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*Research Article*

# Tsallis Holographic Dark Energy with Granda-Oliveros Scale in $(n+1)$-Dimensional FRW Universe

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Based on Tsallis holographic dark energy model recently proposed by using the general model of the Tsallis entropy expression, we reconstruct cosmographic parameters, $q, j, \kappa, l$, and we study their evolution in spatially flat $(n+1)$-dimensional Friedmann-Robertson-Walker universe using Granda-Oliveros scale. Our results show that the universe is in an accelerating expansion mode described by phantom-like behavior. We go further and study the state finder operators and the *Om* diagnostic to understand the behavior of our model. The stability of the system is also studied by using the square of speed of sound showing that our model is stable over the low range of red-shift considered. The results indicate that the entropy formalism will play an important role in understanding the dynamics of our universe.

## 1. Introduction

The study of accelerated expansion of the universe becomes one of the main hot topics in cosmology in the last few years [1–3]. Many theories are considered to explain this behavior. Actually, scientists believe that this accelerating behavior is mainly due to some repulsive gravity at large scale which is caused by a nonstandard component with negative pressure representing around two-thirds of universe known as dark energy *DE*. This accelerating behavior is characterized by equation of state (EoS) parameter which lies in a narrow region around the value EoS = -1. The region corresponding to EoS parameter < -1 belongs to what is known as phantom dark energy (PDE) with an energy having positive energy density, but negative pressure. The PDE is theoretically proposed by some models like braneworlds models or scalar-tensor gravity model [4–11].

Recent observations by Planck 2015, Baryon Acoustic Oscillations (BAO), Supernova Type Ia (SNIa), Large Scale Galaxy Surveys (LSS), and Weak Lensing (WL), show that $\Lambda CDM$ is the best candidate to explain the present acceleration of the universe [1–3, 12].

An interesting work by M. Li [13] assumed a new dark energy model called holographic dark energy model (HDE) based on what is called holographic principle. In this model the future event horizon of the universe can be used as infrared cutoff. Actually, This model gives a good explanation for accelerated expansion nature of the universe and that fits the current observations. Tsallis and Cirto assumed some quantum modification for HDE by assuming that the black hole horizon entropy could be given by [14]

$$S_\delta = \gamma A^\delta, \tag{1}$$

where $\gamma$ is an unknown constant and $\delta$ denotes the nonadditivity parameter chosen to have a positive value. By choosing $\delta = 1$ and $\gamma = 1/4G$ the Bekenstein entropy is easily recovered [14].

The holographic principle, which states that the number of degrees of freedom of a physical system should be scaled with its bounding area rather than with its volume [13], should be constrained by an infrared cutoff. Cohen et al. [15] proposed a relation between the system entropy $S$ and the IR $L$ and UV $(\Lambda)$ cutoffs as [16]

$$L^3\Lambda^3 \leq S^{3/4}. \tag{2}$$

After combining (1) with (3), one finds

$$\Lambda^4 \leq \gamma (4\pi)^\sigma L^{2\delta-4}, \tag{3}$$

where $\Lambda^4$ stands for the energy density of vacuum which is denoted as $\rho_D$, the energy density of dark energy. Using this inequality, a new Tsallis holographic dark energy density (THDE) is established as [14]

$$\rho_D = BL^{2\delta-4}, \tag{4}$$

where $B$ is an unknown parameter [14]. In this work we will use the IR cutoff $L$ as Granda-Oliveros ($GO$) scale $L_{GO}$ which has the following form [17]:

$$L_{GO} = \left(\beta\dot{H} + \alpha H^2\right)^{-1/2}, \tag{5}$$

where $\alpha$ and $\beta$ are two positive constant parameters and H is the Hubble parameter. By combining (4) with (23) one can write THDE in $GO$ scale as

$$\rho_D = B\left(\beta\dot{H} + \alpha H^2\right)^{-\delta+2}. \tag{6}$$

For flat case, which we are going to consider in this study, the best estimated values for $\alpha$ and $\beta$ are $\alpha \sim 0.8502$ and $\beta \sim 0.4817$ as mentioned in [18]. Actually, dark energy theories use Granda-Oliveros scale as IR scale depending only on local quantities; that way it is possible to avoid the causality problem; moreover it is also possible to obtain the accelerated expansion mode of the universe [17].

