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Research Article

A Cosmological Scenario from the Starobinsky Model within the $f(R,T)$ Formalism

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In this paper we derive a novel cosmological model from the $f(R,T)$ theory of gravitation, for which R is the Ricci scalar and T is the trace of the energy-momentum tensor. We consider the functional form $f(R,T) = f(R)+f(T)$, with $f(R)$ being the Starobinsky Model, named $R + \alpha R^2$, and $f(T) = 2\gamma T$, with α and γ being constants. We show that a hybrid expansion law form for the scale factor is a solution for the derived Friedmann-like equations. In this way, the model is able to predict both the decelerated and the accelerated regimes of expansion of the universe, with the transition redshift between these stages being in accordance with recent observations. We also apply the energy conditions to our material content solutions. Such an application makes us able to obtain the range of acceptability for the free parameters of the model, named α and γ.

1. Introduction

The $f(R)$ theories of gravity [1, 2] are an optimistic alternative to the shortcomings General Relativity (GR) faces as the underlying gravitational theory, such as those discussed by Padmanabhan [3]; Antoniadis et al. [4]; Demorest et al. [5]; Bull et al. [6]. They can account for the cosmic acceleration [7, 8] with no need for a cosmological constant, providing good match between theory and cosmological observations [9–11].

Particularly, in standard cosmology, derived from GR, the dark energy and dark matter should compose ~ 95% of the universe [12], but their nature is still dubious [13, 14].

Another crucial trouble surrounding GR is the difficulty in quantizing it. Attempts to do so have been proposed [15–17] and can, in future, provide us a robust and trustworthy model of gravity-quantum mechanics unification.

Meanwhile it is worthwhile to attempt to consider the presence of quantum effects in gravitational theories. Those effects can rise from the consideration of terms proportional to the trace of the energy-momentum tensor T in the gravitational part of the $f(R)$ action, yielding the $f(R,T)$ gravity theories [18]. Those theories were also motivated by the fact that although $f(R)$ gravity is well behaved in cosmological scales, the solar system regime seems to rule out most of the $f(R)$ models proposed so far [19–22]. Furthermore, rotation curves of spiral galaxies were constructed in $f(R)$ gravity, but the results did not favour the theory, as it can be checked in Chiba [23]; Dolgov & Kawasaki [24]; Olmo [25]. The structure and cosmological properties of the modified gravity starting from $f(R)$ theory to power-counting renormalizable covariant gravity were presented by Nojiri & Odintsov [26]. The review in Nojiri et al. [27] describes the cosmological developments regarding inflation, bounce, and late-time evolution in $f(R)$, $f(\mathcal{G})$, and $f(\mathcal{T})$ modified theories of gravity, with $\mathcal{G}$ and $\mathcal{T}$ being the Gauss-Bonnet and torsion scalars.

Despite its recent elaboration, $f(R,T)$ gravity has already been applied to a number of areas, such as cosmology

([28–34] and astrophysics [35–42]). Particularly, solar system tests have been applied to $f(R,T)$ gravity [43] and the dark matter issue was analysed by Zaregonbadi et al. [44]. The late-time behaviour of cosmic fluids consisting of collisional self-interacting dark matter and radiation was discussed by Zubair et al. [45]. Moreover, considering the metric and the affine connection as independent field variables, the Palatini formulation of the $f(R,T)$ gravity can be seen in [46, 47].

By investigating the features of an $f(R,T)$ or $f(R)$ model, one realizes the strong relation they have with the functional form of the chosen functions for $f(R,T)$ and $f(R)$, as well as with their free parameters values. In fact, a reliable method to constraint those "free" parameters to values that yield realistic models can be seen in Correa & Moraes [48] and Correa et al. [49] for these theories, respectively.

In the $f(R)$ gravity, a reliable and reputed functional form was proposed by A.A. Starobinsky as [50, 51]

$$f(R) = R + \alpha R^2, \tag{1}$$

which is known as Starobinsky Model (SM), with α a constant. It predicts a quadratic correction of the Ricci scalar to be inserted in the gravitational part of the Einstein-Hilbert action.

