Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2011, Article ID 149091, 10 pages
doi:10.1155/2011/149091

Research Article

Nonexistence Results for the Cauchy Problem for Nonlinear Ultraparabolic Equations

Sebti Kerbal[1] and Mokhtar Kirane[2]

[1] *Department of Mathematics and Statistics, Sultan Qaboos University, P.O. Box 36, Al-Khodh, Muscat 123, Oman*
[2] *Laboratoire de Mathématiques, Image et Applications, Pôle Sciences et Technologies, Université de La Rochelle, Avenue M. Crépeau, 17042 La Rochelle, France*

Correspondence should be addressed to Sebti Kerbal, skerbal@hotmail.com

Received 23 March 2011; Revised 21 May 2011; Accepted 14 June 2011

Academic Editor: Toka Diagana

Nonexistence of global solutions to ultraparabolic equations and systems is presented. Our results fill a gap in the literature on ultraparabolic equations. The method of proof we use relies on a choice of a suitable test function in the weak formulation of the solutions of the problems under-study.

1. Introduction

In this paper, we will present first nonexistence results for the two-time nonlinear equation

$$\mathcal{L}u := u_{t_1} + u_{t_2} - \Delta\left(|u|^m\right) = |u|^p, \tag{1.1}$$

posed for $(t_1, t_2, x) \in Q = (0, +\infty) \times (0, +\infty) \times \mathbb{R}^d$, $d \in \mathbb{N}$, and subject to the initial conditions

$$u(t_1, 0; x) = \varphi_1(t_1; x), \qquad u(0, t_2; x) = \varphi_2(t_2; x). \tag{1.2}$$

Here $p > 1$, $m > 0$ are real numbers. Then we extend our results to systems of the form

$$\begin{gathered} u_{t_1} + u_{t_2} - \Delta\left(|u|^m\right) = |v|^p, \\ v_{t_1} + v_{t_2} - \Delta\left(|v|^n\right) = |u|^q, \end{gathered} \tag{1.3}$$

for $(t_1, t_2, x) \in Q$, subject to the initial conditions

$$\begin{aligned} u(t_1, 0, x) &= \varphi_1(t_1, x), \qquad u(0, t_2, x) = \varphi_2(t_2, x), \\ v(t_1, 0, x) &= \psi_1(t_1, x), \qquad v(0, t_2, x) = \psi_2(t_2, x), \end{aligned} \tag{1.4}$$

and where $p > 1$, $q > 1$, $m > 0$, and $n > 0$ are real numbers. We take the nonlinearities $|u|^p$ in (1.1) and $(|v|^p, |u|^q)$ in (1.3) as prototypes; we could consider much more general nonlinearities.

Before we present our results, let us dwell a while on the existing literature on nonlinear ultraparabolic parabolic equations known also as pluri-parabolic equations or multitime parabolic equations which we are aware of. These types of equations started in the case of linear equations with Kolmogoroff [1] in 1934; he introduced them in order to describe the probability density of a system with 2d degrees of freedom. A lot of generalizations have been made by a large number of authors since then. Nonlinear ultraparabolic equations arise in the kinetic theory of gases [2, 3]. Some stochastic processes models lead also to ultraparabolic equations [4–7]. The analysis of nonlinear ultraparabolic equations have been studied first by Ugowski [8] who studied differential inequalities of parabolic type with multidimensional time; he established, for example, a maximum principle which is very useful for applications. His results were reformulated, in a less general setting, by Walter in [9]. Many nice works on different aspects on ultraparabolic nonlinear equations have been conducted by Lavrenyuk and his collaborators [10–12], Lanconelli and his collaborators [13, 14], and Citti et al. [15]; see also [16, 17]. In the absence of diffusion, an interesting application is mentioned in [18]. Our equation and system have their applications in diffusion theory in porous media.

For better positioning of our results, let us recall the pioneering results of Fujita [19] and their complementary results by Hayakawa [20], Kobayashi et al. [21], and Samarskii et al. [22] concerning nonexistence results for the equation

$$u_t - \Delta(u^m) = |u|^p, \quad t > 0, \ x \in \mathbb{R}^d, \tag{1.5}$$

which corresponds to (1.1) in the absence of t_2 and with $t_1 = t$.

