Research Article

Product of Extended Cesàro Operator and Composition Operator from Lipschitz Space to $F(p, q, s)$ Space on the Unit Ball

Yu-Xia Liang and Ze-Hua Zhou

Department of Mathematics, Tianjin University, Tianjin 300072, China

Correspondence should be addressed to Ze-Hua Zhou, zehuazhou2003@yahoo.com.cn

Received 16 January 2011; Accepted 16 March 2011

Academic Editor: Ljubisa Kocinac

Copyright © 2011 Y.-X. Liang and Z.-H. Zhou. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper characterizes the boundedness and compactness of the product of extended Cesàro operator and composition operator from Lipschitz space to $F(p, q, s)$ space on the unit ball of $\mathbb{C}^n$.

1. Introduction

Let $\mathbb{B}$ be the unit ball in the n -dimensional complex space $\mathbb{C}^n$, the closure of $\mathbb{B}$ will be written as $\overline{\mathbb{B}}$. By dv we denote the Lebesgue measure on $\mathbb{B}$ normalized so that $v(\mathbb{B}) = 1$ and by $d\sigma$ the normalized rotation invariant measure on the boundary $S = \partial\mathbb{B}$ of $\mathbb{B}$. Let $H(\mathbb{B})$ be the class of all holomorphic functions on $\mathbb{B}$ and $S(\mathbb{B})$ the collection of all the holomorphic self-mappings of $\mathbb{B}$. Denote by $A(\mathbb{B})$ the unit ball algebra of all continuous functions on $\mathbb{B}$ that are holomorphic on $\mathbb{B}$.

For $f \in H(\mathbb{B})$, let

$$\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z) \quad (1.1)$$

be the radial derivative of f .

We recall that the α -Bloch space $\mathcal{B}^\alpha$ for $\alpha \geq 0$ consists of all $f \in H(\mathbb{B})$ such that

$$\mathcal{B}_\alpha(f) = \sup_{z \in \mathbb{B}} (1 - |z|^2)^\alpha |\Re f(z)| < \infty. \quad (1.2)$$

The expression $\mathcal{B}_\alpha(f)$ defines a seminorm while the natural norm is given by $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + \mathcal{B}_\alpha(f)$. This norm makes $\mathcal{B}^\alpha$ into a Banach space. When $\alpha = 1$, $\mathcal{B}_1 = \mathcal{B}$ is the well known Bloch space.

For $\alpha \in (0, 1)$, $\mathcal{L}_\alpha(\mathbb{B})$ denotes the holomorphic Lipschitz space of order α which is the set of all $f \in H(\mathbb{B})$ such that, for some $C > 0$,

$$|f(z) - f(w)| \leq C|z - w|^\alpha \quad (1.3)$$

for every $z, w \in \mathbb{B}$. It is clear that each space $\mathcal{L}_\alpha(\mathbb{B})$ contains the polynomials and is contained in the ball algebra $A(\mathbb{B})$. It is well known that $\mathcal{L}_\alpha(\mathbb{B})$ is endowed with a complete norm $\|\cdot\|_{\mathcal{L}_\alpha}$ that is given by

$$\|f\|_{\mathcal{L}_\alpha} = |f(0)| + \sup_{z \neq w; z, w \in \mathbb{B}} \left\{ \frac{|f(z) - f(w)|}{|z - w|^\alpha} \right\}. \quad (1.4)$$

See [1, 2] for more information of the Lipschitz spaces on $\mathbb{B}$.

For $a \in \mathbb{B}$, let $g(z, a) = \log |\varphi_a(z)|^{-1}$ be Green's function on $\mathbb{B}$ with logarithmic singularity at a , where φ_a is the Möbius transformation of $\mathbb{B}$ with $\varphi_a(0) = a$, $\varphi_a(a) = 0$, and $\varphi_a = \varphi_a^{-1}$.

