}{8}A_1\epsilon^{(1)} \right. \\
 &\quad \left. + \frac{11}{8}A_2\epsilon^{(2)} + \frac{7}{4}\epsilon^{(1)} - \epsilon^{(2)} \right) \Bigg] \\
 &+ \frac{1}{6} \left[q_1^3 \left(\frac{57}{8} - \frac{45}{8}A_1 + \frac{155}{16}A_2 + \frac{115}{8}\epsilon^{(1)} \right. \right. \\
 &\quad \left. \left. - \frac{15}{4}\epsilon^{(2)} \right) \right]
 \end{aligned}$$

FIGURE 16: Correlation between μ and $\omega_{1,2}$ for binary system Achird-III.

FIGURE 17: Combined graph for binary systems Achird-I, II, and III.

$$\begin{aligned}
 & + 3q_1^2 q_2 \left(-\frac{3\sqrt{3}}{8} + \frac{21\sqrt{3}}{8} A_1 - \frac{61\sqrt{3}}{8} A_2 \right. \\
 & \quad \left. - \frac{61\sqrt{3}}{12} \varepsilon^{(1)} + \frac{7\sqrt{3}}{4} \varepsilon^{(2)} \right) \\
 & + 3q_1 q_2^2 \left(-\frac{33}{8} - \frac{15}{2} A_1 + \frac{5}{12} A_2 + \frac{25}{12} \varepsilon^{(1)} \right. \\
 & \quad \left. + \frac{5}{8} \varepsilon^{(2)} \right) \\
 & + q_2^3 \left(-\frac{9\sqrt{3}}{8} + \frac{33\sqrt{3}}{8} A_1 - \frac{69\sqrt{3}}{16} A_2 \right. \\
 & \quad \left. - \frac{23\sqrt{3}}{8} \varepsilon^{(1)} - \frac{11\sqrt{3}}{4} \varepsilon^{(2)} \right) \Bigg] \\
 & + \frac{1}{24} \left[q_1^4 \left(\frac{183}{2} - \frac{75}{4} A_1 + \frac{1185}{2} A_2 + 290 \varepsilon^{(1)} \right. \right. \\
 & \quad \left. \left. - \frac{25}{2} \varepsilon^{(2)} \right) \right]
 \end{aligned}$$

FIGURE 18: Combined graph for binary systems Luyten-I, II, and III.

FIGURE 19: Combined graph for binary systems Alpha Cen AB-I, II, and III.

FIGURE 20: Combined graph for binary systems Kruger60-I, II, and III.

FIGURE 21: Combined graph for binary systems Xi-Booties-I, II, and III.

$$\begin{aligned}
& + 4q_1^3 q_2 \left(-\frac{75\sqrt{3}}{16} + \frac{17\sqrt{3}}{8} A_1 - \frac{21\sqrt{3}}{2} A_2 \right. \\
& \quad \left. - \frac{35\sqrt{3}}{8} \varepsilon^{(1)} + \frac{21\sqrt{3}}{6} \varepsilon^{(2)} \right) \\
& + 6q_1^2 q_2^2 \left(\frac{123}{16} + \frac{85}{6} A_1 - \frac{105}{8} A_2 \right. \\
& \quad \left. + \frac{235}{12} \varepsilon^{(1)} + \frac{75}{12} \varepsilon^{(2)} \right) \\
& + 4q_1 q_2^3 \left(\frac{135\sqrt{3}}{16} - \frac{11\sqrt{3}}{4} A_1 + \frac{15\sqrt{3}}{4} A_2 \right. \\
& \quad \left. + \frac{33\sqrt{3}}{8} \varepsilon^{(1)} - \frac{21\sqrt{3}}{5} \varepsilon^{(2)} \right) \\
& + q_2^4 \left(\frac{9}{16} - \frac{135}{4} A_1 - \frac{1575}{32} A_2 + \frac{1781}{16} \varepsilon^{(1)} \right. \\
& \quad \left. - \frac{45}{2} \varepsilon^{(2)} \right) \Bigg].
\end{aligned}
\tag{A.5}$$

In a similar manner, $g(q_1, q_2)$, $\alpha(q_1, q_2)$, and $h(q_1, q_2)$ are expanded in the same line of action as it was in the case of $f(q_1, q_2)$ as mentioned above, applying Taylor's theorem, whose coefficients are calculated as follows:

