Research Article

One-Signed Periodic Solutions of First-Order Functional Differential Equations with a Parameter

Ruyun Ma and Yanqiong Lu

Department of Mathematics, Northwest Normal University, Lanzhou 730070, China

Correspondence should be addressed to Ruyun Ma, ruyun.ma@126.com

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We study one-signed periodic solutions of the first-order functional differential equation $u'(t) = -a(t)u(t) + \lambda b(t)f(u(t - \tau(t)))$, $t \in \mathbb{R}$ by using global bifurcation techniques. Where $a, b \in C(\mathbb{R}, [0, \infty))$ are ω -periodic functions with $\int_0^\omega a(t)dt > 0$, $\int_0^\omega b(t)dt > 0$, τ is a continuous ω -periodic function, and $\lambda > 0$ is a parameter. $f \in C(\mathbb{R}, \mathbb{R})$ and there exist two constants $s_2 < 0 < s_1$ such that $f(s_2) = f(0) = f(s_1) = 0$, $f(s) > 0$ for $s \in (0, s_1) \cup (s_1, \infty)$ and $f(s) < 0$ for $s \in (-\infty, s_2) \cup (s_2, 0)$.

1. Introduction

In recent years, there has been considerable interest in the existence of periodic solutions of the following equation:

$$u'(t) = -a(t)u(t) + \lambda b(t)f(u(t - \tau(t))), \quad (1.1)$$

where $a, b \in C(\mathbb{R}, [0, \infty))$ are ω -periodic functions, and $\int_0^\omega a(t)dt > 0$, $\int_0^\omega b(t)dt > 0$, τ is a continuous ω -periodic function, $\lambda > 0$ is a parameter. (1.1) has been proposed as a model for a variety of physiological processes and conditions including production of blood cells, respiration, and cardiac arrhythmias; see, for example, [1–12] and the references therein. Roughly speaking, $u(t)$ represents the number of adult (sexually mature) members in a population at time t , $a(t)$ is the per capita death rate, and $f(u(t - \tau(t)))$ is the rate at which new members are recruited into the population at time t (τ is the age at which members mature, and it is assumed that the birth rate at a given time depends only on the adult population size). The most famous models of this type are

- (i) the Nicholson's blowflies equation proposed in [1] to explain the oscillatory population fluctuations observed by A. J. Nicholson in 1957 in his studies of the sheep blowfly *Lucilia cuprina*:

$$u'(t) = -au(t) + p \cdot u(t-h)e^{-\gamma u(t-h)}, \quad a, p, \gamma, h > 0; \quad (1.2)$$

- (ii) the model for blood cell populations proposed by Mackey and Glass in [2]

$$u'(t) = -au(t) + p \frac{u(t-h)}{1 + [u(t-h)]^n}, \quad a, p, \gamma, h > 0, \quad n > 1; \quad (1.3)$$

- (iii) the model for the survival of red blood cells in an animal proposed by Wazewska-Czyzewska and Lasota in [3]

$$u'(t) = -au(t) + p \cdot e^{-\gamma u(t-h)}, \quad a, p, \gamma, h > 0. \quad (1.4)$$

Recently, Cheng and Zhang [7] studied the existence of positive ω -periodic solutions of the functional equation (1.1) under the assumptions:

- (H1) $f \in C([0, \infty), [0, \infty))$, and $f(s) > 0$ for $s > 0$;
 (H2) $a, b \in C(\mathbb{R}, [0, \infty))$ are ω -periodic functions, $\int_0^\omega a(t)dt > 0$, $\int_0^\omega b(t)dt > 0$, $\tau \in C(\mathbb{R}, \mathbb{R})$ is a ω -periodic function;
 (H3) there exist $f_0, f_\infty \in (0, \infty)$ such that

$$f_0 = \lim_{|s| \rightarrow 0} \frac{f(s)}{s}, \quad f_\infty = \lim_{|s| \rightarrow \infty} \frac{f(s)}{s}. \quad (1.5)$$

They proved the following.