In this work we will study the Tsallis holographic together with (n+1)-dimensional universe to explore the nature of our universe.

This paper is organized as follows: In the next section the cosmological model is considered. In Section 2 cosmography is discussed. In Section 3, we study statefinder operators of our model. In Section 4 reconstructed cosmological parameters are deduced and then the stability of our model is assumed through the study of the square of speed of sound. Finally, the conclusion is presented.

## 2. The Cosmological Model

In this section, we are going to give a brief review about $(n+1)$-dimensional FRW universe with Granda-Oliveros cutoff. The metric of $(n+1)$-dimensional Lorentzian isotropic and homogeneous space-time is described by [19]

$$ds^2 = -dt^2 + a(t)^2 g_{ij}dx^i dx^j, \tag{7}$$

where $i, j = 1, \ldots, n$, $t$ is the cosmic time and $g_{ij}$ is the metric of an $n$-dimensional manifold $M$ with curvature $k = 1, 0, -1$ for open, flat, and closed universe. Since $M$ is an $n$-hyperboloid, then the flat space has metric on the form [19]

$$ds^2 = \frac{1}{1-kr^2}dr^2 + r^2 d\Omega_{n-1}^2, \tag{8}$$

where $r$ is the radial variable and is assumed to have a positive value and $d\Omega_{n-1}^2$ is the metric of $(n-1)$-dimensional sphere. Inserting (7) and (8) into the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{9}$$

where $G_{\mu\nu}$ is the Einstein tensor, $G$ the universal gravitational constant, $\Lambda$ the cosmological constant, and $T_{\mu\nu}$ the energy momentum tensor of an ideal fluid, we deduced

$$\dot{H} - \frac{k}{a^2} = \frac{8\pi G}{n-1}\rho_{tot}\left(1+\omega_D\right), \tag{10}$$

and

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n(n-1)}\rho_{tot}, \tag{11}$$

which is the Friedmann equation in (n+1)-dimensional universe. Here, the Hubble parameter is represented by $H = \dot{a}(t)/a(t)$, the dot represents the first derivative with respect to cosmic time $t$, and the total energy density of the universe $\rho_{tot} = \rho_D + \rho_m$, $\rho_D$ and $\rho_m$ are the energy densities of $DE$ and $DM$, respectively. Assuming a flat FRW universe $k = 0$, for DE-dominated universe filled with THDE, one can write after some algebraic steps

$$H = \left(\frac{\delta t\left((n-1)n/B\right)^{1/(2-\delta)}}{\beta(\delta-2)}\right)^{(\delta-2)/\delta}, \tag{12}$$

and this is the mathematical expression for the Hubble parameter for our model.

In all the analyses below we chose $n = 3, 4, 5$ (red, blue, and black lines, respectively), also we consider the approximation $8\pi G = 1$ and $B = 2.1$, present value of fractional $DE$ is $\Omega_D = 0.70$ [14], and present value of Hubble is $H_0 = 73 kms^{-1}Mpc^{-1}$ [20].

## 3. Cosmography

To get more information about the universe evolution we study the higher order time derivative of the scale factor $a$ called cosmographic parameters in framework of $(n+1)$-dimensional FRW universe. In a cosmological terms the first five time derivatives of the scale factor $a$ are used to define Hubble, deceleration $q$, jerk $j$, snap $\kappa$, and lerk $l$ parameters, respectively, as [21]

$$H = \frac{1}{a}\dot{a}, \tag{13}$$

$$q = -\frac{1}{H^2}\frac{da^{(2)}}{a}, \tag{14}$$

$$j = \frac{1}{H^3}\frac{da^{(3)}}{a} = (1+z)\frac{dq}{dz} + q(1+2q), \tag{15}$$

$$\kappa = \frac{1}{H^4}\frac{da^{(4)}}{a} = -(1+z)\frac{dj}{dz} - j(2+3q), \tag{16}$$

$$l = \frac{1}{H^5}\frac{da^{(5)}}{a} = -(1+z)\frac{d\kappa}{dz} - \kappa(3+4q). \tag{17}$$

