Research Article

Oscillation Criteria for Certain Second-Order Nonlinear Neutral Differential Equations of Mixed Type

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Some oscillation criteria are established for the second-order nonlinear neutral differential equations of mixed type $[(x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2))^\gamma]'' = q_1(t) x^\gamma(t - \sigma_1) + q_2(t) x^\gamma(t + \sigma_2)$, $t \geq t_0$, where $\gamma \geq 1$ is a quotient of odd positive integers. Our results generalize the results given in the literature.

1. Introduction

This paper is concerned with the oscillatory behavior of the second-order nonlinear neutral differential equation of mixed type

$$[(x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2))^\gamma]'' = q_1(t) x^\gamma(t - \sigma_1) + q_2(t) x^\gamma(t + \sigma_2), \quad t \geq t_0. \quad (1.1)$$

Throughout this paper, we will assume the following conditions hold.

(A_1) p_i, τ_i, and σ_i, $i = 1, 2$, are positive constants;

(A_2) $q_i \in C([t_0, \infty), [0, \infty))$, $i = 1, 2$.

By a solution of (1.1), we mean a function $x \in C([T_x, \infty), \mathbb{R})$ for some $T_x \geq t_0$ which has the property that $(x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2))^\gamma \in C^2([T_x, \infty), \mathbb{R})$ and satisfies (1.1) on $[T_x, \infty)$. As is customary, a solution of (1.1) is called oscillatory if it has arbitrarily large zeros on $[t_0, \infty)$, otherwise, it is called nonoscillatory. Equation (1.1) is said to be oscillatory if all its solutions are oscillatory.

Neutral functional differential equations have numerous applications in electric networks. For instance, they are frequently used for the study of distributed networks containing lossless transmission lines which rise in high speed computers where the lossless transmission lines are used to interconnect switching circuits; see [1].

Recently, many results have been obtained on oscillation of nonneutral continuous and discrete equations and neutral functional differential equations, we refer the reader to the papers [2–35], and the references cited therein.

Philos [2] established some Philos-type oscillation criteria for the second-order linear differential equation

$$\left(r(t)x'(t)\right)' + q(t)x(t) = 0, \quad t \geq t_0. \tag{1.2}$$

In [3–5], the authors gave some sufficient conditions for oscillation of all solutions of second-order half-linear differential equation

$$\left(r(t)\left|x'(t)\right|^{\gamma-1}x'(t)\right)' + q(t)|x(\tau(t))|^{\gamma-1}x(\tau(t)) = 0, \quad t \geq t_0 \tag{1.3}$$

by employing a Riccati substitution technique.

Zhang et al. [15] examined the oscillation of even-order neutral differential equation

$$\left[x(t) + p(t)x(\tau(t))\right]^{(n)} + q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0. \tag{1.4}$$

Some oscillation criteria for the following second-order quasilinear neutral differential equation

$$\left(r(t)\left|z'(t)\right|^{\gamma-1}z'(t)\right)' + q(t)|x(\sigma(t))|^{\gamma-1}x(\sigma(t)) = 0, \quad \text{for } z(t) = x(t) + p(t)x(\tau(t)), \ t \geq t_0 \tag{1.5}$$

were obtained by [12–17].

However, there are few results regarding the oscillatory properties of neutral differential equations with mixed arguments, see the papers [20–24]. In [25], the authors established some oscillation criteria for the following mixed neutral equation:

$$\left(x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2)\right)'' = q_1(t)x(t - \sigma_1) + q_2(t)x(t + \sigma_2), \quad t \geq t_0; \tag{1.6}$$

here q_1 and q_2 are nonnegative real-valued functions. Grace [26] obtained some oscillation theorems for the odd order neutral differential equation

$$\left(x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2)\right)^{(n)} = q_1 x(t - \sigma_1) + q_2 x(t + \sigma_2), \quad t \geq t_0, \tag{1.7}$$

where $n \geq 1$ is odd. Grace [27] and Yan [28] obtained several sufficient conditions for the oscillation of solutions of higher-order neutral functional differential equation of the form

$$(x(t) + cx(t-h) + Cx(t+H))^{(n)} + qx(t-g) + Qx(t+G) = 0, \quad t \geq t_0, \tag{1.8}$$

where q and Q are nonnegative real constants.

Clearly, (1.6) is a special case of (1.1). The purpose of this paper is to study the oscillation behavior of (1.1).

In the sequel, when we write a functional inequality without specifying its domain of validity we assume that it holds for all sufficiently large t.

