Research Article

Generalized Hyers-Ulam Stability of Generalized (N, K) -Derivations

M. Eshaghi Gordji,¹ J. M. Rassias,² and N. Ghobadipour¹

¹ Department of Mathematics, Semnan University, P. O. Box 35195-363, Semnan, Iran

² Section of Mathematics and Informatics, Pedagogical Department, National and Capodistrian University of Athens, 4, Agamemnonos St., Aghia Paraskevi, 15342 Athens, Greece

Correspondence should be addressed to M. Eshaghi Gordji, maj.ess@yahoo.com

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Let $3 \leq n$, and $3 \leq k \leq n$ be positive integers. Let A be an algebra and let X be an A -bimodule. A $\mathbb{C}$ -linear mapping $d : A \rightarrow X$ is called a generalized (n, k) -derivation if there exists a $(k-1)$ -derivation $\delta : A \rightarrow X$ such that $d(a_1 a_2 \cdots a_n) = \delta(a_1) a_2 \cdots a_n + a_1 \delta(a_2) a_3 \cdots a_n + \cdots + a_1 a_2 \cdots a_{k-2} \delta(a_{k-1}) a_k \cdots a_n + a_1 a_2 \cdots a_{k-1} d(a_k) a_{k+1} \cdots a_n + a_1 a_2 \cdots a_k d(a_{k+1}) a_{k+2} \cdots a_n + a_1 a_2 \cdots a_{k+1} d(a_{k+2}) a_{k+3} \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n)$ for all $a_1, a_2, \dots, a_n \in A$. The main purpose of this paper is to prove the generalized Hyers-Ulam stability of the generalized (n, k) -derivations.

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1. Introduction

It seems that the stability problem of functional equations introduced by Ulam [1]. Let $(G_1, \cdot)$ be a group and let $(G_2, *)$ be a metric group with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta > 0$, such that if a mapping $h : G_1 \rightarrow G_2$ satisfies the inequality $d(h(x \cdot y), h(x) * h(y)) < \delta$, for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \rightarrow G_2$ with $d(h(x), H(x)) < \epsilon$, for all $x \in G_1$? In other words, under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equations arises when one replaces the functional equation by an inequality which acts as a perturbation of the equation. In 1941, Hyers [2] gave the first affirmative answer to the question of Ulam for Banach spaces E and E' . Let $f : E \rightarrow E'$ be a mapping between Banach spaces such that

$$\|f(x+y) - f(x) - f(y)\| \leq \delta \quad (1.1)$$

for all $x, y \in E$, and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \rightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \delta \quad (1.2)$$

for all $x \in E$. By the seminal paper of Th. M. Rassias [3] and work of Gadjia [4], if one assumes that E and E' are real normed spaces with E' complete, $f : E \rightarrow E'$ is a mapping such that for each fixed $x \in E$ the mapping $t \mapsto f(tx)$ is continuous in real t for each fixed x in E , and that there exists $\delta \geq 0$ and $p \neq 1$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \delta(\|x\|^p + \|y\|^p) \quad (1.3)$$

for all $x, y \in E$. Then there exists a unique linear map $T : E \rightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\delta\|x\|^p}{|2^p - 2|} \quad (1.4)$$

for all $x \in E$.

On the other hand J. M. Rassias [5] generalized the Hyers stability result by presenting a weaker condition controlled by a product of different powers of norms. If it is assumed that there exist constants $\Theta \geq 0$ and $p_1, p_2 \in \mathbb{R}$ such that $p = p_1 + p_2 \neq 1$, and $f : E \rightarrow E'$ is a map from a norm space E into a Banach space E' such that the inequality

$$\|f(x+y) - f(x) - f(y)\| \leq \Theta\|x\|^{p_1}\|y\|^{p_2} \quad (1.5)$$

for all $x, y \in E$, then there exists a unique additive mapping $T : E \rightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \frac{\Theta}{2 - 2^p}\|x\|^p \quad (1.6)$$

for all $x \in E$. If in addition for every $x \in E$, $f(tx)$ is continuous in real t for each fixed x , then T is linear.