$$\begin{aligned}
r_2^{-1} &= g(q_1, q_2) \\
&= \left(1 + \frac{1}{2} A_1 + \frac{1}{9} A_1 \varepsilon^{(1)} + \frac{1}{9} A_2 \varepsilon^{(2)} + \frac{1}{3} \varepsilon^{(2)} \right) \\
&+ \left[q_1 \left(\frac{1}{2} + \frac{A_1}{4} + \frac{A_2}{2} + \frac{7}{6} A_1 \varepsilon^{(1)} + \frac{7}{6} A_2 \varepsilon^{(2)} + \frac{\varepsilon^{(2)}}{6} \right) \right] \\
&+ \left[q_2 \left(-\frac{\sqrt{3}}{2} - \frac{7\sqrt{3}}{12} A_1 + \frac{\sqrt{3}}{6} A_2 + \frac{\sqrt{3}}{6} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{\sqrt{3}}{6} A_2 \varepsilon^{(2)} + \frac{\sqrt{3}}{9} \varepsilon^{(1)} - \frac{7\sqrt{3}}{18} \varepsilon^{(2)} \right) \right] \\
&+ \frac{1}{2} \left[q_1^2 \left(-\frac{1}{4} - \frac{9}{8} A_1 + \frac{3}{2} A_2 - \frac{111}{36} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{267}{72} A_2 \varepsilon^{(2)} + \varepsilon^{(1)} - \frac{3}{4} \varepsilon^{(2)} \right) \right. \\
&+ 2q_1 q_2 \left(-\frac{3\sqrt{3}}{4} - \frac{7\sqrt{3}}{8} A_1 - \frac{\sqrt{3}}{2} A_2 + \frac{21\sqrt{3}}{12} A_1 \varepsilon^{(1)} \right. \\
& \quad \left. - \frac{21\sqrt{3}}{12} A_2 \varepsilon^{(2)} - \frac{\sqrt{3}}{3} \varepsilon^{(1)} + \frac{2\sqrt{3}}{3} \varepsilon^{(2)} \right) \\
&+ q_2^2 \left(\frac{5}{4} + \frac{21}{8} A_1 - \frac{3}{2} A_2 - \frac{33}{12} A_1 \varepsilon^{(1)} \right. \\
& \quad \left. - \frac{33}{12} A_2 \varepsilon^{(2)} + \frac{7}{4} \varepsilon^{(2)} \right) \Bigg]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} \left[q_1^3 \left(-\frac{21}{8} + \frac{123}{12} A_1 + \frac{35}{4} A_2 + \frac{112}{7} \varepsilon^{(1)} - \frac{85}{12} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + 3q_1^2 q_2 \left(-\frac{3\sqrt{3}}{8} - \frac{31\sqrt{3}}{16} A_1 - \frac{23\sqrt{3}}{16} A_2 \right. \right. \\
& \quad \left. \left. + \frac{25\sqrt{3}}{8} \varepsilon^{(1)} - \frac{15\sqrt{3}}{8} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + 3q_1 q_2^2 \left(\frac{33}{8} - \frac{25}{4} A_1 + \frac{36}{16} A_2 + \frac{125}{16} \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{235}{16} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + q_2^3 \left(-\frac{9\sqrt{3}}{8} - \frac{9\sqrt{3}}{16} A_1 + \frac{75\sqrt{3}}{16} A_2 + \frac{145\sqrt{3}}{16} \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{68\sqrt{3}}{8} \varepsilon^{(2)} \right) \right] \\
& + \frac{1}{24} \left[q_1^4 \left(-\frac{111}{16} - \frac{965}{12} A_1 + \frac{365}{12} A_2 \right. \right. \\
& \quad \left. \left. + \frac{3165}{16} \varepsilon^{(1)} - \frac{295}{18} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + 4q_1^3 q_2 \left(\frac{75\sqrt{3}}{16} + \frac{126\sqrt{3}}{13} A_1 - \frac{837\sqrt{3}}{16} A_2 \right. \right. \\
& \quad \left. \left. - \frac{55\sqrt{3}}{12} \varepsilon^{(1)} - \frac{96\sqrt{3}}{35} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + 6q_1^2 q_2^2 \left(-\frac{123}{16} - \frac{45}{8} A_1 + \frac{156}{16} A_2 \right. \right. \\
& \quad \left. \left. + \frac{25}{8} \varepsilon^{(1)} - \frac{85}{16} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + 4q_1 q_2^3 \left(-\frac{135\sqrt{3}}{16} - \frac{1165\sqrt{3}}{16} A_1 + \frac{765\sqrt{3}}{32} A_2 \right. \right. \\
& \quad \left. \left. + \frac{95\sqrt{3}}{8} \varepsilon^{(1)} - \frac{85\sqrt{3}}{16} \varepsilon^{(2)} \right) \right. \\
& \quad \left. + q_2^4 \left(\frac{9}{16} + \frac{135}{16} A_1 - \frac{269}{16} A_2 \right. \right. \\
& \quad \left. \left. - \frac{145}{16} \varepsilon^{(1)} + \frac{65}{16} \varepsilon^{(2)} \right) \right], \\
r_1^{-3} &= \alpha(q_1, q_2) \\
&= \left(1 + \frac{3}{2} A_2 + \frac{11}{6} A_1 \varepsilon^{(1)} + \frac{11}{6} A_2 \varepsilon^{(2)} + \varepsilon^{(1)} \right) \\
&+ \left[q_1 \left(-\frac{3}{2} - \frac{3}{2} A_1 - \frac{9}{4} A_2 - \frac{67}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{67}{12} A_2 \varepsilon^{(2)} - \frac{3}{2} \varepsilon^{(1)} - \varepsilon^{(2)} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[q_2 \left(-\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} A_1 + \frac{\sqrt{3}}{12} A_2 - \frac{39\sqrt{3}}{12} A_1 \epsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{49\sqrt{3}}{12} A_2 \epsilon^{(2)} - \frac{13\sqrt{3}}{6} \epsilon^{(1)} + \frac{\sqrt{3}}{3} \epsilon^{(2)} \right) \right] \\
& + \frac{1}{2} \left[q_1^2 \left(\frac{3}{4} + \frac{15}{2} A_1 - \frac{15}{8} A_2 + \frac{95}{8} A_1 \epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{95}{8} A_2 \epsilon^{(2)} - \frac{5}{4} \epsilon^{(1)} + 5 \epsilon^{(2)} \right) \right. \\
& \quad + 2q_1 q_2 \left(\frac{15\sqrt{3}}{4} + \frac{5\sqrt{3}}{4} A_1 + \frac{85\sqrt{3}}{8} A_2 \right. \\
& \quad \left. + \frac{305\sqrt{3}}{24} A_1 \epsilon^{(1)} + \frac{135\sqrt{3}}{8} A_2 \epsilon^{(2)} \right. \\
& \quad \left. + \frac{95\sqrt{3}}{12} \epsilon^{(1)} + \frac{5\sqrt{3}}{3} \epsilon^{(2)} \right) \\
& \quad + q_2^2 \left(\frac{33}{4} - \frac{15}{2} A_1 + \frac{195}{8} A_2 + \frac{125}{8} A_1 \epsilon^{(1)} \right. \\
& \quad \left. + \frac{1495}{24} A_2 \epsilon^{(2)} + \frac{65}{4} \epsilon^{(1)} - 5 \epsilon^{(2)} \right) \Big] \\
& + \frac{1}{6} \left[q_1^3 \left(\frac{75}{8} - \frac{25}{8} A_1 - \frac{65}{4} A_2 + \frac{115}{12} \epsilon^{(1)} - \frac{65}{8} \epsilon^{(2)} \right) \right. \\
& \quad + 3q_1^2 q_2 \left(-\frac{45\sqrt{3}}{8} - \frac{135\sqrt{3}}{32} A_1 + \frac{35\sqrt{3}}{8} A_2 \right. \\
& \quad \left. + \frac{5\sqrt{3}}{12} \epsilon^{(1)} - \frac{15\sqrt{3}}{4} \epsilon^{(2)} \right) \\
& \quad + 3q_1 q_2^2 \left(-\frac{255}{8} - \frac{135}{4} A_1 + \frac{65}{8} A_2 \right. \\
& \quad \left. + \frac{75}{12} \epsilon^{(1)} - \frac{5}{8} \epsilon^{(2)} \right) \\
& \quad + q_2^3 \left(-\frac{135\sqrt{3}}{8} - \frac{25\sqrt{3}}{4} A_1 - \frac{47\sqrt{3}}{8} A_2 \right. \\
& \quad \left. + \frac{217\sqrt{3}}{16} \epsilon^{(1)} + \frac{395\sqrt{3}}{32} \epsilon^{(2)} \right) \Big] \\
& + \frac{1}{24} \left[q_1^4 \left(-\frac{855}{16} - \frac{1255}{32} A_1 + \frac{335}{16} A_2 \right. \right. \\
& \quad \left. \left. - \frac{1195}{8} \epsilon^{(1)} - \frac{965}{16} \epsilon^{(2)} \right) \right. \\
& \quad + 4q_1^3 q_2 \left(-\frac{315\sqrt{3}}{16} - \frac{545\sqrt{3}}{32} A_1 - \frac{165\sqrt{3}}{16} A_2 \right. \\
& \quad \left. + \frac{35\sqrt{3}}{8} \epsilon^{(1)} - \frac{65\sqrt{3}}{12} \epsilon^{(2)} \right)
\end{aligned}$$