In his article [19] corresponding to $m = 1$, Fujita proved that

(i) if $1 < p < 1 + 2/d$, then no global positive solutions for any nonnegative initial data u_0 exist;

(ii) if $p > 1 + 2/d$, global small data solutions exist while global solutions for large data do not exist.

The borderline case $p = 1 + 2/d$ has been decided by Hayakawa [20] for $d = 1, 2$ and then by Kobayashi et al. [21] for any $d \geq 1$; In case $m = 1$, the exponent $p_{\text{crit}} = 1 + 2/d$ is called the critical exponent.

For (1.5), Samarskii et al. [22] showed that the critical exponent is $p_{\text{crit}} = m + 2/d$.

The aim of this paper is to obtain the critical exponent in the sense of Fujita for (1.1) and for system (1.3). Moreover, we present critical exponents for systems of two equations.

2. Results

Solutions to (1.1) subject to conditions (1.2) are meant in the following weak sense.

Definition 2.1. A function $u \in L^m_{\text{loc}}(Q) \cap L^p_{\text{loc}}(Q)$ is called a weak solution to (1.1) if

$$\begin{aligned}
&\int_Q |u|^p \varphi \, dP + \int_S u(0,t_2;x)\varphi(0,t_2;x)dP_2 + \int_S u(t_1,0;x)\varphi(t_1,0;x)dP_1 \\
&\quad = -\int_Q u\varphi_{t_1} dP - \int_Q u\varphi_{t_2} dP - \int_Q |u|^m \Delta\varphi \, dP
\end{aligned} \tag{2.1}$$

for any test function $\varphi \in C_0^\infty(Q)$; $S = \mathbb{R}_+ \times \mathbb{R}^d$, $P = (t_1, t_2, x)$ and $P_1 = (t_1, x)$, $P_2 = (t_2, x)$.

Note that every weak solution is classical near the points (t_1, t_2, x) where $u(t_1, t_2, x)$ is positive.

Two words about the local existence of solutions are in order: as it is a rule, one regularizes (1.1) by adding first a vanishing diffusion term as follows:

$$\mathcal{L}_\varepsilon u = \mathcal{L}u - \varepsilon D_{t_1 t_1}, \quad \varepsilon > 0, \tag{2.2}$$

and then by regularizing the degenerate term $\Delta(|u|^m)$; so, the regular equation

$$u_{t_1} + u_{t_2} - \Delta\left(\min\{ku, |u|^m\}\right) - \varepsilon D_{t_1 t_1} = |u|^p, \quad \varepsilon > 0, \ k = 1, 2, \ldots \tag{2.3}$$

is obtained. Consequently, one obtains, for small time t_1, (ε, k)-uniform estimates of solutions, namely, estimates which are independent on the "parabolicity" constants of the equation as it is clearly explained in [23], see also [10, 24].

Our main first result is dealing with (1.1) subject to (1.2); it is given by the following theorem.

Theorem 2.2. *Assume that* $\int_S u(0,t_2;x)dP_2 + \int_S u(t_1,0;x)dP_1 > 0$. *If* $1 \le m < p \le m + 2m/(2+d)$, *then Problem* (1.1)-(1.2) *does not admit global weak solutions.*

Proof. Our strategy of proof is to use the weak formulation of the solution with a suitable choice of the test function which we learnt from [25]. Assume u is a global solution.

If we write

$$u\varphi_{t_i} = u\varphi^{1/p}\varphi^{-1/p}\varphi_{t_i}, \quad i = 1, 2, \tag{2.4}$$

and estimate $\int_Q u\varphi_{t_i} dP$ using the ε-Young inequality, we obtain

$$\int_Q u\varphi_{t_i} dP \le \varepsilon \int_Q |u|^p \varphi dP + C_\varepsilon \int_Q \varphi^{-1/(p-1)} |\varphi_{t_i}|^{p/(p-1)} dP. \tag{2.5}$$

Similarly, we have

$$\int_Q |u|^m \Delta\varphi dP \leq \varepsilon \int_Q |u|^p \varphi dP + C_\varepsilon \int_Q \varphi^{-m/(p-m)} \left|\Delta\varphi\right|^{p/(p-m)} dP, \tag{2.6}$$

where $p > m$.