2. Main Results

In the following, we give our results.

Theorem 2.1. *Assume that* $\sigma_i > \tau_i$, $i = 1, 2$. *If*

$$\limsup_{t \to \infty} \int_t^{t+\sigma_2-\tau_2} (t + \sigma_2 - \tau_2 - s)Q_2(s)ds > \left(2^{\gamma-1}\right)^2 \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}}\right), \tag{2.1}$$

$$\limsup_{t \to \infty} \int_{t-\sigma_1+\tau_1}^{t} (s - t + \sigma_1 - \tau_1)Q_1(s)ds > \left(2^{\gamma-1}\right)^2 \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}}\right), \tag{2.2}$$

where

$$Q_i(t) = \min\{q_i(t - \tau_1), q_i(t), q_i(t + \tau_2)\}, \tag{2.3}$$

for $i = 1, 2$, *then every solution of* (1.1) *oscillates.*

Proof. Let x be a nonoscillatory solution of (1.1). Without loss of generality, we assume that there exists $t_1 \geq t_0$ such that $x(t) > 0$, $x(t-\tau_1) > 0$, $x(t+\tau_2) > 0$, $x(t-\sigma_1) > 0$, and $x(t+\sigma_2) > 0$ for all $t \geq t_1$. Setting

$$z(t) = (x(t) + p_1 x(t - \tau_1) + p_2 x(t + \tau_2))^\gamma,$$
$$y(t) = z(t) + p_1^\gamma z(t - \tau_1) + \frac{p_2^\gamma}{2^{\gamma-1}} z(t + \tau_2). \tag{2.4}$$

Thus $z(t) > 0$, $y(t) > 0$, and

$$z''(t) = q_1(t)x^\gamma(t - \sigma_1) + q_2(t)x^\gamma(t + \sigma_2) \geq 0. \tag{2.5}$$

Then, $z'(t)$ is of constant sign, eventually. On the other hand,

$$
\begin{aligned}
y''(t) = {}& q_1(t)x^\gamma(t - \sigma_1) + q_2(t)x^\gamma(t + \sigma_2) \\
& + p_1^\gamma q_1(t - \tau_1)x^\gamma(t - \tau_1 - \sigma_1) + p_1^\gamma q_2(t - \tau_1)x^\gamma(t - \tau_1 + \sigma_2) \\
& + \frac{p_2^\gamma}{2^{\gamma-1}} q_1(t + \tau_2)x^\gamma(t + \tau_2 - \sigma_1) \\
& + \frac{p_2^\gamma}{2^{\gamma-1}} q_2(t + \tau_2)x^\gamma(t + \tau_2 + \sigma_2).
\end{aligned}
\tag{2.6}
$$

Note that $g(u) = u^\gamma, \gamma \geq 1, u \in (0, \infty)$ is a convex function. Hence, by the definition of convex function, we obtain

$$
a^\gamma + b^\gamma \geq \frac{1}{2^{\gamma-1}}(a + b)^\gamma.
\tag{2.7}
$$

Using inequality (2.7), we get

$$
\begin{aligned}
& x^\gamma(t - \sigma_1) + p_1^\gamma x^\gamma(t - \tau_1 - \sigma_1) \geq \frac{1}{2^{\gamma-1}}\left(x(t - \sigma_1) + p_1 x(t - \tau_1 - \sigma_1)\right)^\gamma, \\
& \frac{1}{2^{\gamma-1}}\left(x(t - \sigma_1) + p_1 x(t - \tau_1 - \sigma_1)\right)^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}} x^\gamma(t + \tau_2 - \sigma_1) \\
& \geq \frac{1}{\left(2^{\gamma-1}\right)^2}\left(x(t - \sigma_1) + p_1 x(t - \tau_1 - \sigma_1) + p_2 x(t + \tau_2 - \sigma_1)\right)^\gamma = \frac{z(t - \sigma_1)}{\left(2^{\gamma-1}\right)^2}.
\end{aligned}
\tag{2.8}
$$

Similarly, we obtain

$$
x^\gamma(t + \sigma_2) + p_1^\gamma x^\gamma(t - \tau_1 + \sigma_2) + \frac{p_2^\gamma}{2^{\gamma-1}} x^\gamma(t + \tau_2 + \sigma_2) \geq \frac{z(t + \sigma_2)}{\left(2^{\gamma-1}\right)^2}.
\tag{2.9}
$$

Thus, from (2.6), we have

$$
y''(t) \geq \frac{1}{\left(2^{\gamma-1}\right)^2}\left(Q_1(t)z(t - \sigma_1) + Q_2(t)z(t + \sigma_2)\right).
\tag{2.10}
$$

In the following, we consider two cases.