$$\begin{aligned}
& + 6q_1^2 q_2^2 \left(\frac{1395}{16} + \frac{8305}{32} A_1 - \frac{545}{12} A_2 \right. \\
& \quad \left. + \frac{95}{8} \epsilon^{(1)} - \frac{65}{12} \epsilon^{(2)} \right) \\
& + 4q_1 q_2^3 \left(\frac{1575\sqrt{3}}{16} + \frac{5655\sqrt{3}}{48} A_1 + \frac{135\sqrt{3}}{16} A_2 \right. \\
& \quad \left. + \frac{105\sqrt{3}}{12} \epsilon^{(1)} - \frac{75\sqrt{3}}{8} \epsilon^{(2)} \right) \\
& + q_2^4 \left(\frac{1665}{16} + \frac{10345}{32} A_1 - \frac{6985}{32} A_2 \right. \\
& \quad \left. - \frac{965}{16} \epsilon^{(1)} + \frac{1055}{32} \epsilon^{(2)} \right) \Big],
\end{aligned}$$

$$r_2^{-3} = h(q_1, q_2)$$

$$\begin{aligned}
& = \left(1 + \frac{3}{2} A_2 + \frac{1}{3} A_1 \epsilon^{(1)} + \frac{1}{3} A_2 \epsilon^{(2)} + \epsilon^{(2)} \right) \\
& + \left[q_1 \left(\frac{3}{2} + \frac{3}{4} A_1 + \frac{3}{2} A_2 + \frac{29}{6} A_1 \epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{29}{6} A_2 \epsilon^{(2)} + \epsilon^{(1)} - \frac{3}{2} \epsilon^{(2)} \right) \right] \\
& + \left[q_2 \left(\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{4} A_1 + \frac{\sqrt{3}}{2} A_2 + \frac{\sqrt{3}}{2} A_1 \epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{\sqrt{3}}{2} A_2 \epsilon^{(2)} + \frac{\sqrt{3}}{3} \epsilon^{(1)} - \frac{13\sqrt{3}}{6} \epsilon^{(2)} \right) \right] \\
& + \frac{1}{2} \left[q_1^2 \left(\frac{3}{4} - \frac{15}{8} A_1 + \frac{15}{2} A_2 + \frac{75}{4} A_1 \epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{75}{4} A_2 \epsilon^{(2)} + 5 \epsilon^{(1)} - \frac{5}{4} \epsilon^{(2)} \right) \right. \\
& \quad + 2q_1 q_2 \left(-\frac{15\sqrt{3}}{4} - \frac{65\sqrt{3}}{8} A_1 - \frac{5\sqrt{3}}{2} A_2 \right. \\
& \quad \left. + \frac{5\sqrt{3}}{12} A_1 \epsilon^{(1)} - \frac{45\sqrt{3}}{8} A_2 \epsilon^{(2)} \right. \\
& \quad \left. + \frac{5\sqrt{3}}{3} \epsilon^{(1)} - \frac{65\sqrt{3}}{12} \epsilon^{(2)} \right) \\
& \quad + q_2^2 \left(\frac{33}{4} + \frac{195}{8} A_1 - \frac{15}{2} A_2 - \frac{65}{4} A_1 \epsilon^{(1)} \right. \\
& \quad \left. - \frac{65}{4} A_2 \epsilon^{(2)} - 5 \epsilon^{(1)} + \frac{65}{4} \epsilon^{(2)} \right) \Big]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} \left[q_1^3 \left(-\frac{75}{8} + \frac{250}{12} A_1 + \frac{435}{16} A_2 - \frac{35}{8} \varepsilon^{(1)} + \frac{85}{12} \varepsilon^{(2)} \right) \right. \\
& + 3q_1^2 q_2 \left(-\frac{45\sqrt{3}}{8} - \frac{185\sqrt{3}}{16} A_1 + \frac{65\sqrt{3}}{8} A_2 \right. \\
& \quad \left. \left. + \frac{35\sqrt{3}}{12} \varepsilon^{(1)} - \frac{85\sqrt{3}}{12} \varepsilon^{(2)} \right) \right. \\
& + 3q_1 q_2^2 \left(\frac{255}{8} + \frac{985}{16} A_1 - \frac{1125}{32} A_2 \right. \\
& \quad \left. + \frac{1255}{32} \varepsilon^{(1)} - \frac{895}{16} \varepsilon^{(2)} \right) \\
& + q_2^3 \left(-\frac{135\sqrt{3}}{8} + \frac{295\sqrt{3}}{16} A_1 - \frac{1155\sqrt{3}}{16} A_2 \right. \\
& \quad \left. + \frac{255\sqrt{3}}{32} \varepsilon^{(1)} - \frac{175\sqrt{3}}{16} \varepsilon^{(2)} \right) \Bigg] \\
& + \frac{1}{24} \left[q_1^4 \left(-\frac{855}{16} - \frac{3575}{32} A_1 + \frac{4725}{32} A_2 \right. \right. \\
& \quad \left. - \frac{785}{12} \varepsilon^{(1)} - \frac{125}{12} \varepsilon^{(2)} \right) \\
& + 4q_1^3 q_2 \left(\frac{315\sqrt{3}}{16} - \frac{485\sqrt{3}}{8} A_1 + \frac{1165\sqrt{3}}{32} A_2 \right. \\
& \quad \left. + \frac{655\sqrt{3}}{32} \varepsilon^{(1)} - \frac{95\sqrt{3}}{8} \varepsilon^{(2)} \right) \\
& + 6q_1^2 q_2^2 \left(\frac{1235}{16} + \frac{565}{16} A_1 - \frac{1035}{32} A_2 \right. \\
& \quad \left. + \frac{665}{32} \varepsilon^{(1)} - \frac{765}{16} \varepsilon^{(2)} \right) \\
& + 4q_1 q_2^3 \left(-\frac{1575\sqrt{3}}{16} - \frac{1285\sqrt{3}}{32} A_1 + \frac{3255\sqrt{3}}{12} A_2 \right. \\
& \quad \left. - \frac{885\sqrt{3}}{12} \varepsilon^{(1)} - \frac{1225\sqrt{3}}{12} \varepsilon^{(2)} \right) \\
& + q_2^4 \left(\frac{1065}{16} + \frac{10865}{32} A_1 - \frac{7655}{32} A_2 \right. \\
& \quad \left. - \frac{1255}{12} \varepsilon^{(1)} - \frac{895}{12} \varepsilon^{(2)} \right) \Bigg]. \tag{A.6}
\end{aligned}$$

