Extradimensional models are achieving their highest popularity nowadays, among other reasons, because they can plausibly explain some standard cosmology issues, such as the cosmological constant and hierarchy problems. In extradimensional models, we can infer that the four-dimensional matter rises as a geometric manifestation of the extra coordinate. In this way, although we still cannot see the extra dimension, we can relate it to physical quantities that are able to exert such a mechanism of matter induction in the observable universe. In this work we propose that scalar fields are those physical quantities. The models here presented are purely geometrical no matter the fact that Lagrangian is assumed and even the scalar fields are contained in the extradimensional metric. The results are capable of describing different observable cosmic features and yield an alternative to ultimately understand the extra dimension and the mechanism in which it is responsible for the creation of matter in the observable universe.

1. Introduction

Extra dimensions have been an useful artefact in Physics for a long time. Five years after the appearance of Einstein’s General Relativity (GR), T. Kaluza has firstly expanded it to encompass an extra dimension [1]. Kaluza could describe both four-dimensional (4D) GR and Maxwell’s electromagnetism equations from an appropriate choice for the five-dimensional (5D) metric, in which the extra terms depend on the electromagnetic potential \( A_a \) as [2]

\[
\mathcal{G}_{AB} = \begin{pmatrix}
\mathcal{G}_{ab} + A_A A_b \phi^2 & A_A \phi^2 \\
A_A \phi^2 & \phi^2
\end{pmatrix},
\]

with \( A, B \) running from 0 to 4, \( a, b \) running from 0 to 3, and \( \phi \) being a scalar field. Such an achievement can be made by substituting (1) in the 5D version of GR field equations for vacuum, i.e.,

\[
\mathcal{G}_{AB} = 0.
\]

In this way, for Kaluza, matter in 4D space is a purely geometric manifestation of a 5D empty space. This confirmation agrees with a goal once espoused by Einstein, that \textit{matter comes from geometry}, and shall be revisited and clarified.

Since Kaluza’s work, the extradimensional concept has been applied to gravitational waves [3, 4], gravitational lensing [5], and cosmology [6], among many other areas [7–11].

In the present article, the Kaluza’s concept of extra dimension will be taken into account. On this regard, it should be reminded that, in the year of 1926, O. Klein has significantly contributed to Kaluza’s idea [12]. In order to account for the fact that apparently we live in a 4D world, Kaluza has proposed the so-called “cylindrical condition”, which consists in the annulment of all derivatives with respect to the extra coordinate. Klein has made the cylindrical condition less artificial, by arguing that the annulment of the derivatives is a consequence of the extra dimension compactification.

An elegant, simple, and useful form of physically interpreting Kaluza-Klein (KK) model came from a series of
works by P.S. Wesson and collaborators [13–21]. Wesson’s idea, which gave rise to the so-called “Induced Matter Model” (IMM) (sometimes referred to as “Space-Time-Matter Model”), was to collect all the terms coming from the extra dimension in (2) and make them pass to its rhs as an (induced) energy-momentum tensor.

The IMM has been widely applied to cosmology, yielding interesting and testable results [22–27].

In what concerns 5D models, a novel form of understanding the extra dimension is being established [28]. It is stated that we can infer from the extra dimension consequences that it has to do with the origin of matter and particle masses; though to fully understand these will require a better knowledge of the scalar field associated with the extra coordinate. The extra dimension should be accepted, then, in this context, as an useful concept.

From the above paragraph perspective, it is plausible to see the extra dimension as directly related to a scalar field, with this scalar field being responsible for inducing matter in the 4D universe. This is the main goal of the present article, in which the models will be constructed from a purely geometric approach; that is, the scalar fields will be considered as a part of the 5D metric in empty space.

Recently, the Higgs field existence was confirmed in laboratory [29, 30], which has optimized the possibility of existence of other scalar fields, such as quintessence, for example, which are named scalar fields responsible for the universe dynamics [31–34]. Scalar field models are also approached in the study of false vacuum transitions [35, 36], which, for instance, concern to statistical mechanics and also cosmology, the latter because the transition from false to true vacua can be interpreted as transitions between different stages of the universe dynamics [37].

Another contexts where scalar fields play a key role can be found during the generation of coherent structures after cosmic inflation [38–40], in Lorentz and CPT breaking systems [41–46], in regular asymptotically AdS reflecting star backgrounds [47], in hydrodynamic fluctuations [48], and in the production of gravitational waves during preheating after inflation [49, 50].

In the next section we will start relating directly a scalar field with the extra dimension by applying the IMM for two different cases of (1). Our proposal is to construct models that relate different scalar fields with the extra dimension and analyse the referred consequences.