Now, using (2.5) and (2.6), we obtain

$$\begin{aligned} &\int_Q |u|^p \varphi\, dP + \int_S u(0,t_2;x)\varphi(0,t_2;x)dP_2 + \int_S u(t_1,0;x)\varphi(t_1,0;x)dP_1 \\ &\quad\leq 2\varepsilon \int_Q |u|^p \varphi\, dP + C_\varepsilon \int_Q \left(\varphi^{-1/(p-1)}\left(\left|\varphi_{t_1}\right|^{p/(p-1)} + \left|\varphi_{t_2}\right|^{p/(p-1)}\right) + \varphi^{-m/(p-m)}\left|\Delta\varphi\right|^{p/(p-m)}\right)dP. \end{aligned} \tag{2.7}$$

If we choose $\varepsilon = 1/4$, then we get the estimate

$$\begin{aligned} &\int_Q |u|^p \varphi\, dP + 2\int_S u(0,t_2;x)\varphi(0,t_2;x)dP_2 + 2\int_S u(t_1,0;x)\varphi(t_1,0;x)dP_1 \\ &\quad\leq C \int_Q \left(\varphi^{-1/(p-1)}\left(\left|\varphi_{t_1}\right|^{p/(p-1)} + \left|\varphi_{t_2}\right|^{p/(p-1)}\right) + \varphi^{-m/(p-m)}\left|\Delta\varphi\right|^{p/(p-m)}\right)dP =: \mathscr{A}(\varphi), \end{aligned} \tag{2.8}$$

for some positive constant C. Observe that the right-hand side of (2.8) is free of the unknown function u.

At this stage, we introduce the smooth nonincreasing function $\theta : \mathbb{R}_+ \to [0,1]$ such that

$$\theta(z) = \begin{cases} 1, & 0 \leq z \leq 1, \\ 0, & 2 \leq z. \end{cases} \tag{2.9}$$

Let us take in (2.8)

$$\varphi(t_1,t_2;x) = \theta^\lambda\left(\frac{t_1}{R^2} + \frac{t_2}{R^2} + \frac{|x|^2}{R^2}\right), \tag{2.10}$$

with $\lambda > max\{p/(p-1), 2p/(p-m)\}$ and R being positive real number.

Let us now pass to the new variables

$$\tau_1 = R^{-2}t_1, \qquad \tau_2 = R^{-2}t_2, \qquad y = R^{-1}x. \tag{2.11}$$

We have

$$\varphi_{t_i} = R^{-2}\varphi_{\tau_i}, \quad i = 1,2, \qquad \Delta_x\varphi = R^{-2}\Delta_y\varphi. \tag{2.12}$$

Whereupon

$$\int_Q |u|^p \varphi \, dP + 2\int_S u(0,t_2;x)\varphi(0,t_2,x)dP_2 + 2\int_S u(t_1,0;x)\varphi(t_1,0,x)dP_1 \\ \leq L\Big(R^{4+d-2p/(p-1)} + R^{4+d-2p/(p-m)}\Big), \tag{2.13}$$

with

$$L := C\int_{\Omega_1}\Big(\theta^{((\lambda-1)p-\lambda)/(p-1)}\left|\theta'\right|^{p/(p-1)} + \left|\theta'\right|^{2p/(p-m)}\theta^{((\lambda-2)p-\lambda m)/(p-m)} \\ +\left|\theta''\right|^{p/(p-m)}\theta^{((\lambda-1)p-\lambda m)/(p-m)}\Big) < +\infty, \tag{2.14}$$

where $\Omega_1 = \{(\tau_1,\tau_2,y) : 1 \leq |\tau_1| + |\tau_2| + y \leq 2\}$.

Now, we want to pass to the limit as $R \to +\infty$ in (2.13) under the constraint $2p/(p-m) - 4 - d \geq 0$. We have to consider two cases.

(i) Either $2p/(p-m) - 4 - d > 0 \Leftrightarrow 1 < p < m + 2m/(2+d) = p_{\text{crit}}$ and in this case, the right-hand side of (2.13) will go to zero while the left-hand side is positive. Contradiction.

