Research Article

On Subclass of \( k \)-Uniformly Convex Functions of Complex Order Involving Multiplier Transformations

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Received 30 December 2011; Revised 28 February 2012; Accepted 13 March 2012

Academic Editor: Ondřej Došlý

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We introduce a subclass of \( k \)-uniformly convex functions of order \( \alpha \) with negative coefficients by using the multiplier transformations in the open unit disk \( U = \{ z \in \mathbb{C} : |z| < 1 \} \).

1. Introduction

Let \( \mathcal{N} \) denote the class of functions of the form:

\[
 f(z) = z^\beta + \sum_{n=2}^{\infty} \beta a_n z^{\beta+n-1}, \quad \beta > 0,
\]

which are analytic and univalent in the open unit disk \( U = \{ z \in \mathbb{C} : |z| < 1 \} \) (see [1]). Also denote by \( \mathcal{M} \) the subclass of \( \mathcal{N} \) consisting of functions of the form:

\[
 f(z) = z^\beta - \sum_{n=2}^{\infty} \beta a_n z^{\beta+n-1}, \quad (a_n \geq 0, \ \beta > 0).
\]
For any integer $m$, we define the multiplier transformations $I_m^\ell f(z)$ (see [2, 3]) of functions $f \in \mathcal{N}(n)$ by

$$I_m^\ell f(z) = z^\beta + \sum_{n=2}^{\infty} \frac{\beta + \ell}{\beta + \ell + n - 1} a_n z^{\beta + n - 1} n \geq 0,$$  

where $Q(n, \beta, \ell) = ((\beta + \ell)/(\beta + \ell + n - 1))^m$.

A function $f \in \mathcal{N}$ is said to be in the class USL$(\alpha, k)$ ($k$-uniformly starlike Functions of order $\alpha$) if it satisfies the condition:

$$\text{Re}\left\{ \frac{zf'(z)^\beta}{f(z)^\beta} - \alpha \right\} > k \left| \frac{zf'(z)^\beta}{f(z)^\beta} - 1 \right|, \quad (0 \leq \alpha < 1, \ k \geq 0), \ z \in U$$

and is said to be in the class UCV$(\alpha, k)$ ($k$-uniformly convex Functions of order $\alpha$) if it satisfies the condition:

$$\text{Re}\left\{ 1 + \frac{zf''(z)^\beta}{f'(z)^\beta} - \alpha \right\} > k \left| \frac{zf''(z)^\beta}{f'(z)^\beta} \right|, \quad (0 \leq \alpha < 1, \ k \geq 0), \ z \in U.$$  

Indeed it follows from (1.4) and (1.5) that

$$f \in \text{UCV}(\alpha, k) \iff zf' \in \text{USL}(\alpha, k).$$

The interesting geometric properties of these function classes were extensively studied by Kanas et al., in [4, 5], motivated by Altintas et al. [6], Murugusundaramoorthy and Srivastava [7], and Murugusundaramoorthy and Magesh [8, 9], Atshan and Kulkarni [10] and Atshan and Buti [11].

Now, we define a new subclass of uniformly convex functions of complex order.

For $0 \leq \alpha < 1, \ k \geq 0, \ u \in \mathbb{C} \setminus \{0\}$, we let $\mathcal{M}_m^\ell(\alpha, \beta, k, u)$ be the class of functions $f$ satisfying (1.2) with the analytic criterion:

$$\text{Re}\left\{ 1 + \frac{1}{u} \left( 1 + \frac{z(I_m^\ell f(z)^\beta)^n}{(I_m^\ell f(z)^\beta)^n} - \alpha \right) \right\} > k \left| 1 + \frac{1}{u} \left( \frac{z(I_m^\ell f(z)^\beta)^n}{(I_m^\ell f(z)^\beta)^n} \right) \right|, \ z \in U,$$

where $I_m^\ell f(z)^\beta$ is given by (1.3).
2. Main Results

First, we obtain the necessary and sufficient condition for functions $f$ in the class $\mathcal{N}_m^\ell(\alpha, \beta, k, \nu)$.