Let $0 < p, s < \infty$, $-n - 1 < q < \infty$, a function $f \in H(\mathbb{B})$ is said to belong to $F(p, q, s)$ if (see, e.g., [3–5])

$$\|f\|_{F(p, q, s)}^p = |f(0)|^p + \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\Re f(z)|^p (1 - |z|^2)^q g^s(z, a) dv(z) < \infty. \quad (1.5)$$

If X is a Banach space of holomorphic functions on a domain Ω and if φ is a (holomorphic) self-map of Ω , the composition operator of symbol φ is defined by $C_\varphi(f) = f \circ \varphi$. The study of composition operators consists in the comparison of the properties of the operator C_φ with that of the function φ itself, which is called the symbol of C_φ . One can characterize boundedness and compactness of C_φ and many other properties. We refer to the books in [6, 7] and to some recent papers in [4, 5, 8] to learn much more on this subject.

Let $h \in H(\mathbb{B})$, the following integral-type operator was first introduced in [9]

$$T_h f(z) = \int_0^1 f(tz) \Re h(tz) \frac{dt}{t}, \quad f \in H(\mathbb{B}), \quad z \in \mathbb{B}. \quad (1.6)$$

This operator is called generalized Cesàro operator. It has been well studied in many papers, see, for example, [3, 9–24] as well as the related references therein.

It is natural to discuss the product of extended Cesàro operator and composition operator. For $h \in H(\mathbb{B})$ and $\varphi \in S(\mathbb{B})$, the product can be expressed as

$$T_h C_\varphi f(z) = \int_0^1 f(\varphi(tz)) \Re h(tz) \frac{dt}{t}, \quad f \in H(\mathbb{B}), \quad z \in \mathbb{B}. \quad (1.7)$$

It is interesting to characterize the boundedness and compactness of the product operator on all kinds of function spaces. Even on the disk of $\mathbb{C}$, some properties are not easily managed; see some recent papers in [18, 25–28].

Building on those foundations, the present paper continues this line of research and discusses the operator in high dimension. The remainder is assembled as follows: in Section 2, we state a couple of lemmas. In Section 3, we characterize the boundedness and compactness of the product $T_h C_\varphi$ of extended Cesàro operator and composition operator from Lipschitz spaces to $F(p, q, s)$ spaces on the unit ball of $\mathbb{C}^n$.

Throughout the remainder of this paper, C will denote a positive constant, the exact value of which will vary from one appearance to the next. The notation $A \asymp B$ means that there is a positive constant C such that $B/C \leq A \leq CB$.

2. Some Lemmas

To begin the discussion, let us state a couple of lemmas, which are used in the proofs of the main results.

Lemma 2.1. *Suppose that $f, h \in H(\mathbb{B})$. Then,*

$$\Re[T_h C_\varphi(f)](z) = f(\varphi(z))\Re h(z). \quad (2.1)$$

Proof. The proof of this Lemma follows by standard arguments (see, e.g., [9, 29, 30]). $\square$

Lemma 2.2 (see [2, 31]). *If $0 < \alpha < 1$, then $\mathcal{B}^{1-\alpha} = \mathcal{L}_\alpha(\mathbb{B})$; furthermore,*

$$\|f\|_{\mathcal{B}^{1-\alpha}} \asymp \|f\|_{\mathcal{L}_\alpha} \quad (2.2)$$

as f varies through $\mathcal{L}_\alpha(\mathbb{B})$.

The following criterion for compactness follows from standard arguments similar to the corresponding lemma in [6]. Hence, we omit the details.