2. Induced Matter Model Applications

In this section we will consider two different cases of KK metric (1) and apply the IMM to them. In both cases, the electromagnetic potential \( A_\mu \) will be neglected.

Case I (\( a = a(t) \) and \( \phi = \phi(t) \)). We are going to start by considering the simplest case for the scalar field of (1), that is, \( \phi = \phi(t) \). The line element can then be written as

\[
\text{d}S^2 = \text{d}t^2 - a(t)^2 \left( \text{d}x^2 + \text{d}y^2 + \text{d}z^2 \right) - \phi(t)^2 \text{d}l^2, \tag{3}
\]

in which \( g_{\mu\nu} \) was taken as the Friedmann-Robertson-Walker metric with null curvature (in accord with observations [51]), \( a(t) \) is the scale factor, which dictates how distances grow in an expanding universe, \( l \) is the extra coordinate, and throughout this work, it will be assumed units such that \( 8\pi G = c = 1 \).

Let us apply the IMM to (3). This consists in evaluating (2), collecting all the extradimensional terms, and relating them with the matter content of the observable universe, that is, matter-energy density \( \rho \) and pressure \( p \).

The Einstein tensor for (3) reads

\[
G_0^0 = 3 \left( \frac{\dot{a}}{a} \right)^2 + 3 \frac{\ddot{a}}{a} \phi^2 \tag{4}
\]

\[
G_1^1 = 2 \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \phi \frac{\ddot{\phi}}{\phi} + 2 \frac{\dot{a} \dot{\phi}}{a \phi} \tag{5}
\]

\[
G_2^2 = G_1^1 \tag{6}
\]

\[
G_3^3 = 3 \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right) \tag{7}
\]

in which dots represent time derivatives. We consider, as in braneworld models, that one cannot find matter in the extra dimension; that is, the energy-momentum tensor of a perfect fluid is written as \( T_\mu^\nu = \text{diag}(\rho, -p, -p, -p, 0) \).

Equations (4), (5), and (7) yield, respectively,

\[
\frac{\dot{a}}{a} = -\frac{\phi}{\dot{\phi}}, \tag{8}
\]

\[
2 \frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2 = -\frac{\phi}{\dot{\phi}}, \tag{9}
\]

with \( \rho = -3(\dot{a}/a)(\dot{\phi}/\phi) \), \( p = 2(\dot{a}/a)(\dot{\phi}/\phi) + \ddot{\phi}/\phi \) and the trivial differential equation

\[
\frac{\dot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 = 0, \tag{10}
\]

whose solution reads

\[
a(t) = a_1 \sqrt{2t - \beta_1}, \tag{11}
\]

with \( a_1 \) and \( \beta_1 \) being constants.

Substituting (11) into (8)-(9) and rearranging them yield

\[
\frac{\dot{\phi}}{\phi} = \frac{3}{\beta_1 - 2t}. \tag{12}
\]

The solution for (12) is

\[
\phi(t) = \gamma_1 - \frac{\delta_1}{\sqrt{2t - \beta_1}}, \tag{13}
\]

with \( \gamma_1 \) and \( \delta_1 \) constants.

Figure 1 shows the time evolution of \( \phi(t) \) for different values of the constants involved.
Figure 1: Case I: time evolution of $\phi$. Dotted (blue) line stands for $\beta_1 = 1$, $\gamma_1 = 10$, and $\delta_1 = 5$, while for the dashed (green) line and solid (red) line, $\beta_2 = 5$, $\gamma_1 = 10$, and $\delta_1 = 1$ and $\beta_1 = 1$, $\gamma_1 = 0$, and $\delta_1 = -1$, respectively.

From (13), the matter-energy density and pressure of the universe in this model read:

\[ \rho(t) = \frac{3\delta_1}{(\beta_1 - 2t)^2 (\delta_1 - \gamma_1 \sqrt{2t - \beta_1})}, \tag{14} \]

\[ \rho(t) = \frac{\delta_1}{(\beta_1 - 2t)^2 (\delta_1 - \gamma_1 \sqrt{2t - \beta_1})}, \tag{15} \]

Case II ($a = a(t, l)$ and $\phi = \phi(t)$). Let us generalize the previous case. If one makes $\phi(t) \rightarrow \phi(t, l)$ in (3), the Einstein tensor stays the same. Therefore, let us make $a(t) \rightarrow a(t, l)$ in (3) as

\[ \frac{d\xi^2}{d\tau^2} - a(t, l)^2 (dx^2 + dy^2 + dz^2) = \phi(t)^2 d\tau^2, \tag{16} \]

i.e., it is assumed that the extra coordinate also influences the scale factor.