Suppose $(G, +)$ is an abelian group, E is a Banach space, and that the so-called admissible control function $\varphi : G \times G \rightarrow \mathbb{R}$ satisfies

$$\tilde{\varphi}(x, y) := 2^{-1} \sum_{n=0}^{\infty} 2^{-n} \varphi(2^n x, 2^n y) < \infty \quad (1.7)$$

for all $x, y \in G$. If $f : G \rightarrow E$ is a mapping with

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y) \quad (1.8)$$

for all $x, y \in G$, then there exists a unique mapping $T : G \rightarrow E$ such that $T(x+y) = T(x) + T(y)$ and $\|f(x) - T(x)\| \leq \tilde{\varphi}(x, x)$, for all $x, y \in G$ (see [6]).

Generalized derivations first appeared in the context of operator algebras [7]. Later, these were introduced in the framework of pure algebra [8, 9].

Definition 1.1. Let A be an algebra and let X be an A -bimodule. A linear mapping $d : A \rightarrow X$ is called

- (i) derivation if $d(ab) = d(a)b + ad(b)$, for all $a, b \in A$;
- (ii) generalized derivation if there exists a derivation (in the usual sense) $\delta : A \rightarrow X$ such that $d(ab) = ad(b) + \delta(a)b$, for all $a, b \in A$.

Every right multiplier (i.e., a linear map h on A satisfying $h(ab) = ah(b)$, for all $a, b \in A$) is a generalized derivation.

Definition 1.2. Let $n \geq 2$, $k \geq 3$ be positive integers. Let A be an algebra and let X be an A -bimodule. A $\mathbb{C}$ -linear mapping $d : A \rightarrow X$ is called

- (i) n -derivation if

$$d(a_1 a_2 \cdots a_n) = d(a_1) a_2 \cdots a_n + a_1 d(a_2) a_3 \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n) \quad (1.9)$$

for all $a_1, a_2, \dots, a_n \in A$;

- (ii) generalized (n, k) -derivation if there exists a $(k-1)$ -derivation $\delta : A \rightarrow X$ such that

$$\begin{aligned} d(a_1 a_2 \cdots a_n) &= \delta(a_1) a_2 \cdots a_n + a_1 \delta(a_2) a_3 \cdots a_n + \cdots + a_1 a_2 \cdots a_{k-2} \delta(a_{k-1}) a_k \cdots a_n \\ &\quad + a_1 a_2 \cdots a_{k-1} d(a_k) a_{k+1} \cdots a_n + a_1 a_2 \cdots a_k d(a_{k+1}) a_{k+2} \cdots a_n \\ &\quad + a_1 a_2 \cdots a_{k+1} d(a_{k+2}) a_{k+3} \cdots a_n + \cdots + a_1 \cdots a_{n-1} d(a_n) \end{aligned} \quad (1.10)$$

for all $a_1, a_2, \dots, a_n \in A$.

By Definition 1.2, we see that a generalized $(2, 3)$ -derivation is a generalized derivation.

For instance, let $\mathcal{A}$ be a Banach algebra. Then we take

$$\mathcal{T} = \begin{bmatrix} 0 & \mathcal{A} & \mathcal{A} & \mathcal{A} \\ 0 & 0 & \mathcal{A} & \mathcal{A} \\ 0 & 0 & 0 & \mathcal{A} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (1.11)$$

$\mathcal{T}$ is an algebra equipped with the usual matrix-like operations. It is easy to check that every linear map from $\mathcal{A}$ into $\mathcal{A}$ is a $(5, 3)$ -derivation, but there are linear maps on $\mathcal{T}$ which are not generalized derivations.

The so-called approximate derivations were investigated by Jun and Park [10]. Recently, the stability of derivations have been investigated by some authors; see [10–13] and references therein. Moslehian [14] investigated the generalized Hyers-Ulam stability of generalized derivations from a unital normed algebra A to a unit linked Banach A -bimodule (see also [15]).

In this paper, we investigate the generalized Hyers-Ulam stability of the generalized (n, k) -derivations.

2. Main Result

In this section, we investigate the generalized Hyers-Ulam stability of the generalized (n, k) -derivations from a unital Banach algebra A into a unit linked Banach A -bimodule. Throughout this section, assume that A is a unital Banach algebra, X is unit linked Banach A -bimodule, and suppose that $3 \leq n$, and $3 \leq k \leq n$.

We need the following lemma in the main results of the present paper.