Hubble parameter has a dimension of the inverse of time, while the rest of the cosmographic parameters are dimensionless parameters. By integrating (12) with respect to

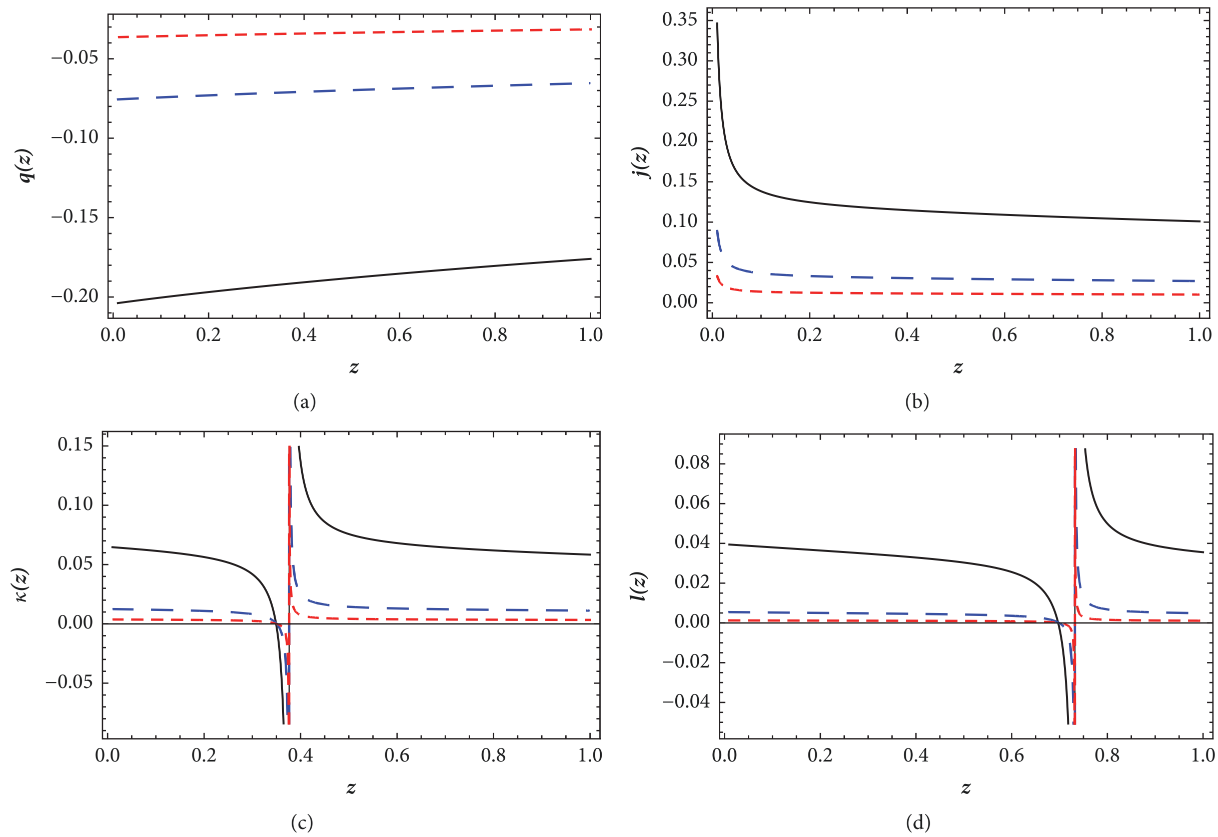


FIGURE 1: Evolution of (a) deceleration, (b) jerk, and (c) snap parameters (d) as function of red-shift $z$ for $\alpha = 0.8502$ and $\beta = 0.4817$.