Case 1. Assume that $z'(t) > 0$. Then, $y'(t) > 0$. In view of (2.10), we see that

$$
y''(t + \tau_2) \geq \frac{1}{\left(2^{\gamma-1}\right)^2} Q_2(t + \tau_2)z(t + \tau_2 + \sigma_2).
\tag{2.11}
$$

Applying the monotonicity of z, we find

$$y(t + \sigma_2) = z(t + \sigma_2) + p_1^\gamma z(t - \tau_1 + \sigma_2) + \frac{p_2^\gamma}{2^{\gamma-1}} z(t + \tau_2 + \sigma_2)$$

$$\leq \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}} \right) z(t + \tau_2 + \sigma_2). \tag{2.12}$$

Combining the last two inequalities, we obtain the inequality

$$y''(t + \tau_2) \geq \frac{Q_2(t + \tau_2)}{(2^{\gamma-1})^2 \left(1 + p_1^\gamma + p_2^\gamma / 2^{\gamma-1} \right)} y(t + \sigma_2). \tag{2.13}$$

Therefore, y is a positive increasing solution of the differential inequality

$$y''(t) \geq \frac{Q_2(t)}{(2^{\gamma-1})^2 \left(1 + p_1^\gamma + p_2^\gamma / 2^{\gamma-1} \right)} y(t - \tau_2 + \sigma_2). \tag{2.14}$$

However, by [11], condition (2.1) contradicts the existence of a positive increasing solution of inequality (2.14).

Case 2. Assume that $z'(t) < 0$. Then, $y'(t) < 0$. In view of (2.10), we see that

$$y''(t - \tau_1) \geq \frac{1}{(2^{\gamma-1})^2} Q_1(t - \tau_1) z(t - \tau_1 - \sigma_1). \tag{2.15}$$

Applying the monotonicity of z, we find

$$y(t - \sigma_1) = z(t - \sigma_1) + p_1^\gamma z(t - \tau_1 - \sigma_1) + p_2^\gamma \frac{1}{2^{\gamma-1}} z(t + \tau_2 - \sigma_1)$$

$$\leq \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}} \right) z(t - \tau_1 - \sigma_1). \tag{2.16}$$

Combining the last two inequalities, we obtain the inequality

$$y''(t - \tau_1) \geq \frac{Q_1(t - \tau_1)}{(2^{\gamma-1})^2 \left(1 + p_1^\gamma + p_2^\gamma / 2^{\gamma-1} \right)} y(t - \sigma_1). \tag{2.17}$$

Therefore, y is a positive decreasing solution of the differential inequality

$$y''(t) \geq \frac{Q_1(t)}{(2^{\gamma-1})^2 \left(1 + p_1^\gamma + p_2^\gamma / 2^{\gamma-1} \right)} y(t + \tau_1 - \sigma_1). \tag{2.18}$$

However, by [11], condition (2.2) contradicts the existence of a positive decreasing solution of inequality (2.18).   □

Remark 2.2. When $\gamma = 1$, Theorem 2.1 involves results of [25, Theorem 1].

Theorem 2.3. *Let $\beta_i = (\sigma_i - \tau_i)/2 > 0$, $i = 1, 2$. Suppose that, for $i = 1, 2$, there exist functions*

$$a_i \in C^1[t_0, \infty), \qquad a_i(t) > 0, \qquad (-1)^i a_i'(t) \le 0, \tag{2.19}$$

such that

$$Q_i(t) \ge \left(2^{\gamma-1}\right)^2 \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}}\right) a_i(t) a_i\left(t + (-1)^i \beta_i\right), \tag{2.20}$$

where Q_i are as in (2.3) for $i = 1, 2$. If the first-order differential inequality

$$v'(t) + (-1)^{i+1} a_i\left(t + (-1)^i \beta_i\right) v\left(t + (-1)^i \beta_i\right) \ge 0 \tag{2.21}$$

has no eventually negative solution for $i = 1$ and no eventually positive solution for $i = 2$, then (1.1) is oscillatory.

Proof. Let x be a nonoscillatory solution of (1.1). Without loss of generality, we assume that there exists $t_1 \ge t_0$ such that $x(t) > 0$, $x(t-\tau_1) > 0$, $x(t+\tau_2) > 0$, $x(t-\sigma_1) > 0$, and $x(t+\sigma_2) > 0$ for all $t \ge t_1$. Define z and y as in Theorem 2.1. Proceeding as in the proof of Theorem 2.1, we get (2.10).