Theorem A. Assume (H1)–(H3) hold. Then for each λ satisfying

$$\frac{1}{\sigma B f_\infty} < \lambda < \frac{1}{A f_0}, \quad \text{or} \quad \frac{1}{\sigma A f_0} < \lambda < \frac{1}{B f_\infty}, \quad (1.6)$$

equation (1.1) has a positive periodic solution, where

$$A = \max_{t \in [0, \omega]} \int_0^\omega G(t, s) b(s) ds, \quad B = \min_{t \in [0, \omega]} \int_0^\omega G(t, s) b(s) ds, \quad \sigma = e^{\int_0^\omega a(t) dt}. \quad (1.7)$$

However, the condition used in [7] is not sharp, and the main results in [7] give no any information about the global structure of the set of positive periodic solutions. Moreover, f satisfied (H1) in [7], so a natural question is what would happen if f is allowed to have some zeros in $\mathbb{R}$? The purpose of this work is to study the global behavior of the components of one-signed solutions of (1.1) under the condition

(H4) $f \in C(\mathbb{R}, \mathbb{R})$; there exist two constants $s_2 < 0 < s_1$ such that $f(s_2) = f(0) = f(s_1) = 0$, $f(s) > 0$ for $s \in (0, s_1) \cup (s_1, \infty)$, and $f(s) < 0$ for $s \in (-\infty, s_2) \cup (s_2, 0)$.

The rest of this paper is organized as follows. In Section 2, we give some notations and the main results. Section 3 is devoted to proving the main results.

2. Statement of the Main Results

Let $Y = \{u \in C(\mathbb{R}, \mathbb{R}) : u(t) = u(t + \omega)\}$ with the norm

$$\|u\|_{\infty} = \max_{t \in [0, \omega]} |u(t)|. \quad (2.1)$$

Then $(Y, \|\cdot\|_{\infty})$ is a Banach space. Let

$$E = \left\{ u \in C^1(\mathbb{R}, \mathbb{R}) : u(t) = u(t + \omega) \right\} \quad (2.2)$$

be the Banach space with the norm $\|u\| = \max\{\|u\|_{\infty}, \|u'\|_{\infty}\}$.

It is well known that (1.1) is equivalent to

$$u(t) = \lambda \int_t^{t+\omega} G(t, s) b(s) f(u(s - \tau(s))) ds := (Au)(t), \quad (2.3)$$

where

$$G(t, s) = \frac{e^{\int_t^s a(\theta) d\theta}}{e^{\int_0^{\omega} a(\theta) d\theta} - 1}, \quad s \in [t, t + \omega]. \quad (2.4)$$

Notice that $\int_0^{\omega} a(t) dt > 0$, we have

$$\frac{1}{\sigma - 1} \leq G(t, s) \leq \frac{\sigma}{\sigma - 1}, \quad (2.5)$$

where $\sigma = e^{\int_0^{\omega} a(t) dt}$, and $0 < 1/\sigma < 1$.

Define that K is a cone in Y by

$$K = \left\{ u \in Y : u(t) \geq 0, \quad u(t) \geq \frac{1}{\sigma} \|u\| \right\}. \quad (2.6)$$

It is not difficult to prove that $A(K) \subset K$ and $A : K \rightarrow K$ is completely continuous.

Let us consider the spectrum of the linear eigenvalue problem

$$u'(t) = -a(t)u(t) + \lambda b(t)u(t - \tau(t)), \quad t \in \mathbb{R}. \quad (2.7)$$

Lemma 2.1. *Assume that (H2) holds. Then the linear problem (2.7) has a unique eigenvalue λ_1 , which is positive and simple, and the corresponding eigenfunction φ is of one sign.*

Proof. It is a direct consequence of the Krein-Rutman Theorem [13, Theorem 19.3]. $\square$

In the rest of the paper, we always assume that

$$\|\varphi\| = 1, \quad \varphi(t) > 0, \quad t \in \mathbb{R}. \quad (2.8)$$

Define $L : E \rightarrow Y$ by setting

$$Lu := u'(t) + a(t)u(t), \quad u \in E. \quad (2.9)$$

Then $L^{-1} : Y \rightarrow E$ is completely continuous.

Let $\zeta, \xi \in C(\mathbb{R}, \mathbb{R})$ be such that

$$f(s) = f_0s + \zeta(s), \quad f(s) = f_\infty s + \xi(s). \quad (2.10)$$

Clearly,

$$\lim_{|s| \rightarrow 0} \frac{\zeta(s)}{s} = 0, \quad \lim_{|s| \rightarrow \infty} \frac{\xi(s)}{s} = 0. \quad (2.11)$$

Let us consider

$$Lu(t) - \lambda b(t)f_0u(t - \tau(t)) = \lambda b(t)\zeta(u(t - \tau(t))) \quad (2.12)$$

as a bifurcation problem from the trivial solution $u \equiv 0$ and

$$Lu(t) - \lambda b(t)f_\infty u(t - \tau(t)) = \lambda b(t)\xi(u(t - \tau(t))) \quad (2.13)$$

as a bifurcation problem from infinity. We note that (2.12) and (2.13) are the same and each of them is equivalent to (1.1).