Lemma 2.3. *Assume that $h \in H(\mathbb{B})$ and $\varphi \in S(\mathbb{B})$. Suppose that X or Y is one of the following spaces $\mathcal{L}_\alpha(\mathbb{B})$, $F(p, q, s)$. Then, $T_h C_\varphi : X \rightarrow Y$ is compact if and only if $T_h C_\varphi : X \rightarrow Y$ is bounded, and for any bounded sequence $\{f_k\}_{k \in \mathbb{N}}$ in X which converges to zero uniformly on compact subsets of $\mathbb{B}$ as $k \rightarrow \infty$, one has $\|T_h C_\varphi f_k\|_Y \rightarrow 0$ as $k \rightarrow \infty$.*

Lemma 2.4 (see [4, 5]). *If $f \in \mathcal{B}^\alpha$, then*

$$|f(z)| \leq C \|f\|_{\mathcal{B}^\alpha}, \quad 0 < \alpha < 1, \quad (2.3)$$

$$|f(z)| \leq C \|f\|_{\mathcal{B}^\alpha} \ln \frac{e}{1 - |z|^2}, \quad \alpha = 1, \quad (2.3')$$

$$|f(z)| \leq C \frac{\|f\|_{\mathcal{B}^\alpha}}{(1 - |z|^2)^{\alpha-1}}, \quad \alpha > 1. \quad (2.3'')$$

The next lemma was obtained in [32].

Lemma 2.5. *If $a > 0$, $b > 0$, then the elementary inequality holds*

$$(a + b)^p \leq \begin{cases} a^p + b^p, & 0 < p < 1, \\ 2^{p-1}(a^p + b^p), & p \geq 1. \end{cases} \quad (2.4)$$

It is obvious that Lemma 2.5 holds for the sum of finite number k , that is,

$$(a_1 + \cdots + a_k)^p \leq C(a_1^p + \cdots + a_k^p), \quad (2.5)$$

where $a_1, \dots, a_k > 0$ and C is a positive constant.

Lemma 2.6 (see [4, 5]). *For $0 < p, s < +\infty$, $-n-1 < q < +\infty$, $q+s > -1$, there exists $C > 0$ such that*

$$\sup_{a \in \mathbb{B}} \int_{\mathbb{B}} \frac{(1 - |w|^2)^p}{|1 - \langle z, w \rangle|^{n+1+q+p}} (1 - |z|^2)^q g^s(z, a) dv(z) \leq C \quad (2.6)$$

for every $\omega \in \mathbb{B}$.

Lemma 2.7 (see [4]). *There is a constant $C > 0$ so that, for all $t > -1$ and $z \in \mathbb{B}$, one has*

$$\int_{\mathbb{B}} \left| \ln \frac{1}{1 - \langle z, w \rangle} \right|^2 \frac{(1 - |w|^2)^t}{|1 - \langle z, w \rangle|^{n+1+t}} dv(z) \leq C \left(\ln \frac{1}{1 - |z|^2} \right)^2. \quad (2.7)$$

Lemma 2.8 (see [4, 5]). *Suppose that $0 < p, s < \infty$, $-n-1 < q < \infty$, and $q+s > -1$. If $f \in F(p, q, s)$, then $f \in \mathcal{B}^{(n+1+q)/p}$, and $\|f\|_{\mathcal{B}^{(n+1+q)/p}} \leq C\|f\|_{F(p, q, s)}$.*

Lemma 2.9. *Let $\{f_k\}_{k \in \mathbb{N}}$ be a bounded sequence in $F(p, q, s)$ which converges to zero uniformly on compact subsets of the unit ball $\mathbb{B}$, where $(n+1+q)/p < 1$. Then, $\lim_{k \rightarrow \infty} \sup_{z \in \mathbb{B}} |f_k(z)| = 0$.*

Proof. It follows from Lemma 2.8 that $F(p, q, s) \subseteq \mathcal{B}^{(n+1+q)/p}$ and $\|f\|_{\mathcal{B}^{(n+1+q)/p}} \leq C\|f\|_{F(p, q, s)}$ for any $f \in F(p, q, s)$. So, when $(n+1+q)/p < 1$, the proof of this lemma is similar to that of Lemma 3.6 of [33], hence the proof is omitted. $\square$

3. The Boundedness and Compactness of the Operator $T_h C_\varphi : \mathcal{L}_\alpha(\mathbb{B}) \rightarrow F(p, q, s)$