the cosmic time, one can write the cosmic scale factor as

$$a = \exp\left(\frac{t^{(\delta-2)/\delta+1}\left(\delta\left((n-1)\,n/B\right)^{1/(2-\delta)}/\beta\left(\delta-2\right)\right)^{(\delta-2)/\delta}}{(\delta-2)/\delta+1}\right). \quad (18)$$

From observational and theoretical views the deceleration parameter $q$ is main indicator about the expansion or contraction of universe according to its sign. The third order derivative, the jerk parameter $j$, represents the time variation of deceleration parameter. Since $q$ is observed, $j$ is used to predict the future of the universe. Actually the higher order derivatives like jerk and snap factors are mainly used to study the dynamics of the universe, because they could be related to the emergence of the sudden future singularities [21].

By using the relation between the cosmic time and the red-shift [22],

$$t = \frac{2}{H_0\left((z+1)^2+1\right)}. \quad (19)$$

Figure 1 represents the evolution of $q$, $j$, and $\kappa$ against the red-shift. In Figure 1(a), we notice that $q < 0$ and that agrees with the observation that constrains the necessary condition for continuous expansion as $-1 < q < 0$. In Figure 1(b), we notice that the jerk parameter at $z \simeq 0$ shows a decreasing behavior until approximately $z \sim 0.05$ and then tends to have a constant value indicating that the rate of change of deceleration is constant, which is away from $j = 1$ that is corresponding to $\Lambda CDM$; this may indicates that our model is not a $\Lambda CDM$. In Figures 1(c) and 1(d) the snap $\kappa$ and lerk factors fluctuate around the zero showing some discontinuity in behavior around $z \sim 0.4$ and $z \sim 0.7$, respectively.

## 4. Statefinder Operators

In this section, we review the geometrical statefinder operators which are used to understand the differences between different dark energy models. The general expressions take the following forms [23]:

$$r = 2q^2 + q - \frac{\dot{q}}{H}, \quad (20)$$

$$s = \frac{r-1}{3\left(q-1/2\right)}. \quad (21)$$

By using (21) and (20), we plot the $s-r$ plane as shown in Figure 2(a), showing a decreasing behavior of our model over the given red-shift range. We observe that trajectory reaches the quadrant where $r$ is positive and $s$ is negative for all considered dimensions. Actually, we should point to the fact that $s > 0$ represents a quintessence-like behavior