The Einstein tensor, in this case, reads

\[ G_0^0 = 3 \left( \frac{\ddot{a}}{a} \right)^2 + 3 \left\{ \frac{\dot{a} \dot{\phi}}{a \phi} - \frac{1}{\phi^2} \left[ \frac{a''}{a} + \left( \frac{a'}{a} \right)^2 \right] \right\}, \tag{17} \]

\[ G_1^1 = 2 \left( \frac{\ddot{a}}{a} \right) + \frac{\dot{a}^2}{a} + \frac{\phi}{\dot{\phi}} + 2 \frac{\ddot{\phi}}{\dot{\phi}}, \tag{18} \]

\[ G_2^2 = G_3^3 = G_4^4, \tag{19} \]

\[ G_4^4 = G_5^5 = 3 \left( \frac{\ddot{a}}{a} \right)^2 - \frac{3}{\phi^2} \left( \frac{a'}{a} \right)^2, \tag{20} \]

\[ G_6^6 = G_7^7 = 3 \left( \frac{a' \dot{\phi}}{a \phi} - \frac{a'}{a} \right), \tag{21} \]

with $' = \partial / \partial l$.

The IMM application in (17), (18), (20), and (21) yields

\[ \left( \frac{\ddot{a}}{a} \right)^2 = - \frac{\ddot{\phi}}{\dot{\phi}} + 1 \left[ \frac{a''}{a} + \left( \frac{a'}{a} \right)^2 \right], \tag{22} \]

\[ 2 \frac{\ddot{a}}{a} = - \frac{\dot{\phi}}{\phi} - \ddot{a} \frac{\dot{\phi}}{a} + 1 \frac{a''}{a}, \tag{23} \]

\[ \frac{\ddot{a}}{a} + \frac{\ddot{a}}{a} + \left( \frac{a'}{a} \right)^2 = 0, \tag{24} \]

\[ \frac{\dot{\phi}}{\phi} = \frac{a'}{a}, \tag{25} \]

Solving (22)-(25) for $a(t, l)$ yields

\[ a(t, l) = \alpha_2 \left[ (2\kappa - 1) t \right]^{(1/2)(3\kappa-1)/(2k-1)} e^{\lambda l}, \tag{26} \]

with $\alpha_2$ being an arbitrary constant and $\kappa$ being the constant of separation of variables.

From solution (26), we can write the Hubble and deceleration parameters, respectively, as

\[ H(t) = \frac{1 - 3\kappa}{1 - 2\kappa 2t}, \tag{27} \]

\[ q(t) = \frac{\kappa - 1}{3\kappa - 1}, \tag{28} \]

with the former being related to the recession velocity of galaxies according to the Hubble's law, $H = \dot{a}/a$, and the latter being defined so that $q < 0$ represents an accelerated expansion of the universe, with $q = -\ddot{a}/a^2$.

In the next section, some cosmological features of Cases I and II of the geometric model described by metric (1) will be presented.

3. Cosmological Features

Let us analyse the solutions obtained in the last section from a cosmological perspective. We will start recalling solutions (14) and (15) of Case I. Remarkably, the relation between these equations is a factor of 3. The relation between $\rho$ and $\rho$ is known as equation of state (EoS) $\omega$ and it is well known that $\omega = 1/3$ for the radiation-dominated era of the universe [52].

Therefore, purely from geometry (see (3)), relativistic matter could be described. Specifically, this is a radiation-dominated universe model. In fact, such an argumentation is confirmed from (11) since in standard model of cosmology, $a(t) \sim \sqrt{t}$ is exactly the time proportionality of the scale factor in a radiation-dominated universe [52].

It is also possible to corroborate (13) as related to a scalar field which describes the radiation-dominated universe. On this regard, in [53] a cosmological model from a primordial scalar field was constructed. The solution they have obtained for the scalar field was $\sim 1/\sqrt{t}$, in agreement with (13).

There is another form of indicating the correlation between solution (13) and the radiation era, by analysing Figure 1. By firstly checking the dotted (blue) and dashed
standard cosmology [51,60,61]. The acceleration, evading the cosmological constant problem of an extra dimension may be the responsible for the cosmic evolution. That is, a scalar field "living" in the extra dimension inducing the counter-intuitive effect of accelerated expansion, together with a scale factor that "feels" the presence of such an extra dimension may be the responsible for the cosmic acceleration, evading the cosmological constant problem of standard cosmology [51, 60, 61].

From Figure 3, one can note that the value of \(l\), i.e., the length scale of the extra coordinate, has a profound influence on the evolution of the scale factor. One realizes that the smaller the value of \(l\), the greater the age of the universe, which is the value of \(t\) for \(a(t,l) = a_0 = 1\) (the present value of the scale factor [52]). According to observations of fluctuations on the temperature of cosmic microwave background radiation [51], the present age of the universe is \(\sim 1.4 \times 10^{10}\) yr. When inserting the natural constants in Case II, constrains to \(l\) might be obtained. One expects such constraints to respect others, obtained by completely different approaches [62–64].