SM has been deeply applied to the cosmological and astrophysical contexts in the literature. Starobinsky showed that a cosmological model obtained from (1) can satisfy cosmological observational tests [51]. On the other hand, the model seems to predict an overproduction of scalarons in the very early universe. In an astrophysical context, SM is also of great importance. In Sharif & Siddiqa [52], the authors have explored the source of a gravitational radiation in SM by considering axially symmetric dissipative dust under geodesic condition. In Resco et al. [53], it has been shown that in SM it is possible to find neutron stars with $2M_\odot$, which raises as an important alternative to some of the GR shortcomings mentioned above [4, 5]. In Astashenok et al. [54], the macroscopical features of quark stars were obtained.

Our proposal in this paper is to construct a cosmological scenario from an $f(R,T)$ functional form whose R−dependence is the same as in the SM, i.e., with a quadratic extra contribution of R, as in (1). The T−dependence will be considered to be linear, as $2\gamma T$, with γ being a constant. Therefore, we will take

$$f(R,T) = R + \alpha R^2 + 2\gamma T. \tag{2}$$

As far as the present authors know, the SM has not been considered for the R−dependence within $f(R,T)$ models for cosmological purposes so far, only in the study of astrophysical compact objects [55–57] and wormholes [58, 59]. We believe this is due to the expected high nonlinearity of the resulting differential equation for the scale factor. Anyhow, the consideration of linear material corrections together with quadratic geometrical terms can imply interesting outcomes in a cosmological perspective as it does in the astrophysical level.

2. The $f(R,T)=R+\alpha R^2+2\gamma T$ Gravity

Following the steps in Harko et al. [18], we can write the $f(R,T) = R + \alpha R^2 + 2\gamma T$ gravity total action as

$$S = \frac{1}{16\pi}\int \left(R + \alpha R^2 + 2\gamma T\right)\sqrt{-g}d^4x + \int L_m \sqrt{-g}d^4x, \tag{3}$$

in which g is the determinant of the metric $g_{\mu\nu}$, L_m is the matter Lagrangian, and we are working with natural units.

By taking $L_m = -p$, with p being the pressure of the universe, the variational principle applied in (3) yields the following field equations:

$$\frac{1}{2}G_{\mu\nu} + \alpha\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)R = 4\pi T_{\mu\nu} + \gamma\left[T_{\mu\nu} + \left(p + \frac{1}{2}T\right)g_{\mu\nu}\right]. \tag{4}$$

In (4), $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$, ρ is the matter-energy density of the universe, and $T = \rho - 3p$. Still in (4), it can be straightforwardly seen that the limit $\alpha = \gamma = 0$ recovers the GR field equations.

Also, the above choice for the matter Lagrangian is usually assumed in the literature as it can be checked in Harko et al. [18]; Moraes et al. [35, 38], among many others.

3. The $f(R,T)=R+\alpha R^2+2\gamma T$ Cosmology

Let us assume a flat Friedmann-Robertson-Walker metric in the field equations above. Such a substitution yields the following Friedmann-like equations:

$$\left(\frac{\dot a}{a}\right)^2 + 6\alpha G\left(a,\dot a,\ddot a,\dddot a\right) = \frac{8\pi}{3}\rho + \gamma\left(\rho - \frac{p}{3}\right), \tag{5}$$

$$\frac{\ddot a}{a} + \frac{1}{2}\left(\frac{\dot a}{a}\right)^2 + 6\alpha S\left(a,\dot a,\ddot a,\dddot a,\ddddot a\right) = -4\pi p + \frac{1}{2}\gamma\left(\rho - p\right), \tag{6}$$

where we are using the following definitions:

$$G\left(a,\dot a,\ddot a,\dddot a\right) \equiv 2\left(\frac{\dot a}{a}\right)^2\left[\frac{\ddot a}{a} + \frac{\dddot a}{a} - \frac{3}{2}\left(\frac{\dot a}{a}\right)^2\right] - \left(\frac{\ddot a}{a}\right)^2, \tag{7}$$

$$S\left(a,\dot a,\ddot a,\dddot a,\ddddot a\right) \equiv \frac{3}{2}\left[\left(\frac{\dot a}{a}\right)^4 + \frac{\ddot a}{a}\right] + 2\left[\frac{\dot a\dddot a}{a^2} - 3\left(\frac{\dot a}{a}\right)^2\frac{\ddot a}{a}\right] + \frac{\ddddot a}{a}. \tag{8}$$

In the equations above, $a = a(t)$ is the scale factor and dots represent time derivatives. Once again, the limit $\alpha = \gamma = 0$ retrieves the standard formalism.