Equation (9) can be expressed in the following forms:

$$\begin{aligned}
H = & \frac{p_1^2 + p_2^2}{2} + (p_1 q_2 - p_2 q_1) + e \cos \nu \frac{(q_1^2 + q_2^2)}{2(1 + e \cos \nu)} \\
& - \frac{1}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}}
\end{aligned}$$

$$\begin{aligned}
& \cdot \left[\frac{(1 - \mu)(1 - \varepsilon^{(1)})}{r_1} + \frac{\mu(1 - \varepsilon^{(2)})}{r_2} \right. \\
& \quad \left. + \frac{(1 - \mu)(1 - \varepsilon^{(1)}) A_1}{2r_1^3} + \frac{\mu(1 - \varepsilon^{(2)}) A_2}{2r_2^3} \right]. \tag{A.7}
\end{aligned}$$

The Hamiltonian of the perturbed motion of infinitesimal around L_4 is obtained after using the values obtained from (14), (15), (16), and (17), substituting in (9), along with substituting the value of r_1^{-1} , r_2^{-1} , r_1^{-3} , and r_2^{-3} from (A.5), (A.6) in (17), and collecting only fourth degree terms of q_1 and q_2 , and we get

$$\begin{aligned}
H = & \frac{p_1^2 + p_2^2}{2} + (p_1 q_2 - p_2 q_1) + e \cos \nu \frac{(q_1^2 + q_2^2)}{2(1 + e \cos \nu)} \\
& - \frac{(1 - \mu)(1 - \varepsilon^{(1)})}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}} \\
& \times \left[\left(1 + \frac{1}{2} A_2 + \frac{11}{18} A_1 \varepsilon^{(1)} + \frac{11}{18} A_2 \varepsilon^{(2)} + \frac{1}{3} \varepsilon^{(1)} \right) \right. \\
& + \left\{ q_1 \left(-\frac{1}{2} - \frac{A_1}{2} - \frac{A_2}{4} - \frac{11}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{3}{4} A_2 \varepsilon^{(2)} - \frac{1}{6} \varepsilon^{(1)} - \frac{1}{3} \varepsilon^{(2)} \right) \right\} \\
& + \left\{ q_2 \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6} A_1 - \frac{7\sqrt{3}}{12} A_2 - \frac{7\sqrt{3}}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. \left. - \frac{7\sqrt{3}}{6} A_2 \varepsilon^{(2)} - \frac{7\sqrt{3}}{18} \varepsilon^{(1)} + \frac{\sqrt{3}}{9} \varepsilon^{(2)} \right) \right\} \\
& + \frac{1}{2} \left\{ q_1^2 \left(-\frac{1}{4} + \frac{3}{2} A_1 - \frac{9}{8} A_2 - \frac{31}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \left. + \frac{11}{24} A_2 \varepsilon^{(2)} - \frac{13}{4} \varepsilon^{(1)} + \varepsilon^{(2)} \right) \\
& + 2q_1 q_2 \left(\frac{3\sqrt{3}}{4} + \frac{2\sqrt{3}}{4} A_1 + \frac{7\sqrt{3}}{8} A_2 \right. \\
& \quad \left. + \frac{51\sqrt{3}}{24} A_1 \varepsilon^{(1)} + \frac{15\sqrt{3}}{8} A_2 \varepsilon^{(2)} \right. \\
& \quad \left. + \frac{7\sqrt{3}}{12} \varepsilon^{(1)} + \frac{1}{\sqrt{3}} \varepsilon^{(2)} \right) \\
& + q_2^2 \left(\frac{5}{4} - \frac{3}{2} A_1 + \frac{21}{8} A_2 + \frac{31}{8} A_1 \varepsilon^{(1)} \right. \\
& \quad \left. + \frac{11}{8} A_2 \varepsilon^{(2)} + \frac{7}{4} \varepsilon^{(1)} - \varepsilon^{(2)} \right) \Bigg]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{6} \left\{ q_1^3 \left(\frac{57}{8} - \frac{45}{8} A_1 + \frac{155}{16} A_2 + \frac{115}{8} \varepsilon^{(1)} - \frac{15}{4} \varepsilon^{(2)} \right) \right. \\
& \quad + 3q_1^2 q_2 \left(-\frac{3\sqrt{3}}{8} + \frac{21\sqrt{3}}{8} A_1 - \frac{61\sqrt{3}}{8} A_2 \right. \\
& \quad \quad \left. \left. - \frac{61\sqrt{3}}{12} \varepsilon^{(1)} + \frac{7\sqrt{3}}{4} \varepsilon^{(2)} \right) \right. \\
& \quad + 3q_1 q_2^2 \left(-\frac{33}{8} - \frac{15}{2} A_1 + \frac{5}{12} A_2 \right. \\
& \quad \quad \left. + \frac{25}{12} \varepsilon^{(1)} + \frac{5}{8} \varepsilon^{(2)} \right) \\
& \quad + q_2^3 \left(-\frac{9\sqrt{3}}{8} + \frac{33\sqrt{3}}{8} A_1 - \frac{69\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. \left. - \frac{23\sqrt{3}}{8} \varepsilon^{(1)} - \frac{11\sqrt{3}}{4} \varepsilon^{(2)} \right) \right\} \\
& + \frac{1}{24} \left\{ q_1^4 \left(\frac{183}{2} - \frac{75}{4} A_1 + \frac{1185}{2} A_2 \right. \right. \\
& \quad \quad \left. + 290 \varepsilon^{(1)} - \frac{25}{2} \varepsilon^{(2)} \right) \\
& \quad + 4q_1^3 q_2 \left(-\frac{75\sqrt{3}}{16} + \frac{17\sqrt{3}}{8} A_1 - \frac{21\sqrt{3}}{2} A_2 \right. \\
& \quad \quad \left. \left. - \frac{35\sqrt{3}}{8} \varepsilon^{(1)} + \frac{21\sqrt{3}}{6} \varepsilon^{(2)} \right) \right. \\
& \quad + 6q_1^2 q_2^2 \left(\frac{123}{16} + \frac{85}{6} A_1 - \frac{105}{8} A_2 \right. \\
& \quad \quad \left. + \frac{235}{12} \varepsilon^{(1)} + \frac{75}{12} \varepsilon^{(2)} \right) \\