(ii) Or $p = p_{\text{crit}}$, and in this case, we get in particular

$$\int_{\mathbb{R}_+^2\times\mathbb{R}^d} |u|^p\varphi\, dP \leq C \Longrightarrow \lim_{R\to+\infty}\int_{C_R}|u|^p\varphi\, dP = 0, \tag{2.15}$$

where $C_R = \{(t_1,t_2;x) \mid R^2 \leq t_1 + t_2 + |x|^2 \leq 2R^2\}$.

Now, to conclude, we rely on the estimate

$$\int_Q |u|^p\varphi\, dP + \int_S u(0,t_2;x)\varphi(0,t_2;x)dP_2 + \int_S u(t_1,0;x)\varphi(t_1,0;x)dP_1 \\ \leq \left(\int_{C_R}|u|^p\varphi\, dP\right)^{1/p}\mathcal{A}(\varphi), \tag{2.16}$$

which is obtained by using the Hölder inequality.

Passing to the limit as $R \to +\infty$ in (2.16), we obtain

$$\int_{\mathbb{R}_+^2\times\mathbb{R}^d}|u|^p dP + \int_S u(0,t_2;x)dP_2 + \int_S u(t_1,0;x)dP_1 = 0. \tag{2.17}$$

Contradiction. □

Remark 2.3. Notice that the critical exponent for the ultraparabolic equation is *smaller* than the one of the corresponding parabolic equation.

2.1. The Case of a 2×2-System with a 2-Dimensional Time

In this section, we extend the analysis of the previous section to the case of a 2×2-system of 2-time equations. More precisely, we consider the system

$$\begin{aligned} u_{t_1} + u_{t_2} - \Delta\left(|u|^m\right) &= |v|^p, \\ v_{t_1} + v_{t_2} - \Delta\left(|v|^n\right) &= |u|^q, \end{aligned} \tag{2.18}$$

for $(t_1, t_2; x) \in Q$, subject to the initial conditions

$$\begin{aligned} u(0, t_2, x) &= \varphi_1(t_2, x), \qquad & u(t_1, 0, x) &= \varphi_2(t_1, x), \\ v(0, t_2, x) &= \psi_1(t_2, x), \qquad & v(t_1, 0, x) &= \psi_2(t_1, x), \end{aligned} \tag{2.19}$$

and where $0 < m < p$, $0 < n < q$, and $p, q > 1$ are real numbers.

To lighten the presentation, let us set

$$\begin{aligned} I_0 &:= \int_S u(t_1, 0; x)\varphi(t_1, 0; x)dP_1 + \int_S u(0, t_2; x)\varphi(0, t_2; x)dP_2, \\ J_0 &:= \int_S v(t_1, 0; x)\varphi(t_1, 0; x)dP_1 + \int_S v(0, t_2; x)\varphi(0, t_2; x)dP_2. \end{aligned} \tag{2.20}$$

Let us start with the following definition.

Definition 2.4. We say that $(u, v) \in (L^q_{\mathrm{loc}}(Q) \cap L^m_{\mathrm{loc}}(Q)) \times (L^p_{\mathrm{loc}}(Q) \cap L^n_{\mathrm{loc}}(Q))$ is a weak solution to system (2.18) if

$$\begin{aligned} \int_Q |v|^p \varphi \, dP + I_0 &= -\int_Q u\varphi_{t_1} dP - \int_Q u\varphi_{t_2} dP - \int_Q |u|^m \Delta\varphi \, dP, \\ \int_Q |u|^q \varphi \, dP + J_0 &= -\int_Q v\varphi_{t_1} dP - \int_Q v\varphi_{t_2} dP - \int_Q |v|^n \Delta\varphi \, dP \end{aligned} \tag{2.21}$$

for any test function $\varphi \in C_0^\infty(Q)$.

Note that every weak solution is classical near the points (t_1, t_2, x) where $u(t_1, t_2, x)$ and $v(t_1, t_2, x)$ are positive.