In the following, we consider two cases.

Case 1. Assume that $z'(t) > 0$. Clearly, $y'(t) > 0$. Then, just as in Case 1 of Theorem 2.1, we find that y is a positive increasing solution of inequality (2.14). Let $b_2(t) = y'(t) + a_2(t) y(t + \beta_2)$. Then $b_2(t) > 0$. Using (2.19) and (2.20), we obtain

$$
\begin{aligned}
b_2'(t) &- \frac{a_2'(t)}{a_2(t)} b_2(t) - a_2(t) b_2(t + \beta_2) \\
&= y''(t) - \frac{a_2'(t)}{a_2(t)} y'(t) - a_2(t) a_2(t + \beta_2) y(t + 2\beta_2) \\
&\ge y''(t) - a_2(t) a_2(t + \beta_2) y(t + 2\beta_2) \\
&\ge y''(t) - \frac{Q_2(t)}{\left(2^{\gamma-1}\right)^2 \left(1 + p_1^\gamma + \left(p_2^\gamma/2^{\gamma-1}\right)\right)} y(t - \tau_2 + \sigma_2) \ge 0.
\end{aligned}
\tag{2.22}
$$

Define $b_2(t) = a_2(t) v(t)$. Then, v is a positive solution of (2.21) for $i = 2$, which is a contradiction.

Case 2. Assume that $z'(t) < 0$. Clearly, $y'(t) < 0$. Then, just as in Case 2 of Theorem 2.1, we find that y is a positive decreasing solution of inequality (2.18). Let $b_1(t) = y'(t) - a_1(t)y(t - \beta_1)$. Then $b_1(t) < 0$. Using (2.19) and (2.20), we obtain

$$
\begin{aligned}
b_1'(t) &- \frac{a_1'(t)}{a_1(t)}b_1(t) + a_1(t)b_1(t - \beta_1) \\
&= y''(t) - \frac{a_1'(t)}{a_1(t)}y'(t) - a_1(t)a_1(t - \beta_1)y(t - 2\beta_1) \\
&\geq y''(t) - a_1(t)a_1(t - \beta_1)y(t - 2\beta_1) \\
&\geq y''(t) - \frac{Q_1(t)}{(2^{\gamma-1})^2 (1 + p_1^\gamma + p_2^\gamma/2^{\gamma-1})}y(t + \tau_1 - \sigma_1) \geq 0.
\end{aligned}
\tag{2.23}
$$

Define $b_1(t) = a_1(t)v(t)$. Then, v is a negative solution of (2.21) for $i = 1$. This contradiction completes the proof of the theorem. $\qquad\square$

Remark 2.4. When $\gamma = 1$, Theorem 2.3 involves results of [25, Theorem 2].

From Theorem 2.3 and the results given in [12], we have the following oscillation criterion for (1.1).

Corollary 2.5. *Let $\beta_i = (\sigma_i - \tau_i)/2 > 0$, $i = 1, 2$. Assume that (2.19) and (2.20) hold for $i = 1, 2$. If*

$$
\liminf_{t \to \infty} \int_{t - \beta_1}^{t} a_1(s - \beta_1)ds > \frac{1}{e},
\tag{2.24}
$$

$$
\liminf_{t \to \infty} \int_{t}^{t + \beta_2} a_2(s + \beta_2)ds > \frac{1}{e},
\tag{2.25}
$$

then (1.1) is oscillatory.

Proof. It is known (see [12]) that condition (2.24) is sufficient for inequality (2.21) (for $i = 1$) to have no eventually negative solution. On the other hand, condition (2.25) is sufficient for inequality (2.21) (for $i = 2$) to have no eventually positive solution. $\qquad\square$

For an application of our results, we give the following example.

Example 2.6. Consider the second-order differential equation

$$
\left[(x(t) + p_1x(t - \tau_1) + p_2x(t + \tau_2))^\gamma\right]'' = q_1x^\gamma(t - \sigma_1) + q_2x^\gamma(t + \sigma_2), \quad t \geq t_0,
\tag{2.26}
$$

where $q_i > 0$ are constants and $\sigma_i > \tau_i$ for $i = 1, 2$.