**Theorem 2.1.** The necessary and sufficient condition for $f$ of the form of (1.2) to be in the class $\mathcal{N}_m^\ell(\alpha, \beta, k, \nu)$ is

$$
\sum_{n=2}^{\infty} (\beta + n - 1) [ (\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)] Q(n, \beta, \ell) a_n \leq (k - \alpha) + (1 - k) (\beta + |\nu|),
$$

(2.1)

where $0 \leq \alpha < 1$, $k \geq 0$, $\nu \in \mathbb{C} \setminus \{0\}$.

**Proof.** Suppose that (2.1) is true for $z \in U$. Then

$$
\text{Re} \left\{ 1 + \frac{1}{|\nu|} \left( 1 + \frac{z \left( I_m^\ell f(z)^\beta \right)^\nu}{(I_m^\ell f(z)^\beta)^\nu - \alpha} \right) \right\} - k \left| 1 + \frac{z \left( I_m^\ell f(z)^\beta \right)^\nu}{(I_m^\ell f(z)^\beta)^\nu - \alpha} \right| > 0,
$$

(2.2)

if

$$
1 + \frac{1}{|\nu|} \left( \frac{(\beta - \alpha) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - \alpha - 1) Q(n, \beta, \ell) a_n |z|^{n-1}}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n |z|^{n-1}} \right) - k \left[ 1 + \frac{1}{|\nu|} \left( \frac{(\beta - 1) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - 2) Q(n, \beta, \ell) a_n |z|^{n-1}}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n |z|^{n-1}} \right) \right] > 0,
$$

(2.3)

that is, if

$$
\sum_{n=2}^{\infty} (\beta + n - 1) [ (\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)] Q(n, \beta, \ell) a_n \leq (k - \alpha) + (1 - k) (\beta + |\nu|).
$$

(2.4)
Conversely, assume that \( f \in \mathcal{M}_n^{\alpha}(\alpha, \beta, k, \nu) \), then

\[
\text{Re} \left\{ 1 + \frac{1}{\nu} \left( 1 + \frac{z \left( t_m^\alpha f(z)^\beta \right)^\nu}{(t_m^\alpha f(z)^\beta)^\nu} - \alpha \right) \right\} > k \left| 1 + \frac{1}{\nu} \left( \frac{z \left( t_m^\alpha f(z)^\beta \right)^\nu}{(t_m^\alpha f(z)^\beta)^\nu} \right) \right|
\]

\[
\text{Re} \left\{ 1 + \frac{1}{\nu} \left( \frac{(\beta - \alpha) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - \alpha - 1) Q(n, \beta, \ell) a_n z^{n-1}}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n z^{n-1}} \right) \right\} > k \left| 1 + \frac{1}{\nu} \left( \frac{(\beta - 1) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - 2) Q(n, \beta, \ell) a_n z^{n-1}}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n z^{n-1}} \right) \right|
\]

(2.5)

Letting \( z \to 1^+ \) along the real axis, we have

\[
1 + \frac{1}{\nu} \left( \frac{(\beta - \alpha) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - \alpha - 1) Q(n, \beta, \ell) a_n}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n} \right)
\]

\[
> k \left[ 1 + \frac{1}{\nu} \left( \frac{(\beta - 1) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - 2) Q(n, \beta, \ell) a_n}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) Q(n, \beta, \ell) a_n} \right) \right].
\]

(2.6)

Hence, by maximum modulus theorem, the simple computation leads to the desired inequality

\[
\sum_{n=2}^{\infty} (\beta + n - 1) [ (\beta + n - 1 + |\nu|) (1 - k) + (k - \alpha) ] Q(n, \beta, \ell) a_n \leq (k - \alpha) + (1 - k) (\beta + |\nu|),
\]

(2.7)

which completes the proof.

\[\square\]

**Corollary 2.2.** Let the function \( f \) defined by (1.2) belong to \( \mathcal{M}_n^{\alpha}(\alpha, \beta, k, \nu) \). Then,

\[
a_n \leq \frac{(k - \alpha) + (1 - k) (\beta + |\nu|)}{(\beta + n - 1) [ (\beta + n - 1 + |\nu|) (1 - k) + (k - \alpha) ] Q(n, \beta, \ell)},
\]