In terms of the Hubble parameter $H = \dot{a}/a$, the values of ρ and p from (5)-(6) are

$$\rho = \frac{F(\gamma)}{2}\left[\gamma\widetilde{F}(H) - (8\pi + \gamma)\widetilde{G}(H)\right], \tag{9}$$

$$p = F(\gamma)\left[4\pi\overline{F}(H) + 3\gamma\overline{G}(H)\right], \tag{10}$$

with the following definitions:

$$\widetilde{F}(H) \equiv -F_4(H) - 6\alpha\left[-3F_2(H) + G_3(H) + 3F_1(H)\right], \tag{11}$$

$$\widetilde{G}(H) \equiv 3\left\{H^2 + 6\left[G_4(H) + F_3(H)\right]\alpha\right\}, \tag{12}$$

$$\overline{F}(H) \equiv F_4(H) + 18\alpha F_2(H) - F_1(H) + \dot{H} + 2\dot{H}^2, \tag{13}$$

$$\overline{G}(H) \equiv -H^2 + \dot{H} + 3\alpha\left[G_1(H) + G_2(H) + 2H^5 + 6H^3\dot{H}\right]. \tag{14}$$

For (9)-(10), we considered $F(\gamma) = 1/(32\pi^2 + 16\pi\gamma + \gamma^2)$. Moreover, F_i and G_i, with $i = 1, 2, 3, 4$, are functions of H and its time derivatives, expressed by the following:

$$F_1(H) = H^2\left(1 + 4\dot{H}\right), \tag{15}$$

$$F_2(H) = H^4 - 4H\ddot{H}, \tag{16}$$

$$F_3(H) = 2\left(H^5 + 3H^3\dot{H}\right), \tag{17}$$

$$F_4(H) = 3H^2 + 2\dot{H}, \tag{18}$$

$$G_1(H) = H^4 + \dot{H}\left(3 + 7\dot{H}\right), \tag{19}$$

$$G_2(H) = -12H\ddot{H} + H^2\left(-3 - 12\dot{H} + 2\ddot{H}\right) - 2\dddot{H}, \tag{20}$$

$$G_3(H) = 3\dot{H}\left(1 + 2\dot{H}\right) + 2\dddot{H}, \tag{21}$$

$$G_4(H) = -2H^4 - \dot{H}^2 + 2H^2\ddot{H}. \tag{22}$$

We can consider, as a solution for (9)-(10), the scale factor in the hybrid expansion law form [60]:

$$a(t) = e^{mt}t^n, \tag{23}$$

with m and n being constants. It can be seen that such a form for the scale factor consists of a product of power law and exponential law functions. Equation (23) mimics the power law and de-Sitter cosmologies as particular cases and can predict the transition from a decelerated to an accelerated regime of the universe expansion. It has been applied to Brans-Dicke models in Ozgur et al. [60], yielding observational constraints to m and n.

From (23), the Hubble and deceleration parameters are

$$H = m + \frac{n}{t}, \tag{24}$$

$$q = -\frac{\ddot{a}}{aH^2} = -1 + \frac{n}{(mt + n)^2}, \tag{25}$$

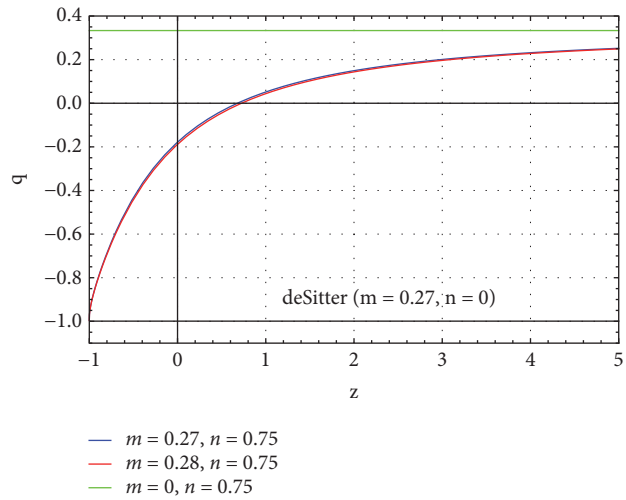

Figure 1: Deceleration parameter evolution in redshift for different values of m and n.

such that the deceleration parameter is defined in such a way that its negative values describe an accelerated expansion of the universe.