It is easy to see that $Q_i(t) = q_i$, $i = 1, 2$. Assume that $\varepsilon > 0$. Let $a_i(t) = (2 + \varepsilon)/(e(\sigma_i - \tau_i))$, $i = 1, 2$. Clearly, (2.19) holds. If

$$q_i > \left[\frac{2}{(e(\sigma_i - \tau_i))} \right]^2 \left(2^{\gamma-1} \right)^2 \left(1 + p_1^\gamma + \frac{p_2^\gamma}{2^{\gamma-1}} \right) \tag{2.27}$$

for $i = 1, 2$, then (2.20) holds. Moreover, we see that

$$\liminf_{t \to \infty} \int_{t-\beta_1}^{t} a_1(s - \beta_1) \, ds = \frac{2 + \varepsilon}{2e} > \frac{1}{e},$$

$$\liminf_{t \to \infty} \int_{t}^{t+\beta_2} a_2(s + \beta_2) \, ds = \frac{2 + \varepsilon}{2e} > \frac{1}{e}. \tag{2.28}$$

Hence by applying Corollary 2.5, we find that (2.26) is oscillatory.
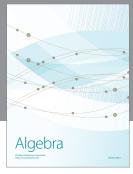
Acknowledgments

References

[1] J. Hale, *Theory of Functional Differential Equations*, vol. 3 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 2nd edition, 1977.

[2] Ch. G. Philos, "Oscillation theorems for linear differential equations of second order," *Archiv der Mathematik*, vol. 53, no. 5, pp. 482–492, 1989.

[3] R. P. Agarwal, S.-L. Shieh, and C.-C. Yeh, "Oscillation criteria for second-order retarded differential equations," *Mathematical and Computer Modelling*, vol. 26, no. 4, pp. 1–11, 1997.

[4] J. Džurina and I. P. Stavroulakis, "Oscillation criteria for second-order delay differential equations," *Applied Mathematics and Computation*, vol. 140, no. 2-3, pp. 445–453, 2003.

[5] J. Baštinec, L. Berezansky, J. Diblík, and Z. Šmarda, "On the critical case in oscillation for differential equations with a single delay and with several delays," *Abstract and Applied Analysis*, vol. 2010, Article ID 417869, 20 pages, 2010.

[6] J. Baštinec, J. Diblík, and Z. Šmarda, "Oscillation of solutions of a linear second-order discrete-delayed equation," *Advances in Difference Equations*, vol. 2010, Article ID 693867, 12 pages, 2010.

[7] J. Diblík, Z. Svoboda, and Z. Šmarda, "Explicit criteria for the existence of positive solutions for a scalar differential equation with variable delay in the critical case," *Computers & Mathematics with Applications*, vol. 56, no. 2, pp. 556–564, 2008.

[8] L. Berezansky, J. Diblík, and Z. Šmarda, "Positive solutions of second-order delay differential equations with a damping term," *Computers & Mathematics with Applications*, vol. 60, no. 5, pp. 1332–1342, 2010.

[9] Y. G. Sun and F. W. Meng, "Note on the paper of J. Džurina and I. P. Stavroulakis," *Applied Mathematics and Computation*, vol. 174, no. 2, pp. 1634–1641, 2006.

[10] B. Baculíková, "Oscillation criteria for second order nonlinear differential equations," *Archivum Mathematicum*, vol. 42, no. 2, pp. 141–149, 2006.
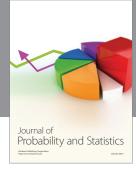
[11] R. G. Koplatadze and T. A. Chanturiya, *Ob ostsillyatsionnykh svoistvakh differentsialnykh uravnenii s otklonyayushchimsya argumentom (Oscillatory Properties of Differential Equations with Deviating Argument)*, Izdat. Tbilis. Univ., Tbilisi, Georgia, 1977.

[12] G. S. Ladde, V. Lakshmikantham, and B. G. Zhang, *Oscillation Theory of Differential Equations with Deviating Arguments*, vol. 110 of *Monographs and Textbooks in Pure and Applied Mathematics*, Marcel Dekker, New York, NY, USA, 1987.

[13] J. Diblík, Z. Svoboda, and Z. Šmarda, "Retract principle for neutral functional differential equations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, no. 12, pp. e1393–e1400, 2009.

[14] L. H. Erbe and Q. Kong, "Oscillation results for second order neutral differential equations," *Funkcialaj Ekvacioj*, vol. 35, no. 3, pp. 545–555, 1992.

[15] Q. Zhang, J. Yan, and L. Gao, "Oscillation behavior of even-order nonlinear neutral differential equations with variable coefficients," *Computers & Mathematics with Applications*, vol. 59, no. 1, pp. 426–430, 2010.