Let us set

$$\sigma_1(p,q)=\frac{q(2-(d+2)p)+4+d}{pq-1},\qquad \sigma_2(p,q)=\frac{q(2-(d+2)p)+(4+d)m}{pq-m},$$
$$\sigma_3(p,q)=\frac{q(2n-(d+2)p)+(4+d)n}{pq-n},\qquad \sigma_4(p,q)=\frac{q(2-(d+2)p)+(4+d)mn}{pq-mn}. \tag{2.22}$$

Theorem 2.5. *Let $p>1$, $q>1$, $p>n$, $q>m$, and assume that*

$$\int_S u(t_1,0;x)dP_1+\int_S u(0,t_2;x)dP_2>0,$$
$$\int_S v(t_1,0;x)dP_1+\int_S v(0,t_2;x)dP_2>0. \tag{2.23}$$

Then system (2.18)-(2.19) *admits no global weak solution whenever*

$$\max\{\sigma_1(p,q),\dots,\sigma_4(p,q),\sigma_1(q,p),\dots,\sigma_4(q,p)\}\le 0. \tag{2.24}$$

Proof. Assume that the solution is global. Using Hölder's inequality, we obtain

$$\int_Q |u|^m|\Delta\varphi|dP=\int_Q |u|^m\varphi^{m/q}\varphi^{-m/q}|\Delta\varphi|dP$$
$$\le\left(\int_Q |u|^q\varphi\, dP\right)^{m/q}\left(\int_Q \varphi^{-m/(q-m)}|\Delta\varphi|^{q/(q-m)}dP\right)^{(q-m)/q}, \tag{2.25}$$

$$\int_Q u\varphi_{t_i}dP\le\left(\int_Q |u|^q\varphi\, dP\right)^{1/q}\left(\int_Q \varphi^{-1/(q-1)}|\varphi_{t_i}|^{(q-1)/q}dP\right)^{(q-1)/q}, \tag{2.26}$$

for $i=1,2$. Similarly, we have

$$\int_Q |v|^n|\Delta\varphi|dP\le\left(\int_Q |v|^p\varphi\, dP\right)^{n/p}\left(\int_Q \varphi^{-n/(p-n)}|\Delta\varphi|^{p/(p-n)}dP\right)^{(p-n)/p}. \tag{2.27}$$

If we set

$$\begin{gathered}\mathcal{I} := \int_Q |u|^q \varphi\, dP, \qquad \mathcal{J} := \int_Q |v|^p \varphi\, dP,\\ \mathcal{A}(p,n) = \left(\int_Q \varphi^{-n/(p-n)} \left|\Delta\varphi\right|^{p/(p-n)} dP\right)^{(p-n)/p},\\ \mathcal{B}_i(q) = \left(\int_Q \varphi^{-1/(q-1)} \left|\varphi_{t_i}\right|^{q/(q-1)} dP\right)^{(q-1)/q},\\ \mathcal{B}(q) = \mathcal{B}_1(q) + \mathcal{B}_2(q),\end{gathered} \tag{2.28}$$

then, using (2.23), inequalities (2.26) and (2.27) in (2.21), we my write

$$\begin{gathered}\mathcal{I} \leq \mathcal{J}^{1/p}\mathcal{B}(p) + \mathcal{J}^{n/p}\mathcal{A}(p,n),\\ \mathcal{J} \leq \mathcal{I}^{1/q}\mathcal{B}(q) + \mathcal{I}^{m/q}\mathcal{A}(q,m),\end{gathered} \tag{2.29}$$

so

$$\mathcal{J}^{n/p} \leq C\left\{\mathcal{I}^{n/pq}\mathcal{B}^{n/q}(q) + \mathcal{I}^{mn/pq}\mathcal{A}^{n/p}(q,m)\right\} \tag{2.30}$$

for some positive constant C.

Whereupon

$$\begin{aligned}\mathcal{I} \leq C\Big\{&\mathcal{I}^{1/pq}\mathcal{B}^{1/p}(q)\mathcal{B}(p) + \mathcal{I}^{m/pq}\mathcal{A}^{1/p}(q,m)\mathcal{B}(p)\\ &+\mathcal{I}^{n/pq}\mathcal{B}^{n/p}(q)\mathcal{A}(p,n) + \mathcal{I}^{mn/pq}\mathcal{A}^{n/p}(q,m)\mathcal{A}(p,n)\mathcal{B}(p)\Big\}.\end{aligned} \tag{2.31}$$