Let $\mathbb{E} = \mathbb{R} \times E$ under the product topology. We add the points $\{(\lambda, \infty) \mid \lambda \in \mathbb{R}\}$ to our space $\mathbb{E}$. Let S^+ denote the set of positive functions in E and $S^- = -S^+$, and $S = S^- \cup S^+$. They are disjoint and open in E . Finally, let $\Phi^\pm = \mathbb{R} \times S^\pm$ and $\Phi = \mathbb{R} \times S$.

Remark 2.2. It is worth remarking that if u is a nontrivial solution of (1.1) and a, b , and f satisfy (H2)–(H4), then $u \in S^\nu$ for some $\nu = \{+, -\}$. To see this, define

$$q(t) = \begin{cases} \frac{f(u(t))}{u(t)}, & u(t) \neq 0, \\ f_0, & u(t) = 0. \end{cases} \quad (2.14)$$

Thus (1.1) is equivalent to

$$u'(t) = -a(t)u(t) + \lambda b(t)q(t - \tau(t))u(t - \tau(t)), \quad t \in \mathbb{R}. \quad (2.15)$$

Obviously, $b(\cdot)q(\cdot - \tau(\cdot))$ satisfies (H2). From Lemma 2.1, the nontrivial solution $u \in S^\nu$ for some $\nu \in \{+, -\}$.

The result of Rabinowitz [14] for (2.12) can be stated as follows: for each $\nu \in \{+, -\}$, there exists a continuum $\mathcal{C}^\nu$ of solutions of (2.12) joining $(\lambda_1/f_0, 0)$ to infinity, and $\mathcal{C}^\nu \setminus \{(\lambda_1/f_0, 0)\} \subset \Phi^\nu$.

The result of Rabinowitz [15] for (2.13) can be stated as follows: for each $\nu \in \{+, -\}$, there exists a continuum $\mathfrak{D}^\nu$ of solutions of (2.13) meeting $(\lambda_1/f_\infty, \infty)$, and $\mathfrak{D}^\nu \setminus \{(\lambda_1/f_\infty, \infty)\} \subset \Phi^\nu$.

Our main result is the following.

Theorem 2.3. *Assume (H2)–(H4) hold. Moreover, suppose that*

(H5) *f satisfies the Lipschitz condition in $[s_2, s_1]$.*

Then

(i) *for $(\lambda, u) \in \mathcal{C}^+ \cup \mathcal{C}^-$,*

$$s_2 < u(t) < s_1, \quad t \in [0, \omega]; \quad (2.16)$$

(ii) *for $(\lambda, u) \in \mathfrak{D}^+ \cup \mathfrak{D}^-$, we have that either*

$$\max_{t \in [0, \omega]} u(t) > s_1 \quad (2.17)$$

or

$$\min_{t \in [0, \omega]} u(t) < s_2. \quad (2.18)$$

Corollary 2.4. *Let (H2)–(H5) hold. Then*

- (i) *if $\lambda \in (\lambda_1/f_\infty, \lambda_1/f_0]$, then (1.1) has at least two solutions u_∞^+ and u_∞^- , such that u_∞^+ is positive on $[0, \omega]$ and u_∞^- is negative on $[0, \omega]$;*
- (ii) *if $\lambda \in (\lambda_1/f_0, \infty)$, then (1.1) has at least four solutions u_∞^+ , u_∞^- , u_0^+ , and u_0^- , such that u_∞^+ , u_0^+ are positive on $[0, \omega]$ and u_∞^- , u_0^- are negative on $[0, \omega]$.*

Corollary 2.5. *Let (H2)–(H5) hold. Then*

- (i) *if $\lambda \in (\lambda_1/f_0, \lambda_1/f_\infty]$, then (1.1) has at least two solutions u_0^+ and u_0^- , such that u_0^+ is positive on $[0, \omega]$ and u_0^- is negative on $[0, \omega]$;*
- (ii) *if $\lambda \in (\lambda_1/f_\infty, \infty)$, then (1.1) has at least four solutions u_∞^+ , u_∞^- , u_0^+ , and u_0^- , such that u_∞^+ , u_0^+ are positive on $[0, \omega]$ and u_∞^- , u_0^- are negative on $[0, \omega]$.*

3. Proof of the Main Results

To prove Theorem 2.3, we give a Proposition.

