Research Article

Extended Cesàro Operators from Logarithmic-Type Spaces to Bloch-Type Spaces

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The boundedness and compactness of the extended Cesàro operator from logarithmic-type spaces to Bloch-type spaces on the unit ball are completely characterized in this paper.

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1. Introduction

Let $\mathbb{B}_n$ be the unit ball of $\mathbb{C}^n$, $\partial\mathbb{B}_n$ the unit sphere of $\mathbb{C}^n$, $H(\mathbb{B}_n)$ the space of all holomorphic functions in $\mathbb{B}_n$, and H^∞ the space of all bounded holomorphic functions on $\mathbb{B}_n$. For $f \in H(\mathbb{B}_n)$, let

$$\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z) \quad (1.1)$$

denote the radial derivative of $f \in H(\mathbb{B}_n)$.

A positive continuous function μ on the interval $[0, 1)$ is called normal if there is $\delta \in [0, 1)$ and s and t , $0 < s < t$ such that

$$\begin{aligned} \frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta, 1), \quad \lim_{r \rightarrow 1} \frac{\mu(r)}{(1-r)^s} = 0, \\ \frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta, 1), \quad \lim_{r \rightarrow 1} \frac{\mu(r)}{(1-r)^t} = \infty. \end{aligned} \quad (1.2)$$

If we say that $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ is normal we will also assume that $\mu(z) = \mu(|z|)$, $z \in \mathbb{B}_n$.

Let $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ be normal. The Bloch-type space $\mathcal{B}_\mu = \mathcal{B}_\mu(\mathbb{B}_n)$ is the space of all functions $f \in H(\mathbb{B}_n)$ such that

$$b_\mu(f) = \sup_{z \in \mathbb{B}_n} \mu(z) |\Re f(z)| < \infty. \quad (1.3)$$

$\mathcal{B}_\mu$ is Banach space with the norm $\|f\|_{\mathcal{B}_\mu} = |f(0)| + b_\mu(f)$. The little Bloch-type space $\mathcal{B}_{\mu,0} = \mathcal{B}_{\mu,0}(\mathbb{B}_n)$ consists of all $f \in H(\mathbb{B}_n)$ such that

$$\lim_{|z| \rightarrow 1} \mu(z) |\Re f(z)| = 0. \quad (1.4)$$

It is easy to see that $\mathcal{B}_{\mu,0}$ is a closed subspace of $\mathcal{B}_\mu$. When $\mu(r) = (1 - r^2)^\alpha$, $\alpha \in (0, \infty)$, we obtain so-called α -Bloch spaces and little α -Bloch spaces, respectively, which for $\alpha = 1$ are reduced to classical Bloch spaces (see, e.g., [1–4] and the references therein). When $\mu(r) = (1 - r^2) \ln(e/(1 - r^2))$, we obtain the logarithmic Bloch space $\mathcal{LB} = \mathcal{LB}(\mathbb{B}_n)$ and the little logarithmic Bloch space $\mathcal{LB}_0 = \mathcal{LB}_0(\mathbb{B}_n)$, respectively, (see [5]). It was shown that f is a multiplier of $\mathcal{B}$ if and only if $f \in H^\infty$ and $f \in \mathcal{LB}$ in [6].

An $f \in H(\mathbb{B}_n)$ is said to belong to the logarithmic-type space $H_{\log}^\infty$, if

$$\|f\|_{H_{\log}^\infty} = \sup_{z \in \mathbb{B}_n} \frac{|f(z)|}{\ln\left(e/(1 - |z|^2)\right)} < \infty. \quad (1.5)$$

It is easy to see that $H_{\log}^\infty$ becomes a Banach space under the norm $\|\cdot\|_{H_{\log}^\infty}$, and that the inclusions $H^\infty \subset \mathcal{B} \subset H_{\log}^\infty$ hold. For some information of the space $H_{\log}^\infty$ see [7, 8].