Theorem 3.1. *Assume that $\alpha \in (0, 1)$, $0 < p, s < \infty$, $-n-1 < q < \infty$, $q+s > -1$, $\varphi \in S(\mathbb{B})$, and $h \in H(\mathbb{B})$. Then, $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded if and only if $h \in F(p, q, s)$.*

Proof. Assume that $h \in F(p, q, s)$. Since $0 < 1 - \alpha < 1$, by Lemmas 2.2 and 2.4, for any $f \in \mathcal{L}_\alpha$, we have

$$|f(z)| \leq C\|f\|_{\mathcal{B}^{1-\alpha}} \leq C\|f\|_{\mathcal{L}_\alpha}. \quad (3.1)$$

Since $|T_h C_\varphi f(0)| = 0$, by using Lemma 2.1 and relations (2.3) and (3.1), we have

$$\begin{aligned} \|T_h C_\varphi f\|_{F(p,q,s)}^p &= \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |f(\varphi(z)) \Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ &\leq C \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \|f\|_{\mathcal{L}^{1-\alpha}}^p \\ &\leq C \|h\|_{F(p,q,s)}^p \|f\|_{\mathcal{L}^\alpha}^p < \infty. \end{aligned} \quad (3.2)$$

Thus $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded.

Conversely, suppose that $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded. Taking the function $f(z) = 1 \in \mathcal{L}_\alpha$, then

$$\begin{aligned} \|T_h C_\varphi f\|_{F(p,q,s)}^p &= |T_h C_\varphi f(0)|^p + \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\Re(T_h C_\varphi f)(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ &= \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |f(\varphi(z)) \Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ &= \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) = \|h\|_{F(p,q,s)}^p. \end{aligned} \quad (3.3)$$

From which, the boundedness of $T_h C_\varphi$ implies that $h \in F(p, q, s)$. This completes the proof of this theorem. $\square$

Next, we characterize the compactness of $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$.

Theorem 3.2. Assume that $\alpha \in (0, 1)$, $0 < p, s < \infty$, $-n - 1 < q < \infty$, $q + s > -1$, $\varphi \in S(\mathbb{B})$, and $h \in H(\mathbb{B})$. Then, $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is compact if and only if $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded, and

$$\lim_{r \rightarrow 1} \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) = 0. \quad (3.4)$$

Proof. Assume that $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded and (3.4) holds. It follows from Theorem 3.1 that $h \in F(p, q, s)$.

Now, let $\{f_j\}_{j \in \mathbb{N}}$ be a bounded sequence of functions in $\mathcal{L}_\alpha$ such that $f_j \rightarrow 0$ uniformly on the compact subsets of $\mathbb{B}$ as $j \rightarrow \infty$. Suppose that $\sup_{j \in \mathbb{N}} \|f_j\|_{\mathcal{L}_\alpha} \leq L$. It follows from (3.4) that, for any $\varepsilon > 0$, there exists $r_0 \in (0, 1)$ such that, for every $r_0 < r < 1$,

$$\sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) < \varepsilon. \quad (3.5)$$

Set $r_0 < r < 1$, then

$$\begin{aligned}
\|T_h C_\varphi f_j\|_{F(p,q,s)}^p &= \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |f_j(\varphi(z))|^p |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\
&\leq \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| \leq r\}} |f_j(\varphi(z))|^p |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\
&\quad + \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |f_j(\varphi(z))|^p |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\
&= I_1 + I_2,
\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}
I_1 &:= \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| \leq r\}} |f_j(\varphi(z))|^p |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z), \\
I_2 &:= \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |f_j(\varphi(z))|^p |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z).
\end{aligned} \tag{3.7}$$

Let $K = \{w : |w| \leq r\}$, then K is a compact subset of $\mathbb{B}$. Since $f_j \rightarrow 0$ uniformly on compact subsets of $\mathbb{B}$ as $j \rightarrow \infty$ and $h \in F(p, q, s)$, we get

$$\begin{aligned}
I_1 &\leq \sup_{w \in K} |f_j(w)|^p \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| \leq r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\
&\leq \|h\|_{F(p,q,s)}^p \sup_{w \in K} |f_j(w)|^p \leq C \sup_{w \in K} |f_j(w)|^p \rightarrow 0, \quad j \rightarrow \infty.
\end{aligned} \tag{3.8}$$