Proposition 3.1. (i) *The first-order boundary value problem*

$$u'(t) + a(t)u(t) = h(t), \quad u(0) = u(\omega) \quad (3.1)$$

has a unique solution for all $h \in L^1[0, \omega]$ if and only if $\int_0^\omega a(s)ds \neq 0$.

(ii) Assume that u is a solution of (3.1). If $h \geq 0$ and $h(\cdot) \not\equiv 0$ on any subinterval of $[0, \omega]$, then $u(t) \int_0^\omega a(s)ds > 0$ on $[0, \omega]$.

Proof. (i) The equation $u'(t) + a(t)u(t) = 0$ has a solution $u(t) = Ce^{-\int_0^t a(s)ds}$, where C is a constant. If $u(t)$ is a nontrivial solution, then by $u(0) = C$, $u(\omega) = Ce^{-\int_0^\omega a(s)ds}$, we can get that $\int_0^\omega a(s)ds = 0$.

On the other hand, from $\int_0^\omega a(s)ds = 0$, we can get that $u'(t) + a(t)u(t) = 0$ has a nontrivial solution $u(t) = Ce^{-\int_0^t a(s)ds}$, where $C \in \mathbb{R} \setminus \{0\}$.

(ii) We claim that $u(t) \neq 0, t \in [0, \omega]$. Suppose on the contrary that there exists $t_0 \in [0, \omega]$, such that $u(t_0) = 0$; it is not difficult to compute that

$$u'(t) + a(t)u(t) = h(t), \quad u(t_0) = u_0 \quad (3.2)$$

has a solution

$$u(t) = \int_{t_0}^t h(s)e^{\int_t^s a(\tau)d\tau} ds. \quad (3.3)$$

Since $h \geq 0$, we have

$$u(0) \leq u(t_0) \leq u(\omega). \quad (3.4)$$

If $h(\hat{t}) > 0, \hat{t} \in [0, t_0)$, then there exists a neighborhood $U(\hat{t}) \subset [0, t_0)$ of $\hat{t}$, such that $h(t) > 0$ on $U(\hat{t})$. Thus, $u(0) = \int_{t_0}^0 h(s)e^{\int_0^s a(\tau)d\tau} ds < 0$; this contradicts with $u(0) = u(\omega)$.

If $h(\bar{t}) > 0, \bar{t} \in (t_0, \omega]$, then there exists a neighborhood $U(\bar{t}) \subset (t_0, \omega]$ of $\bar{t}$, such that $h(t) > 0$ on $U(\bar{t})$. By using a similar way, we can prove that $u(\omega) > 0$, which also contradicts with $u(0) = u(\omega)$.

Hence $u(t) \neq 0$ on $[0, \omega]$. Moreover, it follows that

$$\int_0^\omega \frac{u'(t)}{u(t)} dt + \int_0^\omega a(t)dt = \int_0^\omega \frac{h(t)}{u(t)} dt, \quad (3.5)$$

that is,

$$(\ln u(t))|_0^\omega + \int_0^\omega a(t)dt = \int_0^\omega \frac{h(t)}{u(t)}dt. \quad (3.6)$$

Thus $\int_0^\omega a(t)dt = \int_0^\omega h(t)/u(t)dt$, that is $u \int_0^\omega a(s)ds > 0$. $\square$

Next, we prove Theorem 2.3 and Corollaries 2.4 and 2.5.

Proof of Theorem 2.3. Suppose on the contrary that there exists $(\lambda, u) \in \mathcal{C}^+ \cup \mathcal{C}^- \cup \mathfrak{D}^+ \cup \mathfrak{D}^-$ such that either

$$\max\{u(t) \mid t \in [0, \omega]\} = s_1 \quad (3.7)$$

or

$$\min\{u(t) \mid t \in [0, \omega]\} = s_2. \quad (3.8)$$

We divide the proof into two cases.