Let $g \in H(\mathbb{B}_n)$. The extended Cesàro operator on $H(\mathbb{B}_n)$ is defined by

$$T_g f(z) = \int_0^1 f(tz) \Re g(tz) \frac{dt}{t}, \quad f \in H(\mathbb{B}_n), \quad z \in \mathbb{B}_n. \quad (1.6)$$

This operator is a natural extension of a one-dimensional operator defined in [9]. Some other results on the one-dimensional operator can be found, for example, in [10, 11] (see also the references therein). For some extensions of operator (1.6) on the unit disk see [12–16]. On related operators on the unit polydisk see, for example, [17–21] and references therein. The boundedness and compactness of operator (1.6) between various spaces of holomorphic functions has been extensively studied recently, see, [17, 22–36]. For some integral operators on spaces of harmonic functions see, for example, [37] as well as the references therein. A new extension of operator (1.6) in the unit ball case have been recently introduced by Stević in [38] (see also [5, 39]).

In this paper, we study the extended Cesàro operator from $H_{\log}^\infty$ to Bloch-type spaces $\mathcal{B}_\mu$ and $\mathcal{B}_{\mu,0}$. Sufficient and necessary conditions for the extended Cesàro operator T_g to be bounded and compact are given.

Throughout the paper, constants are denoted by C , they are positive and may not be the same in every occurrence.

2. Main Results and Proofs

In this section, we give our main results and their proofs. Before stating these results, we need some auxiliary results, which are incorporated in the lemmas which follows.

Lemma 2.1. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ are normal. Then $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is compact if and only if $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is bounded and for any bounded sequence $(f_k)_{k \in \mathbb{N}}$ in $H_{\log}^\infty$ which converges to zero uniformly on compact subsets of $\mathbb{B}_n$ as $k \rightarrow \infty$, one has $\|T_g f_k\|_{\mathcal{B}_\mu} \rightarrow 0$ as $k \rightarrow \infty$.*

The proof of Lemma 2.1 follows by standard arguments (see, e.g., Lemmas 3 in [20, 21, 29]). Hence, we omit the details.

Lemma 2.2. *Assume that $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ is normal. A closed set K in $\mathcal{B}_{\mu,0}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z| \rightarrow 1} \sup_{f \in K} \mu(z) |\Re f(z)| = 0. \quad (2.1)$$

This lemma can be found in [5], and its proof is similar to the proof of Lemma 1 in [40]. Hence, it will be omitted.

The following result was proved in [8].

Lemma 2.3. *There exist two functions $f_1, f_2 \in H_{\log}^\infty(\mathbb{B}_1)$ such that*

$$|f_1(z)| + |f_2(z)| \geq \ln \frac{1}{1 - |z|}, \quad z \in \mathbb{B}_1. \quad (2.2)$$

Now we are in a position to state and prove our main results.

Theorem 2.4. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ is normal. Then $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is bounded if and only if*

$$M = \sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z)| \ln \frac{e}{1 - |z|^2} < \infty. \quad (2.3)$$

Moreover, if $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is bounded then the following asymptotic relation holds

$$\|T_g\|_{H_{\log}^\infty \rightarrow \mathcal{B}_\mu} \asymp \sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z)| \ln \frac{e}{1 - |z|^2} < \infty. \quad (2.4)$$

Proof. Assume that (2.3) holds. Then, for any $f \in H_{\log}^\infty$, we have

$$\mu(z) |\Re(T_g f)(z)| = \mu(z) |\Re g(z)| |f(z)| \leq \mu(z) |\Re g(z)| \left(\ln \frac{e}{1 - |z|^2} \right) \|f\|_{H_{\log}^\infty}. \quad (2.5)$$

In addition, it is easy to see that $T_g f(0) = 0$. Therefore we have

$$\|T_g f\|_{\mathcal{B}_\mu} = \sup_{z \in \mathbb{B}_n} \mu(z) |\Re(T_g f)(z)| \leq M \|f\|_{H_{\log}^\infty} \quad (2.6)$$

as desired.