We know that the universe not always has accelerated its expansion [7, 8]. The accelerated regime of expansion is considered to be a late-time phenomenon.

In this way, one can choose the constants m and n in such a way that the power law dominates over exponential law in the early universe and the exponential law dominates over power law at late times. Therefore, the decelerated and accelerated regimes of the universe expansion can be, respectively, well described, as well as the transition between these regimes.

From (25) it is clear that there is a transition from deceleration to acceleration phases of the universe expansion at $t = (1/m)(-n \pm \sqrt{n})$, with $0 < n < 1$. Since the negativity of the second term leads to a negative time, which indicates an nonphysical situation, we conclude that the cosmic transition may have occurred at $t = (\sqrt{n} - n)/m$.

From $a(t)/a_0 = 1/(1 + z)$, with $a_0 = 1$ being the present value of the scale factor and z being the redshift, we obtain the following time-redshift relation:

$$t = \frac{n}{m}W\left[\frac{m}{n}\left(\frac{1}{1+z}\right)^{1/n}\right], \tag{26}$$

where W denotes the Lambert function (also known as "product logarithm").

Using (26), we can plot the deceleration parameter with respect to the redshift, which can be appreciated in Figure 1, in which the values chosen for m and n are in agreement with the observational constraints found in Ozgur et al. [60].

Plotting q as a redshift function has the advantage of checking the reliability of the model, through the redshift value in which the decelerated-accelerated expansion of the universe transition occurs. We will call the transition redshift as z_{tr} and in our model it can be seen that it depends directly on the parameter m. From Figure 1, the transition occurs at $z_{tr} = 0.5836, 0.6777$, corresponding to $m = 0.25, 0.27$,

respectively. The values of the transition redshift z_{tr} for our model are in accordance with the observational data, as one can check in Capozziello et al. [61, 62]; Farooq et al. [63].

Now, let us write the solutions for the material content of our model, named ρ and p. Using (24) in (9)-(10), we have

$$\rho = \frac{F(\gamma)}{2}\left[-\gamma\left(\mathscr{P}_1+\mathscr{P}_2\right)+\mathscr{P}_3 + 3\left(8\pi+\gamma\right)\left(\mathscr{P}_4+\mathscr{P}_5+\mathscr{P}_6\right)\right], \tag{27}$$

$$p = F(\gamma)\left[-\left(\mathscr{G}_1+\mathscr{G}_2+\mathscr{G}_3+\mathscr{G}_4\right)+\mathscr{G}_5\right], \tag{28}$$

where we are using the definitions

$$\mathscr{P}_1 \equiv 32\alpha n\mathscr{F}_3(t)^3 \times \left\{4\mathscr{F}_1(t)\left[1-\frac{\mathscr{F}_1(t)}{2}\right] + n\left[1-\frac{\mathscr{F}_3(t)^{-2}}{2}\right]-2\right\}, \tag{29}$$

$$\mathscr{P}_2 \equiv 18\alpha\mathscr{F}_2(t)^4\left[\mathscr{F}_2(t)^{-2}-1\right], \tag{30}$$

$$\mathscr{P}_3 \equiv 3\mathscr{F}_2(t)^2 - 2n\mathscr{F}_3(t)^2, \tag{31}$$

$$\mathscr{P}_4 \equiv 6\alpha n\mathscr{F}_3(t)^5\left\{2\mathscr{F}_1(t)^2\left[2-3\mathscr{F}_1(t)\right] - n\mathscr{F}_3(t)^{-1}\right\}, \tag{32}$$

$$\mathscr{P}_5 \equiv 12\alpha\mathscr{F}_2(t)^5\left[1-\mathscr{F}_2(t)^{-1}\right], \tag{33}$$

$$\mathscr{G}_1 \equiv \alpha n\mathscr{F}_3(t)^4\mathscr{F}_1(t)^2\left\{9\gamma\mathscr{F}_3(t)\left[6\mathscr{F}_1(t)-4\right] - 2\mathscr{F}_1^{-1}(t)G(\gamma)+G(\gamma)\right\}, \tag{34}$$