& \quad + 4q_1 q_2^3 \left(\frac{135\sqrt{3}}{16} - \frac{11\sqrt{3}}{4} A_1 + \frac{15\sqrt{3}}{4} A_2 \right. \\
& \quad \quad \left. + \frac{33\sqrt{3}}{8} \varepsilon^{(1)} - \frac{21\sqrt{3}}{5} \varepsilon^{(2)} \right) \\
& \quad + q_2^4 \left(\frac{9}{16} - \frac{135}{4} A_1 - \frac{1575}{32} A_2 + \frac{1781}{16} \varepsilon^{(1)} \right. \\
& \quad \quad \left. \left. - \frac{45}{2} \varepsilon^{(2)} \right) \right\} \\
& + \frac{\mu(1 - \varepsilon^{(2)})}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}} \\
& \times \left[\left(1 + \frac{1}{2} A_2 + \frac{11}{18} A_1 \varepsilon^{(1)} - \frac{2}{9} A_1 \varepsilon^{(2)} \right. \right. \\
& \quad \left. \left. - \frac{2}{9} A_2 \varepsilon^{(1)} + \frac{11}{18} A_2 \varepsilon^{(2)} + \frac{1}{3} \varepsilon^{(1)} \right) \right. \\
& \quad + \left\{ q_1 \left(\frac{1}{2} + \frac{A_1}{4} + \frac{A_2}{2} + \frac{7}{6} A_1 \varepsilon^{(1)} - \frac{13}{12} A_1 \varepsilon^{(2)} \right. \right. \\
& \quad \quad \left. \left. - \frac{1}{12} A_2 \varepsilon^{(1)} + \frac{7}{6} A_2 \varepsilon^{(2)} + \frac{\varepsilon^{(2)}}{6} \right) \right\} \\
& \quad + \left\{ q_2 \left(-\frac{\sqrt{3}}{2} - \frac{7\sqrt{3}}{12} A_1 + \frac{\sqrt{3}}{6} A_2 + \frac{\sqrt{3}}{6} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \quad + \frac{\sqrt{3}}{12} A_1 \varepsilon^{(2)} - \frac{3}{4} A_2 \varepsilon^{(1)} + \frac{\sqrt{3}}{6} A_2 \varepsilon^{(2)} \\
& \quad \quad \left. \left. + \frac{\sqrt{3}}{9} \varepsilon^{(1)} - \frac{7\sqrt{3}}{18} \varepsilon^{(2)} \right) \right\} \\
& + \frac{1}{2} \left\{ q_1^2 \left(-\frac{1}{4} - \frac{9}{8} A_1 + \frac{3}{2} A_2 - \frac{111}{36} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \quad - \frac{381}{72} A_1 \varepsilon^{(2)} - \frac{11}{72} A_2 \varepsilon^{(1)} + \frac{267}{72} A_2 \varepsilon^{(2)} \\
& \quad \quad \left. \left. + \varepsilon^{(1)} - \frac{3}{4} \varepsilon^{(2)} \right) \right. \\
& \quad + 2q_1 q_2 \left(-\frac{3\sqrt{3}}{4} - \frac{7\sqrt{3}}{8} A_1 - \frac{\sqrt{3}}{2} A_2 + \frac{21\sqrt{3}}{12} A_1 \varepsilon^{(1)} \right. \\
& \quad \quad + \frac{51\sqrt{3}}{24} A_1 \varepsilon^{(2)} + \frac{7\sqrt{3}}{24} A_2 \varepsilon^{(1)} \\
& \quad \quad \left. \left. - \frac{21\sqrt{3}}{12} A_2 \varepsilon^{(2)} - \frac{\sqrt{3}}{3} \varepsilon^{(1)} + \frac{2\sqrt{3}}{3} \varepsilon^{(2)} \right) \right. \\
& \quad + q_2^2 \left(\frac{5}{4} + \frac{21}{8} A_1 - \frac{3}{2} A_2 - \frac{33}{12} A_1 \varepsilon^{(1)} \right. \\
& \quad \quad \left. \left. - \frac{33}{12} A_2 \varepsilon^{(2)} + \frac{7}{4} \varepsilon^{(2)} \right) \right\} \\
& + \frac{1}{6} \left\{ q_1^3 \left(-\frac{21}{8} + \frac{123}{12} A_1 + \frac{35}{4} A_2 \right. \right. \\
& \quad \quad + \frac{112}{7} \varepsilon^{(1)} - \frac{85}{12} \varepsilon^{(2)} \right) \\
& \quad + 3q_1^2 q_2 \left(-\frac{3\sqrt{3}}{8} - \frac{31\sqrt{3}}{16} A_1 - \frac{23\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. + \frac{25\sqrt{3}}{8} \varepsilon^{(1)} - \frac{15\sqrt{3}}{8} \varepsilon^{(2)} \right) \\
& \quad + 3q_1 q_2^2 \left(\frac{33}{8} - \frac{25}{4} A_1 + \frac{36}{16} A_2 + \frac{125}{16} \varepsilon^{(1)} \right. \\
& \quad \quad \left. \left. - \frac{235}{16} \varepsilon^{(2)} \right) \right. \\
& \quad + q_2^3 \left(-\frac{9\sqrt{3}}{8} - \frac{9\sqrt{3}}{16} A_1 + \frac{75\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. \left. + \frac{145\sqrt{3}}{16} \varepsilon^{(1)} - \frac{68\sqrt{3}}{8} \varepsilon^{(2)} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{24} \left\{ q_1^4 \left(-\frac{111}{16} - \frac{965}{12} A_1 + \frac{365}{12} A_2 \right. \right. \\
& \quad \left. \left. + \frac{3165}{16} \varepsilon^{(1)} - \frac{295}{18} \varepsilon^{(2)} \right) \right. \\
& \quad + 4q_1^3 q_2 \left(\frac{75\sqrt{3}}{16} + \frac{126\sqrt{3}}{13} A_1 - \frac{837\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. - \frac{55\sqrt{3}}{12} \varepsilon^{(1)} - \frac{96\sqrt{3}}{35} \varepsilon^{(2)} \right) \\
& \quad + 6q_1^2 q_2^2 \left(-\frac{123}{16} - \frac{45}{8} A_1 + \frac{156}{16} A_2 \right. \\
& \quad \quad \left. + \frac{25}{8} \varepsilon^{(1)} - \frac{85}{16} \varepsilon^{(2)} \right) \\