(2.8)

where \( 0 \leq \alpha < 1, \ k \geq 0, \ \nu \in \mathbb{C} \setminus \{0\} \), with equality for

\[
f(z)^\beta = z^\beta - \frac{(k - \alpha) + (1 - k) (\beta + |\nu|)}{(\beta + n - 1) [ (\beta + n - 1 + |\nu|) (1 - k) + (k - \alpha) ] Q(n, \beta, \ell)} z^{\beta + n - 1}.
\]

(2.9)
3. Radii of Convexity and Close-to-Convexity

We obtain the radii of convexity and close-to-convexity results for \( f \) functions in the class \( \mathcal{A}_m^\ell(\alpha, \beta, k, \upsilon) \) in the following theorems.

**Theorem 3.1.** Let \( f \in \mathcal{A}_m^\ell(\alpha, \beta, k, \upsilon) \). Then \( f \) is convex of order \( \delta (0 \leq \delta < 1) \) in the disk \( |z| < r = r_1(\alpha, \beta, k, \upsilon, n, \delta) \), where

\[
r_1 = \inf_{n \geq 2} \left[ \frac{(2 - \delta - \beta)}{(3 - \delta - \beta - n)} \left( \frac{(\beta + n - 1 + |\upsilon|) (1 - k) + (k - \alpha)}{(k - \alpha) + (1 - k)(\beta + |\upsilon|)} \right) Q(n, \beta, \ell) \right]^{1/n - 1}.
\]  

**Proof.** Let \( f \in \mathcal{A}_m^\ell(\alpha, \beta, k, \upsilon) \). Then by Theorem 2.1, we have

\[
\sum_{n=2}^{\infty} \left( \frac{(\beta + n - 1) [(\beta + n - 1 + |\upsilon|) (1 - k) + (k - \alpha)]}{(k - \alpha) + (1 - k)(\beta + |\upsilon|)} \right) Q(n, \beta, \ell) a_n \leq 1.
\]  

(3.2)

For \( 0 \leq \delta < 1 \), we need to show that

\[
\frac{|zf''(z)|}{f'(z)^{\beta}} \leq 1 - \delta,
\]  

(3.3)

and we have to show that

\[
\frac{|zf''(z)|}{f'(z)^{\beta}} \leq \frac{(\beta - 1) - \sum_{n=2}^{\infty} (\beta + n - 1) (\beta + n - 2) a_n |z|^{n-1}}{1 - \sum_{n=2}^{\infty} (\beta + n - 1) a_n |z|^{n-1}} \leq 1 - \delta.
\]  

(3.4)

Hence,

\[
\sum_{n=2}^{\infty} \left( \frac{(\beta + n - 1) (3 - \delta - \beta - n)}{(2 - \delta - \beta)} \right) a_n |z|^{n-1} \leq 1.
\]  

(3.5)

This is enough to consider

\[
|z|^{n-1} \leq \frac{(2 - \delta - \beta) [(\beta + n - 1 + |\upsilon|) (1 - k) + (k - \alpha)] Q(n, \beta, \ell)}{(3 - \delta - \beta - n) [(k - \alpha) + (1 - k)(\beta + |\upsilon|)]}.
\]  

(3.6)

Therefore,

\[
|z| \leq \left\{ \frac{(2 - \delta - \beta) [(\beta + n - 1 + |\upsilon|) (1 - k) + (k - \alpha)] Q(n, \beta, \ell)}{(3 - \delta - \beta - n) [(k - \alpha) + (1 - k)(\beta + |\upsilon|)]} \right\}^{1/n - 1}.
\]  

(3.7)