$$\mathscr{G}_2 \equiv n\mathscr{F}_3(t)^2\left[\frac{\alpha}{4}G(\gamma)-3\gamma-8\pi\right], \tag{35}$$

$$\mathscr{G}_3 \equiv \alpha n^2\mathscr{F}_3(t)^2\left[G(\gamma)+18\gamma\right]+\alpha G(\gamma), \tag{36}$$

$$\mathscr{G}_4 \equiv \alpha n\mathscr{F}_3(t)^4 G(\gamma), \tag{37}$$

$$\mathscr{G}_5 \equiv \mathscr{F}_2(t)^2 \times \left\{9\alpha\mathscr{F}_2(t)^2\left[-2\gamma\mathscr{F}_2(t)-\gamma-8\pi\right] - \frac{\alpha}{4}G(\gamma)+3\gamma+12\pi\right\}, \tag{38}$$

with $G(\gamma) = -108\gamma - 288\pi$ and $\mathscr{F}_j$, with $j = 1, 2, 3$, being functions of t only, defined as

$$\mathscr{F}_1(t) = mt + n, \tag{39}$$

$$\mathscr{F}_2(t) = m + \frac{n}{t}, \tag{40}$$

$$\mathscr{F}_3(t) = \frac{1}{t}. \tag{41}$$

The evolution of the energy density, pressure, and corresponding equation of state (EoS) parameter $\omega = p/\rho$ with $m = 0.27$, $n = 0.75$ are shown in Figures 2–4, in which the time units are Gyr.

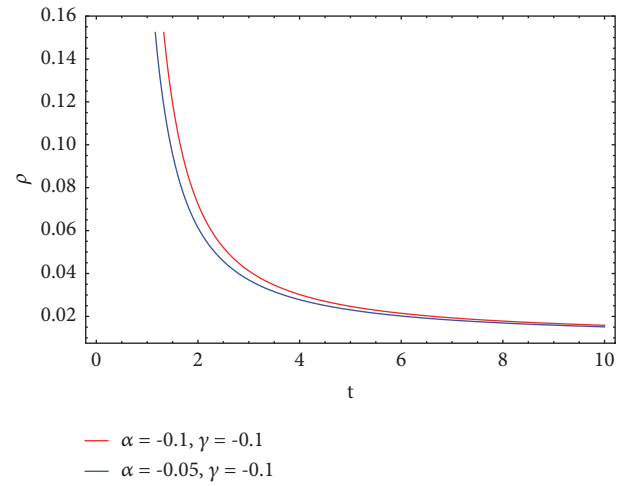

Figure 2: Evolution of ρ with time.

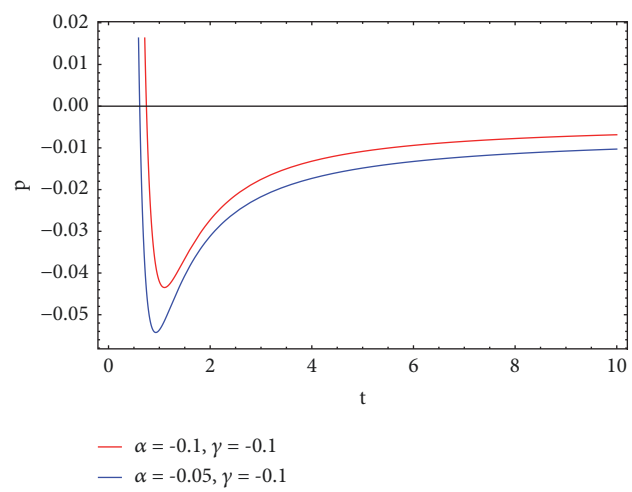

Figure 3: Evolution of p with time.

4. Energy Conditions

Energy conditions (ECs), in the context of a wide class of covariant theories including GR, are relations one demands for the energy-momentum tensor of matter to satisfy in order to try to capture the idea that "energy should be positive". By imposing so, one can obtain constraints to the free parameters of the concerned model.

The standard point-wise ECs are [64–66]

Null energy condition (NEC): $\rho + p \geq 0$.