& \quad + 4q_1 q_2^3 \left(-\frac{135\sqrt{3}}{16} - \frac{1165\sqrt{3}}{16} A_1 + \frac{765\sqrt{3}}{32} A_2 \right. \\
& \quad \quad \left. + \frac{95\sqrt{3}}{8} \varepsilon^{(1)} - \frac{85\sqrt{3}}{16} \varepsilon^{(2)} \right) \\
& \quad \left. + q_2^4 \left(\frac{9}{16} + \frac{135}{16} A_1 - \frac{269}{16} A_2 \right. \right. \\
& \quad \quad \left. \left. - \frac{145}{16} \varepsilon^{(1)} + \frac{65}{16} \varepsilon^{(2)} \right) \right\} \\
& + \frac{(1-\mu)(1-\varepsilon^{(1)}) A_1}{(1+e \cos \nu) \{1+3((A_1+A_2)/2)\}} \\
& \times \left[\left(1 + \frac{3}{2} A_2 + \frac{11}{6} A_1 \varepsilon^{(1)} + \frac{11}{6} A_2 \varepsilon^{(2)} + \varepsilon^{(1)} \right) \right. \\
& \quad + \left\{ q_1 \left(-\frac{3}{2} - \frac{3}{2} A_1 - \frac{9}{4} A_2 - \frac{67}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \quad \left. \left. - \frac{67}{12} A_2 \varepsilon^{(2)} - \frac{3}{2} \varepsilon^{(1)} - \varepsilon^{(2)} \right) \right\} \\
& \quad + \left\{ q_2 \left(-\frac{3\sqrt{3}}{2} + \frac{\sqrt{3}}{2} A_1 + \frac{\sqrt{3}}{12} A_2 - \frac{39\sqrt{3}}{12} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \quad \left. \left. - \frac{49\sqrt{3}}{12} A_2 \varepsilon^{(2)} - \frac{13\sqrt{3}}{6} \varepsilon^{(1)} + \frac{\sqrt{3}}{3} \varepsilon^{(2)} \right) \right\} \\
& \quad + \frac{1}{2} \left\{ q_1^2 \left(\frac{3}{4} + \frac{15}{2} A_1 - \frac{15}{8} A_2 + \frac{95}{8} A_1 \varepsilon^{(1)} \right. \right. \\
& \quad \quad \left. \left. + \frac{95}{8} A_2 \varepsilon^{(2)} - \frac{5}{4} \varepsilon^{(1)} + 5 \varepsilon^{(2)} \right) \right. \\
& \quad + 2q_1 q_2 \left(\frac{15\sqrt{3}}{4} + \frac{5\sqrt{3}}{4} A_1 + \frac{85\sqrt{3}}{8} A_2 \right. \\
& \quad \quad + \frac{305\sqrt{3}}{24} A_1 \varepsilon^{(1)} + \frac{135\sqrt{3}}{8} A_2 \varepsilon^{(2)} \\
& \quad \quad \left. \left. + \frac{95\sqrt{3}}{12} \varepsilon^{(1)} + \frac{5\sqrt{3}}{3} \varepsilon^{(2)} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + q_2^2 \left(\frac{33}{4} - \frac{15}{2} A_1 + \frac{195}{8} A_2 + \frac{125}{8} A_1 \varepsilon^{(1)} \right. \\
& \quad \left. + \frac{1495}{24} A_2 \varepsilon^{(2)} + \frac{65}{4} \varepsilon^{(1)} - 5 \varepsilon^{(2)} \right) \left. \right\} \\
& + \frac{1}{6} \left\{ q_1^3 \left(\frac{75}{8} - \frac{25}{8} A_1 - \frac{65}{4} A_2 + \frac{115}{12} \varepsilon^{(1)} - \frac{65}{8} \varepsilon^{(2)} \right) \right. \\
& \quad + 3q_1^2 q_2 \left(-\frac{45\sqrt{3}}{8} - \frac{135\sqrt{3}}{32} A_1 + \frac{35\sqrt{3}}{8} A_2 \right. \\
& \quad \quad \left. + \frac{5\sqrt{3}}{12} \varepsilon^{(1)} - \frac{15\sqrt{3}}{4} \varepsilon^{(2)} \right) \\
& \quad + 3q_1 q_2^2 \left(-\frac{255}{8} - \frac{135}{4} A_1 + \frac{65}{8} A_2 \right. \\
& \quad \quad \left. + \frac{75}{12} \varepsilon^{(1)} - \frac{5}{8} \varepsilon^{(2)} \right) \\
& \quad + q_2^3 \left(-\frac{135\sqrt{3}}{8} - \frac{25\sqrt{3}}{4} A_1 - \frac{47\sqrt{3}}{8} A_2 \right. \\
& \quad \quad \left. + \frac{217\sqrt{3}}{16} \varepsilon^{(1)} + \frac{395\sqrt{3}}{32} \varepsilon^{(2)} \right) \left. \right\} \\
& + \frac{1}{24} \left\{ q_1^4 \left(-\frac{855}{16} - \frac{1255}{32} A_1 + \frac{335}{16} A_2 \right. \right. \\
& \quad \quad \left. \left. - \frac{1195}{8} \varepsilon^{(1)} - \frac{965}{16} \varepsilon^{(2)} \right) \right. \\
& \quad + 4q_1^3 q_2 \left(\frac{315\sqrt{3}}{16} - \frac{545\sqrt{3}}{32} A_1 - \frac{165\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. + \frac{35\sqrt{3}}{8} \varepsilon^{(1)} - \frac{65\sqrt{3}}{12} \varepsilon^{(2)} \right) \\
& \quad + 6q_1^2 q_2^2 \left(\frac{1395}{16} + \frac{8305}{32} A_1 - \frac{545}{12} A_2 \right. \\
& \quad \quad \left. + \frac{95}{8} \varepsilon^{(1)} - \frac{65}{12} \varepsilon^{(2)} \right) \\
& \quad + 4q_1 q_2^3 \left(\frac{1575\sqrt{3}}{16} + \frac{5655\sqrt{3}}{48} A_1 + \frac{135\sqrt{3}}{16} A_2 \right. \\
& \quad \quad \left. + \frac{105\sqrt{3}}{12} \varepsilon^{(1)} - \frac{75\sqrt{3}}{8} \varepsilon^{(2)} \right) \\
& \quad + q_2^4 \left(\frac{1665}{16} + \frac{10345}{32} A_1 - \frac{6985}{32} A_2 \right. \\
& \quad \quad \left. - \frac{965}{16} \varepsilon^{(1)} + \frac{1055}{32} \varepsilon^{(2)} \right) \left. \right\} \\
& + \frac{\mu(1-\varepsilon^{(2)}) A_2}{(1+e \cos \nu) \{1+3((A_1+A_2)/2)\}}
\end{aligned}$$