Conversely, assume that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is bounded. For $a \in \mathbb{B}_n$, set

$$f_a(z) = \ln \frac{e}{1 - \langle z, a \rangle}. \quad (2.7)$$

It is easy to see that $f_a \in H_{\log}^\infty$ and $\sup_{a \in \mathbb{B}_n} \|f_a\|_{H_{\log}^\infty} < \infty$.

For any $b \in \mathbb{B}_n$, we have

$$\begin{aligned} \infty &> \|T_g f_b\|_{\mathcal{B}_\mu} \\ &= \sup_{z \in \mathbb{B}_n} \mu(z) |\Re(T_g f_b)(z)| \\ &= \sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z)| |f_b(z)| \\ &\geq \mu(b) |\Re g(b)| \ln \frac{e}{1 - |b|^2}, \end{aligned} \quad (2.8)$$

from which (2.3) follows, moreover

$$\sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z)| \ln \frac{e}{1 - |z|^2} \leq C \|T_g\|_{H_{\log}^\infty \rightarrow \mathcal{B}_\mu}. \quad (2.9)$$

From (2.6) and (2.9), we see that (2.4) holds. The proof is completed. $\square$

Theorem 2.5. Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ is normal. Then $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is compact if and only if

$$\lim_{|z| \rightarrow 1} \mu(z) |\Re g(z)| \ln \frac{e}{1 - |z|^2} = 0. \quad (2.10)$$

Proof. Suppose that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is compact. Let $(z_k)_{k \in \mathbb{N}}$ be a sequence in $\mathbb{B}_n$ such that $\lim_{k \rightarrow \infty} |z_k| = 1$. Set

$$f_k(z) = \left(\ln \frac{e}{1 - \langle z, z_k \rangle} \right)^2 \left(\ln \frac{e}{1 - |z_k|^2} \right)^{-1}, \quad k \in \mathbb{N}. \quad (2.11)$$

It is easy to see that $\sup_{k \in \mathbb{N}} \|f_k\|_{H_{\log}^\infty} < \infty$. Moreover $f_k \rightarrow 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \rightarrow \infty$. By Lemma 2.1,

$$\lim_{k \rightarrow \infty} \|T_g f_k\|_{\mathcal{B}_\mu} = 0. \quad (2.12)$$

In addition,

$$\|T_g f_k\|_{\mathcal{B}_\mu} = \sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z) f_k(z)| \geq \mu(z_k) |\Re g(z_k)| \ln \frac{e}{1 - |z_k|^2}, \quad (2.13)$$

which together with (2.12) implies that

$$\lim_{k \rightarrow \infty} \mu(z_k) |\Re g(z_k)| \ln \frac{e}{1 - |z_k|^2} = 0. \quad (2.14)$$

From the above inequality we see that (2.10) holds.

Conversely, assume that (2.10) holds. From Theorem 2.4 we see that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is bounded. In order to prove that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is compact, according to Lemma 2.1, it suffices to show that if $(f_k)_{k \in \mathbb{N}}$ is a bounded sequence in $H_{\log}^\infty$ converging to 0 uniformly on compact subsets of $\mathbb{B}_n$, then

$$\lim_{k \rightarrow \infty} \|T_g f_k\|_{\mathcal{B}_\mu} = 0. \quad (2.15)$$

Let $(f_k)_{k \in \mathbb{N}}$ be a bounded sequence in $H_{\log}^\infty$ such that $f_k \rightarrow 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \rightarrow \infty$. By (2.10) we have that for any $\varepsilon > 0$, there is a constant $\delta \in (0, 1)$, such that

$$\mu(z) |\Re g(z)| \ln \frac{e}{1 - |z_k|^2} < \varepsilon \quad (2.16)$$

whenever $\delta < |z| < 1$. Let $K = \{z \in \mathbb{B}_n : |z| \leq \delta\}$. From (2.10) we see that $g \in \mathcal{B}_\mu$. Equality (2.16) along with the fact that $g \in \mathcal{B}_\mu$ implies