4. Discussion

In the present article, we have constructed Kaluza-Klein models purely from the geometrical sector of a gravitational theory. The material content of the models has arisen as a geometrical manifestation of an extradimensional empty space-time.

Particularly, we have investigated the importance of scalar fields as extradimensional components of the metric. For the sake of generality, two cases for the metric (I), named \(a = a(t)\) and \(a = a(t,l)\), were investigated. In both cases, \(A_\alpha = 0\); models including non-null electromagnetic potentials might be reported as a forthcoming work.

Case I has shown to be able to describe relativistic matter as a geometrical manifestation of a 5D empty space-time. The features of a radiation-dominated universe could be observed in the scale factor solution that evolves as a standard Friedmann-Robertson-Walker universe dominated by radiation, while in standard cosmology, the solutions for a radiation-dominated era are obtained from the assumption of an EoS like \(p = \rho/3\), in the present model, such an EoS was obtained (rather than assumed), as one can check (14)–(15). Beyond that, the solution (13) for the scalar field responsible for inducing matter in the 4D observable universe agrees with that obtained in [53], for which the authors have constructed a cosmological model from a primordial scalar field.

In Case II, the induced cosmological solutions led to an accelerated expansion of the universe. This was reflected in
the negative values of $q$, which, for a range of values for the free parameter $\kappa$, was in agreement with observations [57]. In this manner, this model has shown that it is possible to describe the cosmic acceleration as a purely geometrical effect, evading the cosmological constant problem [51, 60, 61].

It is interesting to note that, for $\kappa = 0$ in Case II, the radiation-dominated universe is retrieved, since $q \rightarrow 1$ as $\kappa \rightarrow 0$ and 1 is the deceleration parameter value at the radiation-dominated stage, according to standard cosmology [52].

The same happens if one applies the cylindrical condition in Case II. In fact, in KK theory, when recovering 4D gravity and Maxwell’s equations from the cylindrical condition application to (1)-(2), the energy-momentum tensor induced by extradimensional geometrical terms is the one of electromagnetism, i.e., $T^M_{ab} = g_{ab} F_{cd} F^{cd}/4 - F_{da} F_{bc}$, with $F_{ab} \equiv \partial_a A_b - \partial_b A_a$. At the same time, it can be shown that the cylindrical condition application yields the EoS of radiation [2]. Such features were obtained in the present model, as one can see that Case I can be obtained from the cylindrical condition application in Case II.

One may wonder about the detectability of the extra coordinate. As we mentioned in the Introduction, a new concept to understand the extra dimension has been established, in which the extra coordinate has to do with the creation of matter in the observable universe [28]. As pointed by the authors in [28], the extra dimension should be in this sense accepted as a useful concept, and to better understand the way it induces matter in the 4D observable universe requires a better knowledge of the scalar field associated with the extra coordinate. That is why we have inserted in the extra part of the metric a scalar field $\phi$.

Such a scalar field has shown to indeed have an important role in inducing different kinds of matter in the 4D universe. When the scale factor $a$ depends on $t$ only, $\phi$ has shown to be responsible for inducing relativistic matter in the observable universe. Remarkably, when $a$ depends on $t$ and $l$, the field induces the fluid responsible for accelerating the expansion of the universe in standard model, i.e., dark energy. This was shown from the negative values obtained for $q$ with no need of a cosmological constant.

In this way, from the approached perspective of the interpretation of the extra dimension, those matter fields in the observable universe are nothing but manifestations of the extra coordinate, recalling that the model does not need any kind of matter Lagrangian. In other words, by facing the extra coordinate as responsible for inducing matter in the 4D universe, as it was assumed throughout this paper, the appearance of the matter fields, such as radiation and dark energy, is nothing but manifestations and evidences for the existence of the extra dimension in the present model.

We would like to emphasize that by using scalar fields as extradimensional metric components we can open a new window to explain some standard cosmology issues. Therefore, in order to deal with these questions, we shall present in near future an approach in which the line element has a background with two interacting scalar fields. In this case, we hope that, from such an inclusion, a greater number of new cosmological effects and phenomenological observations can be described.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

P. H. R. S. Moraes would like to thank São Paulo Research Foundation (FAPESP), Grant 2015/08476-0, for financial support. R. A. C. Correa is partially supported by FAPESP (Foundation for Support to Research of the State of São Paulo) under Grants nos. 2016/03276-5 and 2017/26646-5.

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