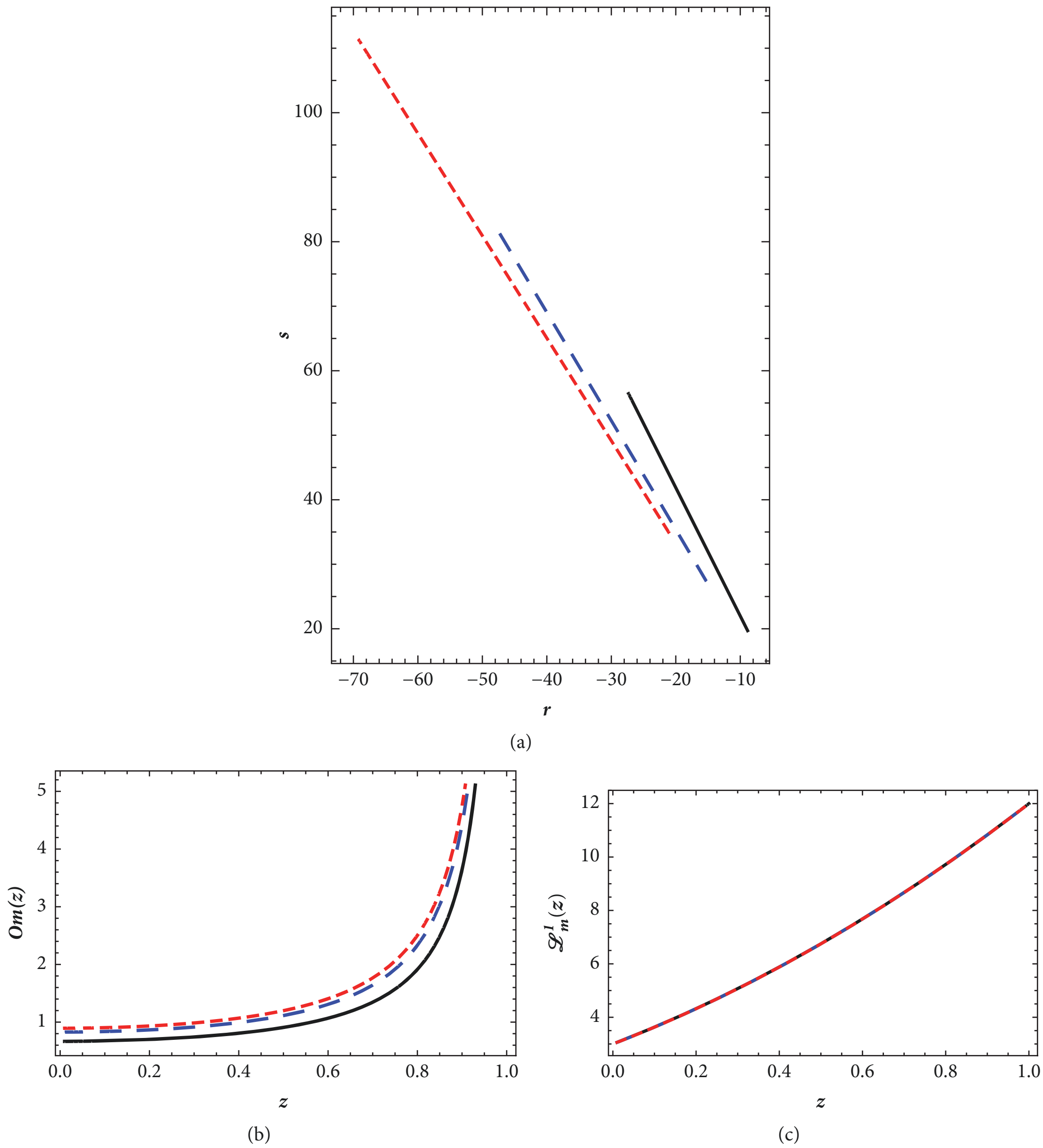


FIGURE 2: (a) The $rs$ trajectories, (b) $Om$, (c) $\mathscr{L}_m^1$ as function of red-shift $z$ for $\alpha = 0.8502$ and $\beta = 0.4817$.

but $s < 0$ represents a phantom-like behavior of the model while the point (1,0) represents $\Lambda CDE$ [23]. Now, we study a diagnostic parameter tool called $Om$ [24]. This parameter is used to study different stages of the universe. The positive slope values of $Om$ indicate phantom-like behavior while its negative values correspond to the quintessence region. The $Om$ diagnostic is defined as

$$Om(z) = \frac{H(z)^2/H_0^2 - 1}{(z+1)^3 - 1}. \tag{22}$$

From Figure 2(b) a positive slope value of $Om$ indicates a phantom-like behavior representing a universe expansion without bound. By using the first order derivative of $Om$ we can define another diagnostic parameter $\mathscr{L}_m^1$ as [21]

$$\begin{aligned} \mathscr{L}_m^1 &= 2z\left(z^2 + 3z + 3\right) h(z)\, h_1(z) \\ &\quad + 3(z+1)^2\left(1 - h(z)^2\right). \end{aligned} \tag{23}$$

where $h(z) = H(z)/H_0$ and $h^1 = dh/dz$. The evolution of $\mathscr{L}_m^1$ versus $z$ is shown in Figure 3(c). One can observe that the change in dimension $n$ has no effect on the behavior $\mathscr{L}_m^1$. We notice that $\mathscr{L}_m^1$ shows an increasing behavior indicating that our model is not a $\Lambda CDM$ model because $\mathscr{L}_m^1 \neq 0$.