$$\begin{aligned} \|T_g f_k\|_{\mathcal{B}_\mu} &= \sup_{z \in \mathbb{B}_n} \mu(z) |\Re(T_g f_k)(z)| \\ &= \sup_{z \in \mathbb{B}_n} \mu(z) |\Re g(z) f_k(z)| \\ &\leq \left(\sup_{\{z \in \mathbb{B}_n : |z| \leq \delta\}} + \sup_{\{z \in \mathbb{B}_n : \delta \leq |z| < 1\}} \right) \mu(z) |\Re g(z)| |f_k(z)| \\ &\leq \|g\|_{\mathcal{B}_\mu} \sup_{z \in K} |f_k(z)| + \sup_{\{z \in \mathbb{B}_n : \delta \leq |z| < 1\}} \mu(z) |g(z)| \left(\ln \frac{e}{1 - |z|^2} \right) \|f_k\|_{H_{\log}^\infty} \\ &\leq \|g\|_{\mathcal{B}_\mu} \sup_{z \in K} |f_k(z)| + C\varepsilon. \end{aligned} \quad (2.17)$$

Observe that K is a compact subset of $\mathbb{B}_n$, so that

$$\lim_{k \rightarrow \infty} \sup_{z \in K} |f_k(z)| = 0. \quad (2.18)$$

Therefore

$$\limsup_{k \rightarrow \infty} \|T_g f_k\|_{\mathcal{B}_\mu} \leq C\varepsilon. \quad (2.19)$$

Since $\varepsilon > 0$ is an arbitrary positive number it follows that the last limit is equal to zero. Therefore, $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_\mu$ is compact. The proof is completed. $\square$

Remark 2.6. From [24] and Theorems 2.4 and 2.5, we see that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}$ is bounded if and only if $T_g : \mathcal{B} \rightarrow \mathcal{B}$ is bounded; $T_g : H_{\log}^\infty \rightarrow \mathcal{B}$ is compact if and only if $T_g : \mathcal{B} \rightarrow \mathcal{B}$ is compact.

Theorem 2.7. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \rightarrow [0, \infty)$ is normal. Then the following statements are equivalent:*

- (a) $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_{\mu,0}$ is bounded;
- (b) $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_{\mu,0}$ is compact;
- (c)

$$\lim_{|z| \rightarrow 1} \mu(z) |\Re g(z)| \ln \frac{e}{1 - |z|^2} = 0. \quad (2.20)$$

Proof. (b) $\Rightarrow$ (a). This implication is obvious.

(a) $\Rightarrow$ (c). Assume that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_{\mu,0}$ is bounded. Now we prove that (2.20) holds. Note that (2.20) is equivalent with

$$\lim_{|z| \rightarrow 1} \mu(z) |\Re g(z)| \ln \frac{1}{1 - |z|} = 0. \quad (2.21)$$

Hence we only need to show that (2.21) holds. This can be done by contradiction. Now assume that the condition (2.21) does not hold. If it was, then it would exist $\varepsilon_0 > 0$ and a sequence $(z^{(j)})_{j \in \mathbb{N}} \in \mathbb{B}_n$, such that $z^{(j)} \rightarrow \partial \mathbb{B}_n$, and

$$\mu(z^{(j)}) |\Re g(z^{(j)})| \ln \frac{1}{1 - |z^{(j)}|} \geq \varepsilon_0 > 0 \quad (2.22)$$

for sufficiently large j . We may assume that $\lim_{j \rightarrow \infty} z^{(j)} = (1, 0, \dots, 0)$ and also

$$1 - |z^{(j)}| > \frac{1 - |z_1^{(j)}|}{2}, \quad j \in \mathbb{N}. \quad (2.23)$$