Setting \( z = r_1(\alpha, \beta, k, \upsilon, n, \delta) \) in (3.7), we get the radius of convexity, which completes the proof of Theorem 3.1. \( \square \)
Theorem 3.2. Let \( f \in \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \). Then \( f \) is close-to-convex of order \( \delta (0 \leq \delta < 1) \) in the disk \(|z| < r = r_2(\alpha, \beta, k, \nu, n, \delta)\), where

\[
r_2 = \inf_{n \geq 2} \left[ \frac{(\beta + n - 1) \left[ (\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha) \right] Q(n, \beta, \ell)}{(k - \alpha) + (1 - k)(\beta + |\nu|)} \right]^{1/n-1}.
\]

Proof. Let \( f \in \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \). Then by Theorem 2.1, we have

\[
\sum_{n=2}^{\infty} \frac{\beta \beta + n - 1 + |\nu|)(1 - k) + (k - \alpha) Q(n, \beta, \ell) a_n}{(k - \alpha) + (1 - k)(\beta + |\nu|)} \leq 1.
\]

For \( 0 \leq \delta < 1 \), we need to show that

\[
\left| \frac{f'(z)^\beta}{z^{\beta-1}} - 1 \right| \leq 1 - \delta.
\]

and we have to show that

\[
\left| \frac{f'(z)^\beta}{z^{\beta-1}} - 1 \right| \leq (\beta - 1) + \sum_{n=2}^{\infty} \beta (\beta + n - 1) a_n |z|^{n-1} \leq 1 - \delta.
\]

Hence,

\[
\sum_{n=2}^{\infty} \frac{\beta (\beta + n - 1)}{(2 - \delta - \beta)} a_n |z|^{n-1} \leq 1.
\]

This is enough to consider

\[
|z|^{n-1} \leq \frac{(2 - \delta - \beta) \left[ (\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha) \right] Q(n, \beta, \ell)}{\beta \left[ (k - \alpha) + (1 - k)(\beta + |\nu|) \right]}.
\]

Therefore,

\[
|z| \leq \left\{ \frac{(2 - \delta - \beta) \left[ (\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha) \right] Q(n, \beta, \ell)}{\beta \left[ (k - \alpha) + (1 - k)(\beta + |\nu|) \right]} \right\}^{1/n-1}.
\]

Setting \( z = r_2(\alpha, \beta, k, \nu, n, \delta) \) in (3.4), we get the radius of close-to-convexity, which completes the proof of Theorem 3.2.

4. Extreme Points

The extreme points of the class \( \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \) are given by the following theorem.
Theorem 4.1. Let

\[ f_1(z)^\beta = z^\beta, \]
\[ f_n(z)^\beta = z^\beta - \beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{(\beta + n - 1)[(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)]Q(n, \beta, \ell)}z^{\beta+n-1}, \]

for \( n = 2, 3, 4, \ldots \)

Then, \( f \in \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \) if and only if it can be expressed in the form:

\[ f(z)^\beta = \sum_{n=1}^{\infty} Y_n f_n(z)^\beta, \]

where \( Y_n \geq 0 \) and

\[ \sum_{n=1}^{\infty} Y_n = 1. \]

Proof. Suppose that \( f \) can be expressed as in (4.2). Our goal is to show that \( f \in \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \). By (4.2), we have that

\[
\begin{align*}
f(z)^\beta &= \sum_{n=1}^{\infty} Y_n f_n(z)^\beta = Y_1 f_1(z)^\beta + \sum_{n=2}^{\infty} Y_n f_n(z)^\beta \\
&= Y_1 f_1(z)^\beta + \sum_{n=2}^{\infty} Y_n \left(z^\beta - \beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{(\beta + n - 1)[(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)]Q(n, \beta, \ell)}z^{\beta+n-1}\right) \\
&= \sum_{n=1}^{\infty} Y_n z^\beta - \sum_{n=2}^{\infty} \beta Y_n \left(\frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{(\beta + n - 1)[(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)]Q(n, \beta, \ell)}z^{\beta+n-1}\right)
\end{align*}
\]