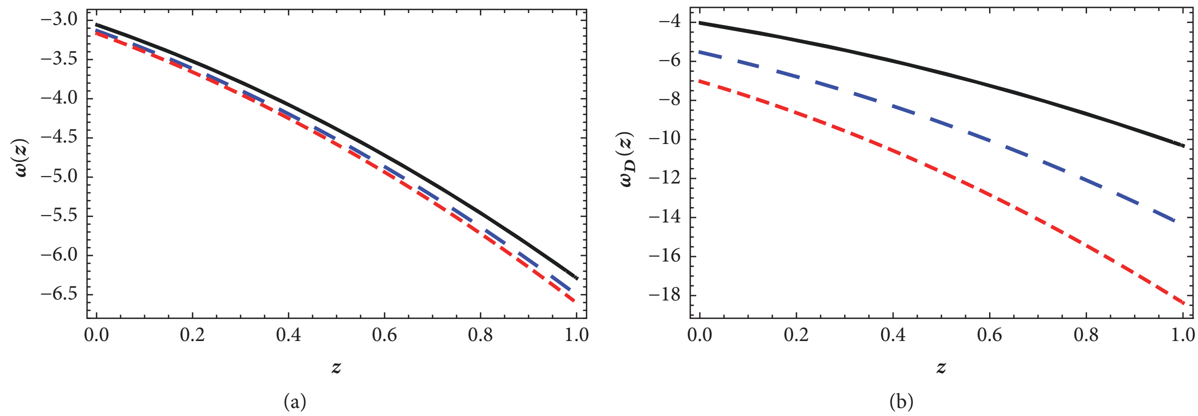


FIGURE 3: Evolution of EoS as function of red-shift $z$ for $\alpha = 0.8502$ and $\beta = 0.4817$, (a) noninteracting and (b) interacting cases.

## 5. Reconstructed Cosmological Parameters

Now, we are going to study some reconstructed cosmological parameters for both noninteracting and interacting cases [25]; the continuity equation is given by

$$\dot{\rho} + 3H\rho_{tot}(1+\omega) = 0, \tag{24}$$

and for dark-energy-dominated universe one can write (24) [26] as

$$\dot{\rho_D} + 3H\rho_D(1+\omega_D) = 0. \tag{25}$$

For interacting case, one finds

$$\dot{\rho_D} + 3H\rho_D(1+\omega_D) = -Q, \tag{26}$$

where $Q$ represents the interaction term which can be considered as an arbitrary function of cosmological parameters, like the Hubble parameter $H$ and energy densities of the model. We assume that classical interaction form for dark-energy-dominated universe $Q = 3bH\rho_D$ [27], where $b$ is the coupling parameter between dark matter and dark energy and is constrained by experiment to be less than 0.025. Using (25) and (26)

$$\omega = -\left(\frac{\dot{\rho_D}}{3H\rho_D} + 1\right) \tag{27}$$

and for interacting case

$$\omega_I = -\left(\frac{\dot{\rho_D} + 3H\rho_D(1+b)}{3H\rho_D}\right). \tag{28}$$

Figure 3 shows the evolution of EoS as a function of $z$ for both noninteracting and interacting cases; one notice that $\omega < -1$ indicating a phantom-like behavior and the universe expands without limit.