According to Lemma 2.3 we know that there exist two functions $h_1, h_2 \in H_{\log}^\infty(\mathbb{B}_1)$ such that

$$|h_1(z_1)| + |h_2(z_1)| \geq \ln \frac{1}{1 - |z_1|}, \quad z_1 \in \mathbb{B}_1. \quad (2.24)$$

Let

$$F_1(z) = h_1(z_1), \quad F_2(z) = h_2(z_1), \quad z = (z_1, \dots, z_n) \in \mathbb{B}_n. \quad (2.25)$$

Then clearly $F_1, F_2 \in H_{\log}^\infty$. By the boundedness of $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_{\mu,0}$ we have

$$T_g F_1, T_g F_2 \in \mathcal{B}_{\mu,0}. \quad (2.26)$$

On the other hand,

$$\begin{aligned} & \mu(z^{(j)}) \left(\left| \Re(T_g F_1)(z^{(j)}) \right| + \left| \Re(T_g F_2)(z^{(j)}) \right| \right) \\ &= \mu(z^{(j)}) \left| \Re g(z^{(j)}) \right| \left(\left| F_1(z^{(j)}) \right| + \left| F_2(z^{(j)}) \right| \right) \\ &= \mu(z^{(j)}) \left| \Re g(z^{(j)}) \right| \left(\left| h_1(z_1^{(j)}) \right| + \left| h_2(z_1^{(j)}) \right| \right) \\ &\geq \mu(z^{(j)}) \left| \Re g(z^{(j)}) \right| \ln \frac{1}{1 - |z_1^{(j)}|} \\ &\geq C \mu(z^{(j)}) \left| \Re g(z^{(j)}) \right| \ln \frac{1}{1 - |z^{(j)}|} \\ &\geq C \varepsilon_0 > 0 \end{aligned} \quad (2.27)$$

for sufficiently large j . Since $z^{(j)} \rightarrow \partial \mathbb{B}_n$, from the above inequality we obtain that $T_g F_1, T_g F_2 \notin \mathcal{B}_{\mu,0}$, which is a contradiction.

(c) $\Rightarrow$ (b). From (1.5) we have that

$$\mu(z) |\Re(T_g f)(z)| \leq \mu(z) |\Re g(z)| \left(\ln \frac{e}{1 - |z|^2} \right) \|f\|_{H_{\log}^\infty}. \quad (2.28)$$

Taking the supremum in the above inequality over all $f \in H_{\log}^\infty$ such that $\|f\|_{H_{\log}^\infty} \leq 1$, then letting $|z| \rightarrow 1$, by (2.20) we arrive at

$$\lim_{|z| \rightarrow 1} \sup_{\|f\|_{H_{\log}^\infty} \leq 1} \mu(z) |\Re(T_g f)(z)| = 0. \quad (2.29)$$

From this and by employing Lemma 2.2, we see that $T_g : H_{\log}^\infty \rightarrow \mathcal{B}_{\mu,0}$ is compact. The proof is completed. $\square$

From Theorems 2.4, 2.5, and 2.7, we have the following corollary.

Corollary 2.8. *Let $g \in H(\mathbb{B}_n)$. Then*

(1) $T_g : H_{\log}^{\infty} \rightarrow \mathcal{LB}$ is bounded if and only if

$$\sup_{z \in \mathbb{B}_n} (1 - |z|^2) |\Re g(z)| \left(\ln \frac{e}{1 - |z|^2} \right)^2 < \infty. \quad (2.30)$$

(2) $T_g : H_{\log}^{\infty} \rightarrow \mathcal{LB}$ is compact if and only if $T_g : H_{\log}^{\infty} \rightarrow \mathcal{LB}_0$ is bounded if and only if
 $T_g : H_{\log}^{\infty} \rightarrow \mathcal{LB}_0$ is compact if and only if

$$\lim_{|z| \rightarrow 1} (1 - |z|^2) |\Re g(z)| \left(\ln \frac{e}{1 - |z|^2} \right)^2 = 0. \quad (2.31)$$

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