Weak energy condition (WEC): $\rho \geq 0$, $\rho + p \geq 0$.

Strong energy condition (SEC): $\rho + 3p \geq 0$.

Dominant energy condition (DEC): $\rho \geq |p|$.

Generally the above ECs are formulated from the Raychaudhuri equation, which describes the behaviour of space-like, time-like, or light-like curves in gravity.

In the present model the energy-momentum tensor has the form of a perfect fluid. So, we will use the above relations

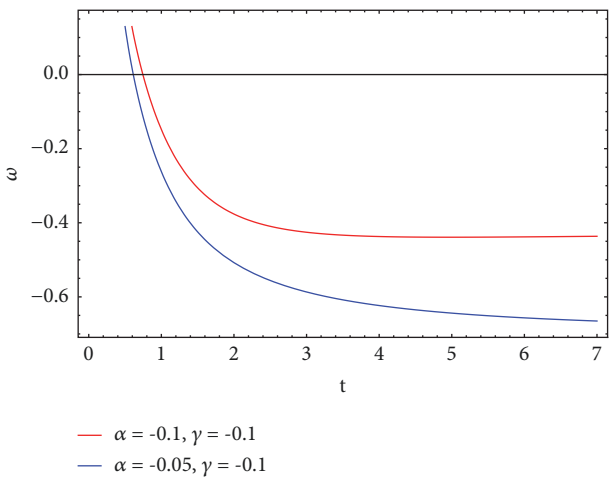

Figure 4: Evolution of ω with time.

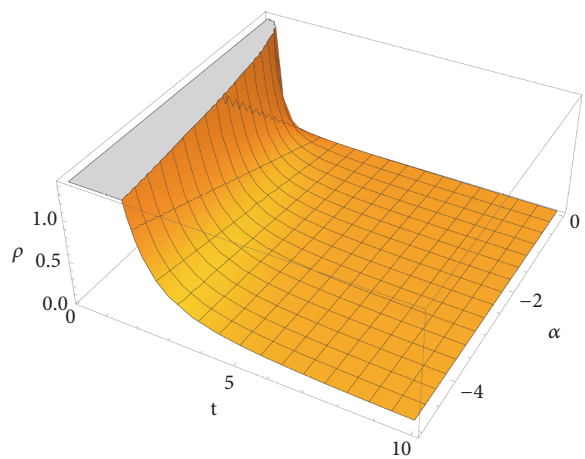

Figure 5: Evolution of ρ.

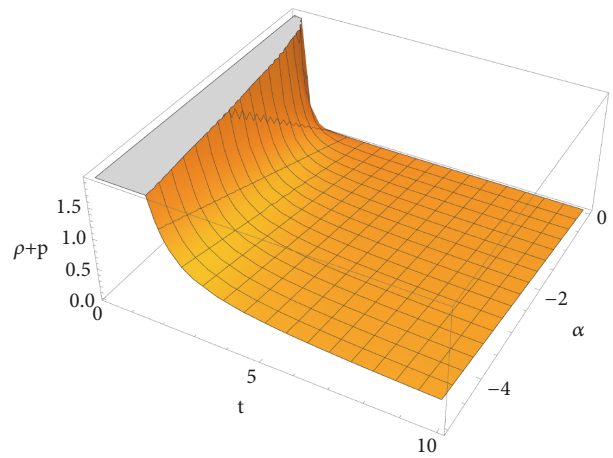

Figure 6: Validation of NEC.

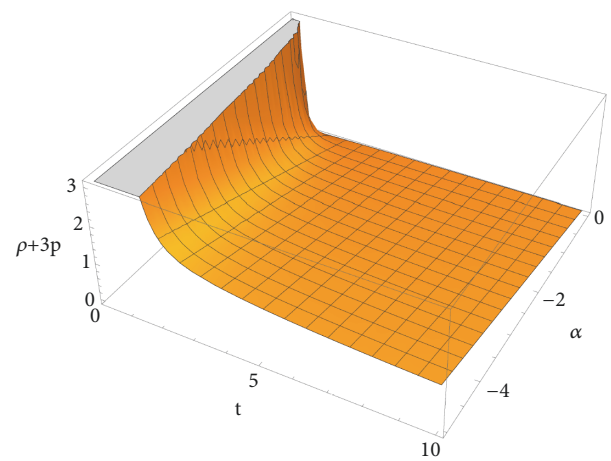

Figure 7: Violation of SEC.

for analysing the ECs in $f(R,T)$ theory. The behaviour of the ECs with $m = 0.27, n = 0.75$, and $\alpha = -0.02$ is given in Figures 5–8.