On the other hand, by (3.5) and Lemmas 2.2 and 2.4, it follows that

$$\begin{aligned}
I_2 &\leq C \|f_j\|_{\mathcal{B}^{1-\alpha}}^p \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\
&\leq C \|f_j\|_{\mathcal{L}_\alpha}^p \varepsilon \leq CL^p \varepsilon.
\end{aligned} \tag{3.9}$$

Since ε is arbitrary, from the above inequalities, we get

$$\lim_{j \rightarrow \infty} \|T_h C_\varphi f_j\|_{F(p,q,s)} = 0. \tag{3.10}$$

Hence, by (3.10) and Lemma 2.3, we conclude that $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is compact.

For the converse direction, we suppose that $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is compact. It is obvious that $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded.

Now, we prove (3.4). Setting the test functions $f_l^{(m)}(z) = z_l^m$ for fixed $l \in \{1, \dots, n\}$, where $z = (z_1, \dots, z_n)$ and $m = 1, 2, \dots$. It is easy to check that $\|f_l^{(m)}\|_{\mathcal{L}_\alpha} \leq C$, and $f_l^{(m)} \rightarrow 0$

uniformly on the compact subsets of $\mathbb{B}$ as $m \rightarrow \infty$. Write $\varphi = (\varphi_1, \dots, \varphi_n)$, since $T_h C_\varphi : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is compact, by Lemma 2.3, it follows that, as $m \rightarrow \infty$,

$$\left\| T_h C_\varphi f_l^{(m)} \right\|_{F(p, q, s)}^p = \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\varphi_l(z)|^{mp} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \rightarrow 0. \quad (3.11)$$

Note that $|\varphi(z)|^2 = |\varphi_1(z)|^2 + \dots + |\varphi_n(z)|^2 \leq (|\varphi_1(z)| + \dots + |\varphi_n(z)|)^2$; by the relation (3.11) and Lemma 2.5, we have

$$\begin{aligned} & \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\varphi(z)|^{mp} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & \leq \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} \left(\sum_{l=1}^n |\varphi_l(z)| \right)^{mp} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & \leq C \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} \left(\sum_{l=1}^n |\varphi_l(z)|^{mp} \right) |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \rightarrow 0, \quad m \rightarrow \infty. \end{aligned} \quad (3.12)$$

This means that, for every $\varepsilon > 0$, there is $m_0 \in \mathbb{N}$ such that, for every $r \in (0, 1)$,

$$\begin{aligned} & r^{m_0 p} \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & = \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} r^{m_0 p} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & \leq \sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\varphi(z)|^{m_0 p} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & \leq \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} |\varphi(z)|^{m_0 p} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) \\ & < \varepsilon. \end{aligned} \quad (3.13)$$

Thus, when $r > 2^{-(1/m_0 p)}$, by the above inequality, we obtain

$$\sup_{a \in \mathbb{B}} \int_{\{|\varphi(z)| > r\}} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) d\nu(z) < 2\varepsilon. \quad (3.14)$$

From which, the desired result (3.4) holds. This completes the proof of this theorem. $\square$

Remark 3.3. When $\varphi(z) = z$, the product of extended Cesàro operator $T_h C_\varphi$ is the generalized extended Cesàro operator T_h ; thus, by Theorems 3.1 and 3.2, we have the following two corollaries.

Corollary 3.4. Assume that $\alpha \in (0, 1)$, $0 < p, s < \infty$, $-n - 1 < q < \infty$, $q + s > -1$, and $h \in H(\mathbb{B})$. Then, $T_h : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded if and only if $h \in F(p, q, s)$.