The fractional energy densities for DE, DM, and curvature parameter are given by the following quantities [28]:

$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{8\pi G(t)\rho_D}{6H^2}, \tag{29}$$

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi G(t)\rho_m}{6H^2}, \tag{30}$$

$$\Omega_k = \frac{\rho_k}{\rho_{cr}} = \frac{k}{a^2H^2}, \tag{31}$$

where $\rho_{cr}$ indicates the critical energy density and G is the gravitational constant. By using (29) and (24), one can calculate the derivative of $\Omega_D$ with respect to the cosmic time $t$. Since $\Omega_D' = \dot{\Omega}_D/H$, we can establish a mathematical expression for the fractional DE density as

$$\Omega_D' = -\frac{2\Omega_D}{H(z)^2} - 3\Omega_D(\omega(z)+1) \tag{32}$$

and for interacting case

$$\Omega_{DI}' = -\frac{2\Omega_D}{H(z)^2} - 3\Omega_D(\omega_{DI}(z)+1). \tag{33}$$

We notice from Figure 4(a) for noninteracting and Figure 4(b) for interacting cases that $\Omega_D' > 0$; this positive increasing behavior indicates that the dark energy now is greater than before.

## 6. Square Speed of Sound

$$V_D^2 < 0 \tag{34}$$

Now we analyze the stability of our model by considering the quantity $V_D^2$. It is known that the model is stable for $V_D^2 > 0$ while unstable for $V_D^2 < 0$; the general form of $V_D^2$ is defined as follows [29].

$$V_D^2 = \frac{\dot{P_D}}{\dot{\rho_D}} = \dot{\omega_D}\frac{\rho_D(z)}{\dot{\rho_D}} + \omega_D \tag{35}$$

The growth of $V_D^2$ against $z$ is considered in Figure 5(a) for noninteracting and Figure 5(b) for interacting cases; we observe that $V_D^2 > 0$, showing that the model under study is stable.

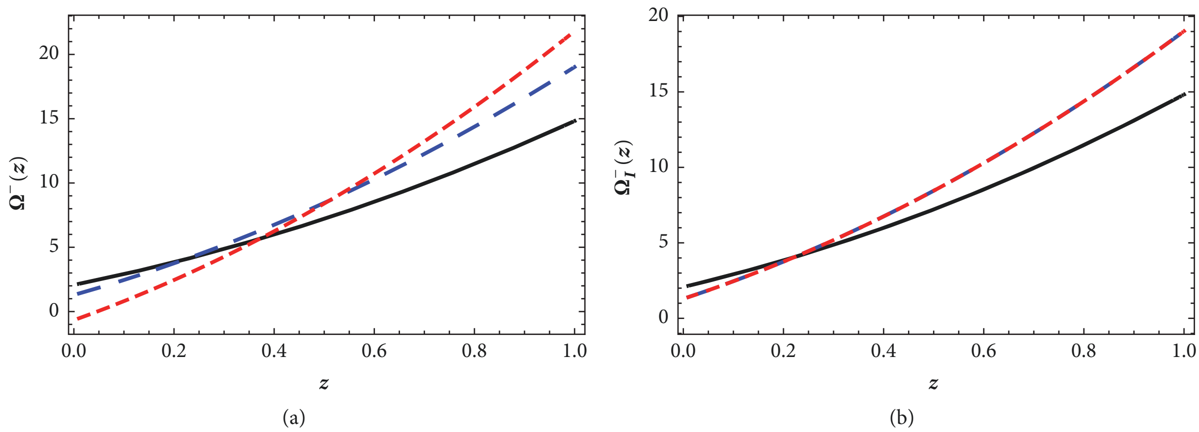


FIGURE 4: Evolution of fraction dark energy as function of red-shift $z$ for $\Omega_D = 0.70$ and $H_0 = 73kms^{-1}Mpc^{-1}$, (a) noninteracting and (b) interacting cases.