$$\begin{aligned}
& \times \left[\left(1 + \frac{3}{2}A_2 + \frac{1}{3}A_1\epsilon^{(1)} + \frac{1}{3}A_2\epsilon^{(2)} + \epsilon^{(2)} \right) \right. \\
& + \left\{ q_1 \left(\frac{3}{2} + \frac{3}{4}A_1 + \frac{3}{2}A_2 + \frac{29}{6}A_1\epsilon^{(1)} + \frac{29}{6}A_2\epsilon^{(2)} \right. \right. \\
& \quad \left. \left. + \epsilon^{(1)} - \frac{3}{2}\epsilon^{(2)} \right) \right\} \\
& + \left\{ q_2 \left(\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{4}A_1 + \frac{\sqrt{3}}{2}A_2 + \frac{\sqrt{3}}{2}A_1\epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{\sqrt{3}}{2}A_2\epsilon^{(2)} + \frac{\sqrt{3}}{3}\epsilon^{(1)} - \frac{13\sqrt{3}}{6}\epsilon^{(2)} \right) \right\} \\
& + \frac{1}{2} \left\{ q_1^2 \left(\frac{3}{4} - \frac{15}{8}A_1 + \frac{15}{2}A_2 + \frac{75}{4}A_1\epsilon^{(1)} \right. \right. \\
& \quad \left. \left. + \frac{75}{4}A_2\epsilon^{(2)} + 5\epsilon^{(1)} - \frac{5}{4}\epsilon^{(2)} \right) \right. \\
& + 2q_1q_2 \left(-\frac{15\sqrt{3}}{4} - \frac{65\sqrt{3}}{8}A_1 - \frac{5\sqrt{3}}{2}A_2 \right. \\
& \quad \left. + \frac{5\sqrt{3}}{12}A_1\epsilon^{(1)} - \frac{45\sqrt{3}}{8}A_2\epsilon^{(2)} \right. \\
& \quad \left. + \frac{5\sqrt{3}}{3}\epsilon^{(1)} - \frac{65\sqrt{3}}{12}\epsilon^{(2)} \right) \\
& + q_2^2 \left(\frac{33}{4} + \frac{195}{8}A_1 - \frac{15}{2}A_2 - \frac{65}{4}A_1\epsilon^{(1)} \right. \\
& \quad \left. - \frac{65}{4}A_2\epsilon^{(2)} - 5\epsilon^{(1)} + \frac{65}{4}\epsilon^{(2)} \right) \left. \right\} \\
& + \frac{1}{6} \left\{ q_1^3 \left(-\frac{75}{8} + \frac{250}{12}A_1 + \frac{435}{16}A_2 \right. \right. \\
& \quad \left. \left. - \frac{35}{8}\epsilon^{(1)} + \frac{85}{12}\epsilon^{(2)} \right) \right. \\
& + 3q_1^2q_2 \left(-\frac{45\sqrt{3}}{8} - \frac{185\sqrt{3}}{16}A_1 + \frac{65\sqrt{3}}{8}A_2 \right. \\
& \quad \left. + \frac{35\sqrt{3}}{12}\epsilon^{(1)} - \frac{85\sqrt{3}}{12}\epsilon^{(2)} \right) \\
& + 3q_1q_2^2 \left(\frac{255}{8} + \frac{985}{16}A_1 - \frac{1125}{32}A_2 \right. \\
& \quad \left. + \frac{1255}{32}\epsilon^{(1)} - \frac{895}{16}\epsilon^{(2)} \right) \\
& + q_2^3 \left(-\frac{135\sqrt{3}}{8} + \frac{295\sqrt{3}}{16}A_1 - \frac{1155\sqrt{3}}{16}A_2 \right. \\
& \quad \left. + \frac{255\sqrt{3}}{32}\epsilon^{(1)} - \frac{175\sqrt{3}}{16}\epsilon^{(2)} \right) \left. \right\} \\
& + \frac{1}{24} \left\{ q_1^4 \left(-\frac{855}{16} - \frac{3575}{32}A_1 + \frac{4725}{32}A_2 \right. \right. \\
& \quad \left. \left. - \frac{785}{12}\epsilon^{(1)} - \frac{125}{12}\epsilon^{(2)} \right) \right. \\
& + 4q_1^3q_2 \left(\frac{315\sqrt{3}}{16} - \frac{485\sqrt{3}}{8}A_1 + \frac{1165\sqrt{3}}{32}A_2 \right. \\
& \quad \left. + \frac{655\sqrt{3}}{32}\epsilon^{(1)} - \frac{95\sqrt{3}}{8}\epsilon^{(2)} \right) \\
& + 6q_1^2q_2^2 \left(\frac{1235}{16} + \frac{565}{16}A_1 - \frac{1035}{32}A_2 \right. \\
& \quad \left. + \frac{665}{32}\epsilon^{(1)} - \frac{765}{16}\epsilon^{(2)} \right) \\
& + 4q_1q_2^3 \left(-\frac{1575\sqrt{3}}{16} - \frac{1285\sqrt{3}}{32}A_1 + \frac{3255\sqrt{3}}{12}A_2 \right. \\
& \quad \left. - \frac{885\sqrt{3}}{12}\epsilon^{(1)} - \frac{1225\sqrt{3}}{12}\epsilon^{(2)} \right) \\
& + q_2^4 \left(\frac{1065}{16} + \frac{10865}{32}A_1 - \frac{7655}{32}A_2 \right. \\
& \quad \left. - \frac{1255}{12}\epsilon^{(1)} - \frac{895}{12}\epsilon^{(2)} \right) \left. \right\}. \tag{A.8}
\end{aligned}$$