Corollary 3.5. Assume that $\alpha \in (0, 1)$, $0 < p, s < \infty$, $-n - 1 < q < \infty$, $q + s > -1$, and $h \in H(\mathbb{B})$. Then, $T_h : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is compact if and only if $T_h : \mathcal{L}_\alpha \rightarrow F(p, q, s)$ is bounded, and

$$\lim_{r \rightarrow 1} \sup_{a \in \mathbb{B}} \int_{|z| > r} |\Re h(z)|^p (1 - |z|^2)^q g^s(z, a) = 0. \quad (3.15)$$

Acknowledgments

The authors would like to thank the editor and referees for carefully reading the paper and providing corrections and suggestions for improvements. Z.-H. Zhou was supported in part by the National Natural Science Foundation of China (Grant nos. 10971153 and 10671141).

References

- [1] W. Rudin, *Function Theory in the Unit Ball of $\mathbb{C}^n$* , vol. 241 of *Fundamental Principles of Mathematical Science*, Springer, Berlin, Germany, 1980.
- [2] K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, vol. 226 of *Graduate Texts in Mathematics*, Springer, New York, NY, USA, 2005.
- [3] S. Li and S. Stević, "Compactness of Riemann-Stieltjes operators between $F(p, q, s)$ spaces and α -Bloch spaces," *Publicationes Mathematicae Debrecen*, vol. 72, no. 1-2, pp. 111–128, 2008.
- [4] Z.-H. Zhou and R.-Y. Chen, "Weighted composition operators from $F(p, q, s)$ to Bloch type spaces on the unit ball," *International Journal of Mathematics*, vol. 19, no. 8, pp. 899–926, 2008.
- [5] H.-G. Zeng and Z.-H. Zhou, "An estimate of the essential norm of a composition operator from $F(p, q, s)$ to β^α in the unit ball," *Journal of Inequalities and Applications*, vol. 2010, Article ID 132970, 22 pages, 2010.
- [6] C. C. Cowen and B. D. MacCluer, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, Fla, USA, 1995.
- [7] J. H. Shapiro, *Composition Operators and Classical Function Theory*, Universitext: Tracts in Mathematics, Springer, New York, NY, USA, 1993.
- [8] Z. Zhou and J. Shi, "Compactness of composition operators on the Bloch space in classical bounded symmetric domains," *The Michigan Mathematical Journal*, vol. 50, no. 2, pp. 381–405, 2002.
- [9] Z. Hu, "Extended Cesàro operators on mixed norm spaces," *Proceedings of the American Mathematical Society*, vol. 131, no. 7, pp. 2171–2179, 2003.
- [10] K. Avetisyan and S. Stević, "Extended Cesàro operators between different Hardy spaces," *Applied Mathematics and Computation*, vol. 207, no. 2, pp. 346–350, 2009.
- [11] Z.-S. Fang and Z.-H. Zhou, "Extended Cesàro operators on BMOA spaces in the unit ball," *Journal of Mathematical Inequalities*, vol. 4, no. 1, pp. 27–36, 2010.
- [12] Z.-S. Fang and Z.-H. Zhou, "Extended Cesàro operators from generally weighted Bloch spaces to Zygmund space," *Journal of Mathematical Analysis and Applications*, vol. 359, no. 2, pp. 499–507, 2009.
- [13] Z.-S. Fang and Z.-H. Zhou, "Extended Cesàro operators on Zygmund spaces in the unit ball," *Journal of Computational Analysis and Applications*, vol. 11, no. 3, pp. 406–413, 2009.
- [14] D. Gu, "Extended Cesàro operators from logarithmic-type spaces to Bloch-type spaces," *Abstract and Applied Analysis*, vol. 2009, Article ID 246521, 9 pages, 2009.
- [15] Z. Hu, "Extended Cesàro operators on Bergman spaces," *Journal of Mathematical Analysis and Applications*, vol. 296, no. 2, pp. 435–454, 2004.
- [16] S. Li and S. Stević, "Riemann-Stieltjes operators on Hardy spaces in the unit ball of $\mathbb{C}^n$," *Bulletin of the Belgian Mathematical Society*, vol. 14, no. 4, pp. 621–628, 2007.
- [17] S. Li and S. Stević, "Riemann-Stieltjes-type integral operators on the unit ball in $\mathbb{C}^n$," *Complex Variables and Elliptic Equations*, vol. 52, no. 6, pp. 495–517, 2007.
- [18] S. Li and S. Stević, "Riemann-Stieltjes operators between different weighted Bergman spaces," *Bulletin of the Belgian Mathematical Society*, vol. 15, no. 4, pp. 677–686, 2008.
- [19] S. Li and S. Stević, "Cesàro-type operators on some spaces of analytic functions on the unit ball," *Applied Mathematics and Computation*, vol. 208, no. 2, pp. 378–388, 2009.