Lemma 2.1 (see [16]). *Let U, V be linear spaces and let $f : U \rightarrow V$ be an additive mapping such that $f(\lambda x) = \lambda f(x)$, for all $x \in U$ and all $\lambda \in \mathbb{T}^1 := \{\lambda \in \mathbb{C}; |\lambda| = 1\}$. Then the mapping f is $\mathbb{C}$ -linear.*

Now we prove the generalized Hyers-Ulam stability of generalized (n, k) -derivations.

Theorem 2.2. *Suppose $f : A \rightarrow X$ is a mapping with $f(0) = 0$ for which there exists a map $g : A \rightarrow X$ with $g(0) = 0$ and a function $\varphi : A^{n+2} \rightarrow \mathbb{R}^+$ such that*

$$\begin{aligned} & \max \{ \|f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\ & \quad - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\ & \quad - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\|, \\ & \|g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\ & \quad - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\| \} \\ & \leq \varphi(a, b, a_1, a_2, \dots, a_n), \end{aligned} \quad (2.1)$$

$$\tilde{\varphi}(a, b, a_1, a_2, \dots, a_n) := 2^{-1} \sum_{i=0}^{\infty} 2^{-i} \varphi(2^i a, 2^i b, 2^i a_1, \dots, 2^i a_n) < \infty \quad (2.2)$$

for all $a, b, a_1, a_2, \dots, a_n \in A$ and all $\lambda \in \mathbb{T}^1$. Then there exists a unique generalized (n, k) -derivation $d : A \rightarrow X$ such that

$$\|f(a) - d(a)\| \leq \tilde{\varphi}(a, a, 0, 0, \dots, 0) \quad (2.3)$$

for all $a \in A$.

Proof. By (2.1) we have

$$\begin{aligned} & \|f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\ & \quad - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\ & \quad - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\| \\ & \leq \varphi(a, b, a_1, a_2, \dots, a_n), \end{aligned} \quad (2.4)$$

$$\begin{aligned}
& \|g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\
& \quad - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n\| \\
& \leq \varphi(a, b, a_1, a_2, \dots, a_n)
\end{aligned} \tag{2.5}$$

for all $a, b, a_1, a_2, \dots, a_n \in A$ and all $\lambda \in \mathbb{T}^1$. Setting $a_1, a_2, \dots, a_n = 0$ and $\lambda = 1$ in (2.4), we have

$$\|f(a+b) - f(a) - f(b)\| \leq \varphi(a, b, 0, 0, \dots, 0) \tag{2.6}$$

for all $a, b \in A$. One can use induction on n to show that

$$\|2^{-m} f(2^m a) - f(a)\| \leq 2^{-1} \sum_{i=0}^{m-1} 2^{-i} \varphi(2^i a, 2^i a, 0, \dots, 0) \tag{2.7}$$

for all $n \in \mathbb{N}$ and all $a \in A$, and that

$$\|2^{-m} f(2^m a) - 2^{-l} f(2^l a)\| \leq 2^{-1} \sum_{i=l}^{m-1} 2^{-i} \varphi(2^i a, 2^i a, 0, \dots, 0) \tag{2.8}$$

for all $m > l$ and all $a \in A$. It follows from the convergence (2.2) that the sequence $2^{-m} f(2^m a)$ is Cauchy. Due to the completeness of X , this sequence is convergent. Set

$$d(a) := \lim_{m \rightarrow \infty} 2^{-m} f(2^m a). \tag{2.9}$$

Putting $a_1, a_2, \dots, a_n = 0$ and replacing a, b by $2^m a, 2^m b$, respectively, in (2.4), we get

$$\|2^{-m} f(2^m(\lambda a + \lambda b)) - 2^{-m} \lambda f(2^m a) - 2^{-m} \lambda f(2^m b)\| \leq 2^{-m} \varphi(2^m a, 2^m b, 0, 0, \dots, 0) \tag{2.10}$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. Taking the limit as $m \rightarrow \infty$ we obtain

$$d(\lambda a + \lambda b) = \lambda d(a) + \lambda d(b) \tag{2.11}$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. So by Lemma 2.1, the mapping d is $\mathbb{C}$ -linear.