Equating the coefficient of the second order form and the expression of Hamilton function (A.8), we get

$$\begin{aligned}
H_2 &= \frac{p_1^2 + p_2^2}{2} + (P_1q_2 - P_2q_1) + e \cos \nu \frac{(q_1^2 + q_2^2)}{2(1 + e \cos \nu)} \\
&- \frac{1}{(1 + e \cos \nu) \{1 + 3((A_1 + A_2)/2)\}} \\
&\times \left[\left(-\frac{1}{8} + \frac{9}{8}A_1 - \frac{9}{16}A_2 - \frac{73}{24}A_1\epsilon^{(1)} \right. \right. \\
& \quad \left. + \frac{11}{48}A_2\epsilon^{(2)} - \frac{27}{16}\mu A_1 + \frac{27}{16}\mu A_2 + \frac{17}{8}\mu\epsilon^{(1)} \right. \\
& \quad \left. - \frac{7}{9}\mu\epsilon^{(2)} + \frac{1}{2}\epsilon^{(2)} - \frac{13}{8}\epsilon^{(1)} \right) q_1^2 \\
& + \sqrt{3} \left(\frac{3}{4} + \frac{17}{4}A_1 + \frac{7}{8}A_2 + \frac{139}{24}A_1\epsilon^{(1)} + \frac{15}{8}A_2\epsilon^{(2)} \right. \\
& \quad \left. - \frac{41}{8}\mu A_1 - \frac{41}{8}\mu A_2 - \frac{1}{6}\mu\epsilon^{(1)} + \mu\epsilon^{(2)} - \frac{3\sqrt{3}}{2}\mu \right. \\
& \quad \left. - \frac{1}{3}\epsilon^{(2)} - \frac{7}{12}\epsilon^{(1)} \right) q_1q_2
\end{aligned}$$

$$\begin{aligned}
& + \left(\frac{5}{8} + \frac{27}{8}A_1 + \frac{21}{16}A_2 + \frac{107}{16}A_1\epsilon^{(1)} + \frac{11}{16}A_2\epsilon^{(2)} \right. \\
& - \frac{33}{16}\mu A_1 + \frac{33}{16}\mu A_2 - \frac{7}{8}\mu\epsilon^{(1)} + \frac{11}{8}\mu\epsilon^{(2)} \\
& \left. - \frac{1}{2}\epsilon^{(2)} + \frac{7}{8}\epsilon^{(1)} \right) q_2^2 \Big].
\end{aligned}
\tag{A.9}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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