Case 1 ($\max\{u(t) \mid t \in [0, \omega]\} = s_1$). In this case, we know that

$$0 \leq u(t) \leq s_1, \quad 0 \leq u(t - \tau(t)) \leq s_1, \quad t \in [0, \omega]. \quad (3.9)$$

Let us consider the functional differential equation

$$u'(t) + a(t)u(t) = \lambda b(t)f(u(t - \tau(t))), \quad t \in \mathbb{R}. \quad (3.10)$$

By (H2), (H4) and (H5), there exists $m \geq 0$ such that $b(t)f(s) + ms$ is strictly increasing on s for $s \in [s_2, s_1]$. Then (3.10) can be rewritten to the form

$$Lu + \lambda mu(t - \tau(t)) = \lambda [b(t)f(u(t - \tau(t))) + mu(t - \tau(t))], \quad (3.11)$$

and since $LS_1 - a(t)s_1 = 0 = f(s_1)$,

$$LS_1 - a(t)s_1 + \lambda ms_1 = \lambda [b(t)f(s_1) + ms_1]. \quad (3.12)$$

Subtracting, we get

$$L(s_1 - u) + \lambda m(s_1 - u(t - \tau(t))) - a(t)s_1 \geq 0. \quad (3.13)$$

That is,

$$\begin{aligned} L(s_1 - u) + \lambda ms_1 &\geq 0, \quad t \in [0, \omega], \\ s_1 - u(0) &= s_1 - u(\omega) > 0. \end{aligned} \quad (3.14)$$

From Proposition 3.1, we deduce that $s_1 > u(t), t \in [0, \omega]$, which contradicts with that $\max\{u(t) \mid t \in [0, \omega]\} = s_1$. Hence,

$$u(t) < s_1, \quad t \in [0, \omega]. \quad (3.15)$$

Case 2 ($\min\{u(t) \mid t \in [0, \omega]\} = s_2$). In this case, we know that

$$s_2 \leq u(t) \leq 0, \quad s_2 \leq u(t - \tau(t)) \leq 0, \quad t \in [0, \omega]. \quad (3.16)$$

Let us consider (3.10); by (H2), (H4), and (H5), there exists $m \geq 0$ such that $b(t)f(s) + ms$ is strictly increasing in s for $s \in [s_2, s_1]$. Then

$$Lu + \lambda mu(t - \tau(t)) = \lambda [b(t)f(u(t - \tau(t))) + mu(t - \tau(t))] \quad (3.17)$$

and since $LS_2 - a(t)s_2 = 0 = f(s_2)$,

$$LS_2 - a(t)s_2 + \lambda ms_2 = \lambda [b(t)f(s_2) + ms_2]. \quad (3.18)$$

Subtracting, we get

$$L(s_2 - u) + \lambda m(s_2 - u(t - \tau(t))) - a(t)s_2 \leq 0. \quad (3.19)$$

That is

$$\begin{aligned} L(s_2 - u) + \lambda ms_2 &\leq 0, \quad t \in [0, \omega], \\ s_2 - u(0) &= s_2 - u(\omega) < 0. \end{aligned} \quad (3.20)$$

From Proposition 3.1, we deduce that $s_2 - u(t) < 0, t \in [0, \omega]$, this contradicts with that $\min\{u(t) \mid t \in [0, \omega]\} = s_2$. Therefore,

$$s_2 < u(t), \quad t \in [0, \omega]. \quad (3.21)$$

□

Proof of Corollaries 2.4 and 2.5. Since boundary value problem

$$u'(t) + a(t)u(t) = 0, \quad u(0) = u(\omega) \quad (3.22)$$

has a unique solution $u \equiv 0$, we get

$$(\mathcal{C}^+ \cup \mathcal{C}^- \cup \mathfrak{D}^+ \cup \mathfrak{D}^-) \subset \{(\lambda, u) \in \mathbb{R} \times E \mid \lambda \geq 0\}. \quad (3.23)$$

Take $\Lambda \in \mathbb{R}$ as an interval such that $\Lambda \cap \{\lambda_1/f_\infty\} = \{\lambda_1/f_\infty\}$ and $\mathcal{M}$ as a neighborhood of $(\lambda_1/f_\infty, \infty)$ whose projection on $\mathbb{R}$ lies in Λ and whose projection on E is bounded away from 0. Then by [15, Theorem 1.6, and Corollary 1.8], we have that for each $v \in \{+, -\}$, either

- (1) $\mathfrak{D}^v \setminus \mathcal{M}$ is bounded in $\mathbb{R} \times E$ in which case $\mathfrak{D}^v \setminus \mathcal{M}$ meets $\{(\lambda, 0) \mid \lambda \in \mathbb{R}\}$, or
- (2) $\mathfrak{D}^v \setminus \mathcal{M}$ is unbounded.