Using Hölder's inequality, we may write

$$\begin{aligned}\mathcal{I} \leq C\Big\{&\left(\mathcal{B}^{1/p}(q)\mathcal{B}(p)\right)^{pq/(pq-1)} + \left(\mathcal{A}^{1/p}(q,m)\mathcal{B}(p)\right)^{pq/(pq-m)}\\ &+\left(\mathcal{B}^{n/p}(q)\mathcal{A}(p,n)\right)^{pq/(pq-n)} + \left(\mathcal{A}^{n/p}(q,m)\mathcal{A}(p,n)\right)^{pq/(pq-mn)}\Big\}.\end{aligned} \tag{2.32}$$

At this stage, using the scaled variables (2.11), we obtain

$$\begin{gathered}\mathcal{A}(p,n) = CR^{-2+(4+d)(1-n/p)},\\ \mathcal{B}_i(q) = CR^{-2+(4+d)(1-1/q)}, \quad i = 1,2.\end{gathered} \tag{2.33}$$

Hence, for $\mathcal{I}$, we get the estimate

$$\mathcal{I} \le C\left\{R^{-\sigma_1(p,q)} + R^{-\sigma_2(p,q)} + R^{-\sigma_3(p,q)} + R^{-\sigma_4(p,q)}\right\}. \tag{2.34}$$

Observe that, following the same lines, we can also obtain the following estimate for $\mathcal{J}$:

$$\mathcal{J} \le C\left\{R^{-\sigma_1(q,p)} + R^{-\sigma_2(q,p)} + R^{-\sigma_3(q,p)} + R^{-\sigma_4(q,p)}\right\}. \tag{2.35}$$

To conclude, we have to consider two cases.

Case 1. If $\max\{\sigma_1(p,q),\dots,\sigma_4(p,q),\sigma_1(q,p),\dots,\sigma_4(q,p)\} < 0$ then

$$\begin{gathered}\lim_{R\to+\infty}\mathcal{I} = \int_Q |u|^q dP = 0 \Longrightarrow u = 0,\ p.p. \\ \lim_{R\to+\infty}\mathcal{J} = \int_Q |v|^p dP = 0 \Longrightarrow v = 0,\ p.p.\end{gathered} \tag{2.36}$$

A contradiction.

Case 2. If $\max\{\sigma_1(p,q),\dots,\sigma_4(p,q),\sigma_1(q,p),\dots,\sigma_4(q,p)\} = 0$, we conclude following the same argument used for one equation. □

Acknowledgments

The authors thank the referees for their remarks. This work is supported by Sultan Qaboos University under Grant: IG/SCI/DOMS/11/06. This work has been done during a visit of the second named author to DOMAS, Sultan Qaboos University, Muscat, Oman, which he thanks for its support and hospitality.

References

[1] A. Kolmogoroff, "Zufällige Bewegungen (zur Theorie der Brownschen Bewegung)," *Annals of Mathematics. Second Series*, vol. 35, no. 1, pp. 116–117, 1934.
[2] C. Cercignani, *The Boltzmann Equation and Its Applications*, vol. 67 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 1988.
[3] P.-L. Lions, "On Boltzmann and Landau equations," *Philosophical Transactions of the Royal Society of London. Series A*, vol. 346, no. 1679, pp. 191–204, 1994.
[4] F. Antonelli, E. Barucci, and M. E. Mancino, "Asset pricing with a forward-backward stochastic differential utility," *Economics Letters*, vol. 72, no. 2, pp. 151–157, 2001.
[5] S. Chapman and T. G. Cowling, *The Mathematical Theory of Nonuniform Gases: An Account of the Kinetic Theory of Viscosity, Thermal Conduction and Diffusion in Gases*, Cambridge Mathematical Library, Cambridge University Press, Cambridge, UK, 3rd edition, 1990.
[6] S. Chandresekhar, "Stochastic problems in physics and astronomy," *Reviews of Modern Physics*, vol. 15, pp. 1–89, 1943.
[7] J. J. Duderstadt and W. R. Martin, *Transport Theory*, John Wiley & Sons, New York, NY, USA, 1979.
[8] H. Ugowski, "On differential inequalities of parabolic type with multidimensional time," *Demonstratio Mathematica*, vol. 7, pp. 113–123, 1974.