Using (2.5), (2.2), and the above technique, we get

$$\delta(a) := \lim_{m \rightarrow \infty} 2^{-m} g(2^m a), \tag{2.12}$$

$$\delta(\lambda a + \lambda b) = \lambda \delta(a) + \lambda \delta(b)$$

for all $a, b \in A$ and all $\lambda \in \mathbb{T}^1$. Hence by Lemma 2.1, δ is $\mathbb{C}$ -linear. Moreover, it follows from (2.7) and (2.9) that $\|f(a) - d(a)\| \leq \tilde{\varphi}(a, a, 0, 0, \dots, 0)$, for all $a \in A$. It is known that the additive mapping d satisfying (2.3) is unique [17]. Putting $\lambda = 1$, $a = b = 0$, and replacing $a_1, a_2, \dots, a_n$ by $2^m a_1, 2^m a_2, \dots, 2^m a_n$, respectively, in (2.4), we get

$$\begin{aligned} & \left\| f(2^{nm} a_1 a_2 \cdots a_n) - 2^{(n-1)m} a_1 \cdots a_{k-1} f(2^m a_k) a_{k+1} \cdots a_n - 2^{(n-1)m} a_1 \cdots a_k f(2^m a_{k+1}) a_{k+2} \cdots a_n \right. \\ & \quad - \cdots - 2^{(n-1)m} a_1 \cdots a_{n-1} f(2^m a_n) - 2^{(n-1)m} g(2^m a_1) a_2 \cdots a_n \\ & \quad - 2^{(n-1)m} a_1 g(2^m a_2) a_3 \cdots a_n - \cdots - 2^{(n-1)m} a_1 a_2 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \left. \right\| \\ & \leq \varphi(0, 0, 2^m a_1, 2^m a_2, \dots, 2^m a_n), \end{aligned} \quad (2.13)$$

whence

$$\begin{aligned} & \left\| 2^{-nm} f(2^{nm} a_1 a_2 \cdots a_n) - 2^{-m} a_1 \cdots a_{k-1} f(2^m a_k) a_{k+1} \cdots a_n \right. \\ & \quad - 2^{-m} a_1 \cdots a_k f(2^m a_{k+1}) a_{k+2} \cdots a_n - \cdots - 2^{-m} a_1 \cdots a_{n-1} f(2^m a_n) - 2^{-m} g(2^m a_1) a_2 \cdots a_n \\ & \quad - 2^{-m} a_1 g(2^m a_2) a_3 \cdots a_n - \cdots - 2^{-m} a_1 a_2 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \left. \right\| \\ & \leq 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \dots, 2^m a_n) \end{aligned} \quad (2.14)$$

for all $a_1, a_2, \dots, a_n \in A$. By (2.9), $\lim_{m \rightarrow \infty} 2^{-nm} f(2^{nm} a) = d(a)$ and by the convergence of series (2.2), $\lim_{m \rightarrow \infty} 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \dots, 2^m a_n) = 0$. Let m tend to ∞ in (2.14). Then

$$\begin{aligned} d(a_1 a_2 \cdots a_n) &= a_1 \cdots a_{k-1} d(a_k) a_{k+1} \cdots a_n + a_1 \cdots a_k d(a_{k+1}) a_{k+2} \cdots a_n \\ & \quad + \cdots + a_1 \cdots a_{n-1} d(a_n) + \delta(a_1) a_2 \cdots a_n + a_1 \delta(a_2) a_3 \cdots a_n \\ & \quad + \cdots + a_1 a_2 \cdots a_{k-2} \delta(a_{k-1}) a_k \cdots a_n \end{aligned} \quad (2.15)$$

for all $a_1, a_2, \dots, a_n \in A$.

Next we claim that δ is a $(k-1)$ -derivation. Putting $\lambda = 1$, $a = b = 0$, and replacing $a_1, a_2, \dots, a_n$ by $2^m a_1, 2^m a_2, \dots, 2^m a_n$, respectively, in (2.5), we get

$$\begin{aligned} & \left\| g(2^{nm} a_1 a_2 \cdots a_n) - 2^{(n-1)m} g(2^m a_1) a_2 \cdots a_n - 2^{(n-1)m} a_1 g(2^m a_2) a_3 \cdots a_n \right. \\ & \quad - 2^{(n-1)m} a_1 a_2 g(2^m a_3) a_4 \cdots a_n - \cdots - 2^{(n-1)m} a_1 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \left. \right\| \\ & \leq \varphi(0, 0, 2^m a_1, 2^m a_2, \dots, 2^m a_n), \end{aligned} \quad (2.16)$$