Moreover, if (1) occurs and $\mathfrak{D}^v \setminus \mathcal{M}$ has a bounded projection on $\mathbb{R}$, then $\mathfrak{D}^v \setminus \mathcal{M}$ meets $(\lambda_k/f_\infty, \infty)$, where $\lambda_k \neq \lambda_1$ is another eigenvalue of (2.7).

Obviously, Theorem 2.3 (ii) implies that (1) does not occur. So $\mathfrak{D}^+ \setminus \mathcal{M}$ is unbounded.

Remark 2.2 guarantees that $\mathfrak{D}^+$ is a component of solutions of (2.12) in S^+ which meets $(\lambda_1/f_\infty, \infty)$, and consequently $\text{Proj}_{\mathbb{R}}(\mathfrak{D}^+ \setminus \mathcal{M})$ is unbounded. Thus

$$\text{Proj}_{\mathbb{R}}(\mathfrak{D}^+) \supset \left(\frac{\lambda_1}{f_\infty}, +\infty \right). \quad (3.24)$$

Similarly, we get

$$\text{Proj}_{\mathbb{R}}(\mathfrak{D}^-) \supset \left(\frac{\lambda_1}{f_\infty}, +\infty \right). \quad (3.25)$$

By Theorem 2.3, for any $(\lambda, u) \in (\mathcal{C}^+ \cup \mathcal{C}^-)$,

$$\|u\|_\infty < \max\{s_1, |s_2|\} := s^*. \quad (3.26)$$

(3.26) and (2.12) imply that

$$\|u\| < \max \left\{ s^*, \|a\|_\infty s^* + \lambda \|b\|_\infty \max_{|s| \leq s^*} |f(s)| \right\}, \quad (3.27)$$

which means that the sets $\{(\lambda, u) \in \mathcal{C}^+ \mid \lambda \in [0, d]\}$ and $\{(\lambda, u) \in \mathcal{C}^- \mid \lambda \in [0, d]\}$ are bounded for any fixed $d \in (0, \infty)$. This together with the fact that $\mathcal{C}^+$ and $\mathcal{C}^-$ join $(\lambda_1/f_0, 0)$ to infinity yields, respectively, that

$$\begin{aligned} \text{Proj}_{\mathbb{R}}(\mathcal{C}^+) &\supset \left(\frac{\lambda_1}{f_0}, +\infty \right), \\ \text{Proj}_{\mathbb{R}}(\mathcal{C}^-) &\supset \left(\frac{\lambda_1}{f_0}, +\infty \right). \end{aligned} \quad (3.28)$$

Combining (3.24), (3.25), and (3.28), we conclude the desired results. $\square$

Remark 3.2. The methods used in the proof of Theorem 2.3, Corollaries 2.4, and 2.5 have been used in the study of other kinds of boundary value problems; see [16–18] and the references therein.

Remark 3.3. The conditions in Corollaries 2.4 and 2.5 are sharp. Let us take

$$a(t) \equiv a > 0, \quad \lambda = a, \quad b(t) = 1, \quad f(s) = s + h(s), \quad \tau(t) \equiv 0. \quad (3.29)$$

Let

$$h(s) = \begin{cases} -\frac{2s}{s^2 + 1}, & s \in (-\infty, -1) \cup (1, +\infty), \\ -\frac{2s^3}{s^2 + 1}, & s \in [-1, 1], \end{cases} \quad (3.30)$$

and consider problem

$$u'(t) = -a(t)u(t) + a[u(t) + h(u(t))], \quad t \in [0, \omega], \quad u(0) = u(\omega). \quad (3.31)$$

It is easy to see that $\lambda_1 = a, f_0 = f_\infty = 1$. Since

$$\frac{\lambda_1}{f_\infty} = a = \frac{\lambda_1}{f_0}, \quad (3.32)$$

the conditions of Corollaries 2.4 and 2.5 are not valid. In this case, (3.31) has no nontrivial solution. In fact, if u is a nontrivial solution of (3.31), then

$$0 = \int_0^\omega u'(t) dt = a \int_0^\omega h(u(t)) dt \neq 0, \quad (3.33)$$

which is a contradiction.

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