[9] W. Walter, "Parabolic differential equations and inequalities with several time variables," *Mathematische Zeitschrift*, vol. 191, no. 2, pp. 319–323, 1986.

[10] S. P. Lavrenyuk and M. O. Olīskevich, "A mixed problem for a semilinear ultraparabolic equation in an unbounded domain," *Ukraïns'kiĭ Matematichniĭ Zhurnal*, vol. 59, no. 12, pp. 1661–1673, 2007.

[11] S. Lavrenyuk and N. Protsakh, "Boundary value problem for nonlinear ultraparabolic equation in unbounded with respect to time variable domain," *Tatra Mountains Mathematical Publications*, vol. 38, pp. 131–146, 2007.

[12] N. Protsakh, "Mixed problem for degenerate nonlinear ultraparabolic equation," *Tatra Mountains Mathematical Publications*, vol. 43, pp. 203–214, 2009.

[13] E. Lanconelli, A. Pascucci, and S. Polidoro, "Linear and nonlinear ultraparabolic equations of Kolmogorov type arising in diffusion theory and in finance," in *Nonlinear Problems in Mathematical Physics and Related Topics, II*, vol. 2 of *Int. Math. Ser. (N. Y.)*, pp. 243–265, Kluwer/Plenum, New York, NY, USA, 2002.

[14] A. Pascucci and S. Polidoro, "On the Cauchy problem for a nonlinear Kolmogorov equation," *SIAM Journal on Mathematical Analysis*, vol. 35, no. 3, pp. 579–595, 2003.

[15] G. Citti, A. Pascucci, and S. Polidoro, "Regularity properties of viscosity solutions of a non-Hörmander degenerate equation," *Journal de Mathématiques Pures et Appliquées*, vol. 80, no. 9, pp. 901–918, 2001.

[16] M. Escobedo, J. L. Vázquez, and E. Zuazua, "Entropy solutions for diffusion-convection equations with partial diffusivity," *Transactions of the American Mathematical Society*, vol. 343, no. 2, pp. 829–842, 1994.

[17] A. S. Tersenov, "Ultraparabolic equations and unsteady heat transfer," *Journal of Evolution Equations*, vol. 5, no. 2, pp. 277–289, 2005.

[18] O. Narayan and J. Roychowdhury, "Analyzing oscillators using multitime PDEs," *IEEE Transactions on Circuits and Systems. I*, vol. 50, no. 7, pp. 894–903, 2003.

[19] H. Fujita, "On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$," *Journal of the Faculty of Science. University of Tokyo. Section IA*, vol. 13, pp. 109–124, 1966.

[20] K. Hayakawa, "On nonexistence of global solutions of some semilinear parabolic differential equations," *Proceedings of the Japan Academy*, vol. 49, pp. 503–505, 1973.

[21] K. Kobayashi, T. Sirao, and H. Tanaka, "On the growing up problem for semilinear heat equations," *Journal of the Mathematical Society of Japan*, vol. 29, no. 3, pp. 407–424, 1977.

[22] A. A. Samarskii, V. A. Galaktionov, S. P. Kurdyumov, and A. P. Mikhailov, *Blow-up in Quasilinear Parabolic Equations*, vol. 19 of *de Gruyter Expositions in Mathematics*, Walter de Gruyter, Berlin, Germany, 1995.

[23] A. S. Kalashnikov, "Some problems of the qualitative theory of second-order nonlinear degenerate parabolic equations," *Uspekhi Matematicheskikh Nauk*, vol. 42, no. 2(254), pp. 135–176, 1987, English translation: Russina Math. Surveys, no. 42, pp. 169–222, 1987.

[24] G. Lu, "Existence and uniqueness of global solution for certain nonlinear degenerate parabolic systems," *Acta Mathematicae Applicatae Sinica. English Series*, vol. 12, no. 3, pp. 332–336, 1996.

[25] E. Mitidieri and S. N. Pokhozhaev, "A priori estimates and blow-up of solutions to nonlinear partial differential equations and inequalities," *Proceeding of the Steklov Institute of Mathematics*, vol. 234, no. 3, pp. 1–362, 2001.