[16] Q. Wang, "Oscillation theorems for first-order nonlinear neutral functional differential equations," *Computers & Mathematics with Applications*, vol. 39, no. 5-6, pp. 19–28, 2000.

[17] Z. Han, T. Li, S. Sun, and Y. Sun, "Remarks on the paper [Appl. Math. Comput. 207 (2009) 388–396]," *Applied Mathematics and Computation*, vol. 215, no. 11, pp. 3998–4007, 2010.

[18] L. Liu and Y. Bai, "New oscillation criteria for second-order nonlinear neutral delay differential equations," *Journal of Computational and Applied Mathematics*, vol. 231, no. 2, pp. 657–663, 2009.

[19] R. Xu and F. Meng, "Oscillation criteria for second order quasi-linear neutral delay differential equations," *Applied Mathematics and Computation*, vol. 192, no. 1, pp. 216–222, 2007.

[20] J.-G. Dong, "Oscillation behavior of second order nonlinear neutral differential equations with deviating arguments," *Computers & Mathematics with Applications*, vol. 59, no. 12, pp. 3710–3717, 2010.

[21] J. Džurina and D. Hudáková, "Oscillation of second order neutral delay differential equations," *Mathematica Bohemica*, vol. 134, no. 1, pp. 31–38, 2009.

[22] M. Hasanbulli and Y. V. Rogovchenko, "Oscillation criteria for second order nonlinear neutral differential equations," *Applied Mathematics and Computation*, vol. 215, no. 12, pp. 4392–4399, 2010.

[23] B. Baculíková and J. Džurina, "Oscillation of third-order neutral differential equations," *Mathematical and Computer Modelling*, vol. 52, no. 1-2, pp. 215–226, 2010.

[24] S. H. Saker, "Oscillation of second order neutral delay differential equations of Emden-Fowler type," *Acta Mathematica Hungarica*, vol. 100, no. 1-2, pp. 37–62, 2003.

[25] J. Dzurina, J. Busha, and E. A. Airyan, "Oscillation criteria for second-order differential equations of neutral type with mixed arguments," *Differential Equations*, vol. 38, no. 1, pp. 137–140, 2002.

[26] S. R. Grace, "On the oscillations of mixed neutral equations," *Journal of Mathematical Analysis and Applications*, vol. 194, no. 2, pp. 377–388, 1995.

[27] S. R. Grace, "Oscillations of mixed neutral functional-differential equations," *Applied Mathematics and Computation*, vol. 68, no. 1, pp. 1–13, 1995.

[28] J. Yan, "Oscillations of higher order neutral differential equations of mixed type," *Israel Journal of Mathematics*, vol. 115, pp. 125–136, 2000.

[29] Z. Wang, "A necessary and sufficient condition for the oscillation of higher-order neutral equations," *The Tôhoku Mathematical Journal*, vol. 41, no. 4, pp. 575–588, 1989.

[30] Z. Han, T. Li, S. Sun, and W. Chen, "On the oscillation of second-order neutral delay differential equations," *Advances in Difference Equations*, vol. 2010, Article ID 289340, 8 pages, 2010.

[31] Z. Han, T. Li, S. Sun, C. Zhang, and B. Han, "Oscillation criteria for a class of second order neutral delay dynamic equations of Emden-Fowler type," *Abstract and Applied Analysis*, vol. 2011, Article ID 653689, 26 pages, 2011.

[32] T. Li, Z. Han, P. Zhao, and S. Sun, "Oscillation of even-order neutral delay differential equations," *Advances in Difference Equations*, vol. 2010, Article ID 184180, 9 pages, 2010.

[33] Z. Han, T. Li, S. Sun, and C. Zhang, "An oscillation criteria for third order neutral delay differential equations," *Journal of Applied Analysis*, vol. 16, no. 2, pp. 295–303, 2010.

[34] Z. Han, T. Li, S. Sun, and W. Chen, "Oscillation criteria for second-order nonlinear neutral delay differential equations," *Advances in Difference Equations*, vol. 2010, Article ID 763278, 23 pages, 2010.

[35] S. Sun, T. Li, Z. Han, and Y. Sun, "Oscillation of second-order neutral functional differential equations with mixed nonlinearities," *Abstract and Applied Analysis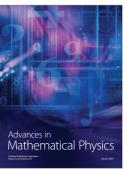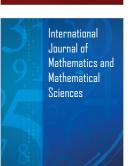*, vol. 2011, Article ID 927690, 15 pages, 2011.