5. Discussion and Conclusions

In this article we have proposed an $f(R,T)$ cosmological model. For the functional form of the function $f(R,T)$, we investigated a quadratic correction to R, as in the well-known SM, together with a linear term on T. The substitution of such an $f(R,T)$ function in the gravitational action generates extra terms in the field equations (4) and, consequently, in the Friedmann-like equations (5) and (6). Those extra terms bring some new information regarding the dynamics of the universe, as we discuss below.

Our solution for the scale factor $a(t)$ is a hybrid expansion law, well described in Ozgur et al. [60]. From (23), we have obtained the cosmological parameters, namely, Hubble factor and deceleration parameter. Specifically, for the deceleration parameter, we could separate it in two phases: one describing the decelerated and the other describing the accelerated regime of the universe expansion. This can be well-checked in Figure 1, in which the transition redshift between these two stages agrees with observational data. Moreover, one can also note that, remarkably, the present ($z = 0$) values for q, named -0.178 for $m = 0.27$ and -0.157 for $m = 0.25$, are also in agreement with observational data [12].

We have also obtained solutions for the material content of the universe, named ρ and p (see (27)-(41)). In Figures 2–4 we plot the evolution of ρ, p, and ω, the EoS parameter. The values chosen for α and γ respect the energy conditions outcomes presented in Section 4. In Figure 3 we see that the pressure of the universe starts in positive values and then assumes negative values. In standard model of cosmology, a negative pressure fluid is exactly the mechanism responsible for accelerating the universe expansion. In the present model, such a behaviour for the pressure was naturally obtained.

It is also worth stressing that the EoS parameter shows a transition from a decelerated to an accelerated regime of the expansion of the universe. This can well be seen in Figure 4, by recalling that, from standard cosmology, the latter regime may happen if $\omega < -1/3$ [67]. Moreover, it can be seen from such a figure that as time passes by, $\omega \longrightarrow -1$, in

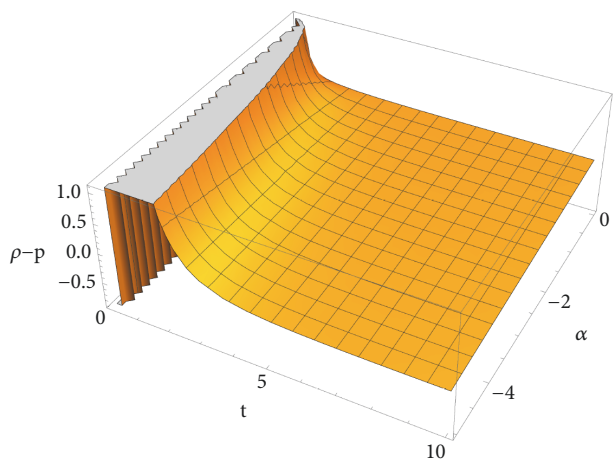

Figure 8: Validation of DEC.

accordance with recent observational data on the cosmic microwave background temperature fluctuations [12].

Furthermore, in Figures 5–8 we plotted the ECs from the material solutions of our cosmological model. Those figures were plotted in terms of γ, for fixed $\alpha = -0.02$. They show the validation of WEC, NEC, and DEC, with a wide range of acceptable values for γ.

On the other hand, Figure 7 shows violation of SEC. However, as it has been deeply discussed by Barcelo & Visser [68] an early and late-time accelerating universe must violate SEC.

As a further work, we can look for the $f(R,T) = R + \alpha R^2 + 2\gamma T$ gravity universe at very early times, particularly investigating the production of scalarons in this model. Moraes & Santos [30] have shown that the trace of the energy-momentum tensor contribution in the theory is higher for the early universe when compared to the late-time contribution. According to Starobinsky [51], some mechanism should work in the early universe to prohibit the scalaron overproduction within SM. The high contribution of the terms proportional to T in the early universe may well be this mechanism.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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