whence

$$\begin{aligned} & \left\| 2^{-nm} g(2^{nm} a_1 a_2 \cdots a_n) - 2^{-m} g(2^m a_1) a_2 \cdots a_n - 2^{-m} a_1 g(2^m a_2) a_3 \cdots a_n \right. \\ & \quad - 2^{-m} a_1 a_2 g(2^m a_3) a_4 \cdots a_n - \cdots - 2^{-m} a_1 \cdots a_{k-2} g(2^m a_{k-1}) a_k \cdots a_n \left. \right\| \\ & \leq 2^{-nm} \varphi(0, 0, 2^m a_1, 2^m a_2, \dots, 2^m a_n) \end{aligned} \quad (2.17)$$

for all $a_1, a_2, \dots, a_n \in A$. Let m tends to ∞ in (2.17). Then

$$\begin{aligned} \delta(a_1 a_2 \cdots a_{k-1} a_k a_{k+1} \cdots a_n) &= \delta(a_1) a_2 \cdots a_n + a_1 \delta(a_2) a_3 \cdots a_n + a_1 a_2 \delta(a_3) a_4 \cdots a_n \\ &+ \cdots + a_1 a_2 \cdots a_{k-2} \delta(a_{k-1}) a_k \cdots a_n \end{aligned} \quad (2.18)$$

for all $a_1, a_2, \dots, a_n \in A$.

Setting $a_k = a_{k+1} = \cdots = a_n = 1$ in (2.18). Hence the mapping δ is $(k-1)$ -derivation. $\square$

Corollary 2.3. Suppose $f : A \rightarrow X$ is a mapping with $f(0) = 0$ for which there exists constant $\theta \geq 0$, $p < 1$ and a map $g : A \rightarrow X$ with $g(0) = 0$ such that

$$\begin{aligned} \max \{ & \| f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\ & - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\ & - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n \|, \\ & \| g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\ & - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n \| \} \\ & \leq \theta \left(\|a\|^p + \|b\|^p + \sum_{i=1}^n \|a_i\|^p \right) \end{aligned} \quad (2.19)$$

for all $a_1, a_2, \dots, a_n \in A$ and all $\lambda \in \mathbb{T}$. Then there exists a unique generalized (n, k) -derivation $d : A \rightarrow X$ such that

$$\|f(a) - d(a)\| \leq \frac{\theta \|a\|^p}{1 - 2^{p-1}} \quad (2.20)$$

for all $a \in A$.

Proof. Put $\varphi(a, b, a_1, a_2, \dots, a_n) = \theta(\|a\|^p + \|b\|^p + \sum_{i=1}^n \|a_i\|^p)$ in Theorem 2.2. $\square$

Corollary 2.4. Suppose $f : A \rightarrow X$ is a mapping with $f(0) = 0$ for which there exists constant $\theta \geq 0$ and a map $g : A \rightarrow X$ with $g(0) = 0$ such that

$$\begin{aligned} \max \{ & \| f(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda f(a) - \lambda f(b) - a_1 \cdots a_{k-1} f(a_k) a_{k+1} \cdots a_n \\ & - a_1 \cdots a_k f(a_{k+1}) a_{k+2} \cdots a_n - \cdots - a_1 \cdots a_{n-1} f(a_n) \\ & - g(a_1) a_2 \cdots a_n - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 a_2 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n \|, \\ & \| g(\lambda a + \lambda b + a_1 a_2 \cdots a_n) - \lambda g(a) - \lambda g(b) - g(a_1) a_2 \cdots a_n \\ & - a_1 g(a_2) a_3 \cdots a_n - \cdots - a_1 \cdots a_{k-2} g(a_{k-1}) a_k \cdots a_n \| \} \\ & \leq \theta \end{aligned} \quad (2.21)$$

for all $a_1, a_2, \dots, a_n \in A$. Then there exists a unique generalized (n, k) -derivation $d : A \rightarrow X$ such that

$$\|f(a) - d(a)\| \leq \theta \quad (2.22)$$

for all $a \in A$.

Proof. Letting $p = 0$ in Corollary 2.3, we obtain the above result of Corollary 2.4. $\square$

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