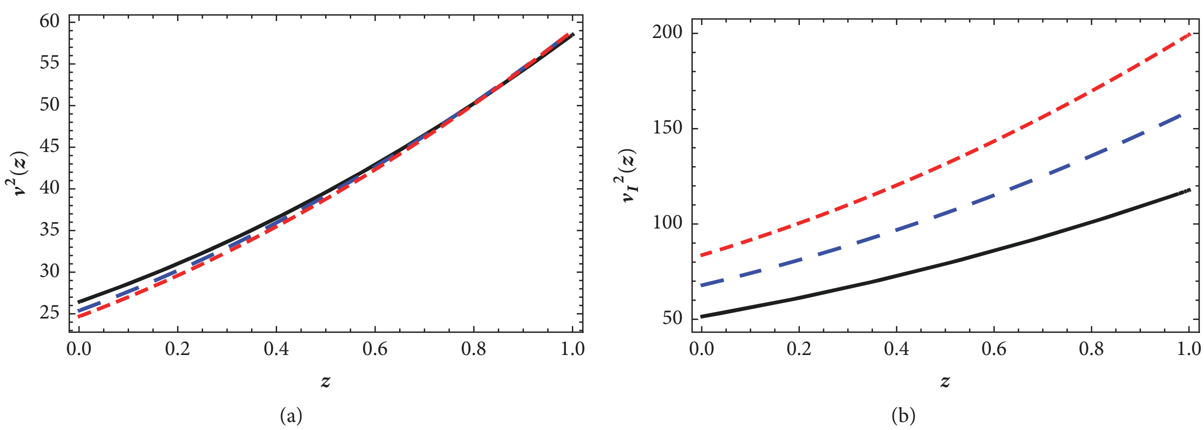


FIGURE 5: Evolution of square speed of sound as function of red-shift $z$ for $\alpha = 0.8502$ and $\beta = 0.4817$, (a) noninteracting and (b) interacting cases.

## 7. Conclusion

In current work, we consider a Tsallis holographic dark energy model (THDEM) to study the evolution of universe in the framework of $(n+1)$-dimensional FRW universe with Granda-Oliveros scale as IR cutoff.

At early universe the energy was high and the universe is small so that our 4 dimensions should be replaced by a higher dimension analysis. Actually the main benefit of this work is to consider a higher dimension treatment by adding more spatial dimensions than the three known ones, making both the scale factor $a$ and Hubble parameter $H$ different and more general than those at normal dimensions. Generally, studying cosmology at higher dimensions is a good treatment to study late time accelerating expansion behavior of our universe. By applying the higher dimension mechanism for Tsallis holographic Dark Energy, assuming spatial dimensions with values n=3,4, and 5, the results obtained fit the observation.

To get further information about the universe behavior and discriminate between different DE models, we study the cosmographic parameters which are the higher order time derivative of scale factor $a$, namely, deceleration $q$, jerk $j$, snap $\kappa$, and lerk $l$ factors. The results show that the universe is in an accelerating expansion mode since $q < 0$. The trajectory of $s-r$ is studied showing a decreasing behavior over the range considered. The diagnostic parameter $Om$, which is obtained from the Hubble parameter, shows a continuous increasing behavior with a positive slope indicating a phantom-like behavior. What is new in this work is the study of the diagnostic parameter $\mathscr{L}_m^1$ which is obtained from first order derivative of $Om$-diagnostic, indicating that our model is not a $\Lambda CDM$ model because $\mathscr{L}_m^1 \neq 0$. Some reconstructed cosmological parameters are established for both noninteracting and interacting cases, where the interaction term is considered to take the form $Q = 3bH\rho_D$ to examine our model with respect to information extracted from observations. For the present values of the cosmic parameters a phantom DE universe $\omega_D < -1$ is observed for both cases. Assuming a phantom scale we notice that $\Omega_D'$ is positive for both cases representing an increase in dark energy with time and this agrees with the cosmological observation. Actually, for a phantom dark-energy-dominated universe both of the cosmological scale factor and Hubble parameters grow rapidly, deceleration parameter becomes very negative, and the universe reaches the accelerated expansion epoch.

Finally, by using the constraints on the *EoS* we studied the stability of the model using $V_D^2$, and we find for both cases that $V_D^2 > 0$ over the red-shift range, so our model is stable which is missed in most of previous work in THDE model found in the literature. What is quite interesting is that our model gives a good consistency with observation so it can be considered to study the universe evolution.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

There are no conflicts of interest regarding the publication of this paper.

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