- [20] J. Xiao, "Riemann-Stieltjes operators on weighted Bloch and Bergman spaces of the unit ball," *Journal of the London Mathematical Society*, vol. 70, no. 1, pp. 199–214, 2004.
- [21] J. Xiao, "Riemann-Stieltjes operators between weighted Bergman spaces," in *Complex and Harmonic Analysis*, pp. 205–212, DEStech, Lancaster, UK, 2007.
- [22] X. Tang, "Extended Cesàro operators between Bloch-type spaces in the unit ball of $\mathbb{C}^n$," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 2, pp. 1199–1211, 2007.
- [23] X. Zhu, "Integral-type operators from iterated logarithmic Bloch spaces to Zygmund-type spaces," *Applied Mathematics and Computation*, vol. 215, no. 3, pp. 1170–1175, 2009.
- [24] Z.-H. Zhou and M. Zhu, "Extended Cesàro operators between generalized Besov spaces and Bloch type spaces in the unit ball," *Journal of Function Spaces and Applications*, vol. 7, no. 3, pp. 209–223, 2009.
- [25] S. Li and S. Stević, "Products of Volterra type operator and composition operator from H^∞ and Bloch spaces to Zygmund spaces," *Journal of Mathematical Analysis and Applications*, vol. 345, no. 1, pp. 40–52, 2008.
- [26] S. Li and S. Stević, "Products of composition and integral type operators from H^∞ to the Bloch space," *Complex Variables and Elliptic Equations*, vol. 53, no. 5, pp. 463–474, 2008.
- [27] H. Y. Li and X. Z. Yang, "Products of integral-type and composition operators from generally weighted Bloch space to $F(p, q, s)$ space," *Faculty of Sciences and Mathematics*, vol. 23, no. 3, pp. 231–241, 2009.
- [28] W. F. Yang, "Volterra composition operators from $F(p, q, s)$ spaces to Bloch-type spaces," *Bulletin of the Malaysian Mathematical Sciences Society*, vol. 34, no. 2, pp. 267–277, 2011.
- [29] S. Stević, "On a new operator from H^∞ to the Bloch-type space on the unit ball," *Utilitas Mathematica*, vol. 77, pp. 257–263, 2008.
- [30] S. Stević, "On a new integral-type operator from the weighted Bergman space to the Bloch-type space on the unit ball," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 154263, 14 pages, 2008.
- [31] D. D. Clahane and S. Stević, "Norm equivalence and composition operators between Bloch/Lipschitz spaces of the ball," *Journal of Inequalities and Applications*, vol. 2006, Article ID 61018, 11 pages, 2006.
- [32] J. H. Shi, "Inequalities for the integral means of holomorphic functions and their derivatives in the unit ball of $\mathbb{C}^n$," *Transactions of the American Mathematical Society*, vol. 328, no. 2, pp. 619–637, 1991.
- [33] S. Ohno, K. Stroethoff, and R. Zhao, "Weighted composition operators between Bloch-type spaces," *The Rocky Mountain Journal of Mathematics*, vol. 33, no. 1, pp. 191–215, 2003.