Now,

\[
\sum_{n=2}^{\infty} \frac{(\beta + n - 1)[(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)]Q(n, \beta, \ell)}{(k - \alpha) + (1 - k)(\beta + |\nu|)} \times \frac{Y_n [(k - \alpha) + (1 - k)(\beta + |\nu|)]}{(\beta + n - 1)[(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)]Q(n, \beta, \ell)}
\]

\[ = \sum_{n=2}^{\infty} Y_n = 1 - Y_1 \leq 1. \]

Thus, \( f \in \mathcal{M}_m^\ell(\alpha, \beta, k, \nu) \).
Conversely, assume that \( f \in \mathcal{A}_m^\ell(\alpha, \beta, k, v) \). Since

\[
a_n \leq \frac{(k - \alpha) + (1 - k)(\beta + |v|)}{(\beta + n - 1) [(\beta + n - 1 + |v|)(1 - k) + (k - \alpha)]\mathcal{Q}(n, \beta, \ell)} a_n \quad (n \geq 2),
\]

we can set

\[
\mathcal{Y}_n = \frac{(\beta + n - 1) [(\beta + n - 1 + |v|)(1 - k) + (k - \alpha)]\mathcal{Q}(n, \beta, \ell)}{(k - \alpha) + (1 - k)(\beta + |v|)} a_n \quad (n \geq 2),
\]

\[
\mathcal{Y}_1 = 1 - \sum_{n=2}^{\infty} \mathcal{Y}_n.
\]

Then,

\[
f(z)^\beta = z^\beta - \sum_{n=2}^{\infty} \beta \mathcal{Y}_n (z^\beta - f_n(z)^\beta)
\]

\[
= z^\beta - \sum_{n=2}^{\infty} \beta \frac{\mathcal{Y}_n [(k - \alpha) + (1 - k)(\beta + |v|)]}{(\beta + n - 1) [(\beta + n - 1 + |v|)(1 - k) + (k - \alpha)]\mathcal{Q}(n, \beta, \ell)} z^{\beta+n-1}
\]

\[
= z^\beta - \sum_{n=2}^{\infty} \mathcal{Y}_n (z^\beta - f_n(z)^\beta)
\]

\[
= z^\beta \left( 1 - \sum_{n=2}^{\infty} \mathcal{Y}_n \right) + \sum_{n=2}^{\infty} \mathcal{Y}_n f_n(z)^\beta
\]

\[
= \mathcal{Y}_1 f_1(z)^\beta + \sum_{n=2}^{\infty} \mathcal{Y}_n f_n(z)^\beta
\]

\[
= \sum_{n=1}^{\infty} \mathcal{Y}_n f_n(z)^\beta.
\]

This completes the proof of Theorem 4.1.

\[
\square
\]

5. Integral Means

In order to find the integral means inequality and to verify the Silverman Conjuncture [12] for \( f \in \mathcal{A}_m^\ell(\alpha, \beta, k, v) \), we need the following definition of subordination and subordination result according to Littlewood [13].

Definition 5.1 (see [13]). Let \( f \) and \( g \) be analytic in \( U \). Then, we say that the function \( f \) is subordinate to \( g \) if there exists a Schwarz function \( w \), analytic in \( U \) with \( w(0) = 0, |w(z)| < 1 \) such that \( f(z) = g(w(z)) \) (\( z \in U \)). We denote this subordination \( f \prec g \) or \( f(z) \prec g(z) \) (\( z \in U \)). In particular, if the function \( g \) is univalent in \( U \), the above subordination is equivalent to \( f(0) = g(0), f(U) \subset g(U) \).
Lemma 5.2 (see [13]). If the functions $f$ and $g$ are analytic in $U$ with $g < f$, then

$$
\int_0^{2\pi} |g(re^{i\theta})|^\eta d\theta \leq \int_0^{2\pi} |f(re^{i\theta})|^\eta d\theta, \quad \eta > 0, \quad z = re^{i\theta}, \quad 0 < r < 1.
$$

(5.1)

Applying Theorem 2.1 with the extremal function and Lemma 5.2, we prove the following theorem.

Theorem 5.3. Let $\eta > 0$. If $f \in \mathcal{N}_m^\ell(\alpha, \beta, k, \nu)$ and \{\Phi(\alpha, \beta, k, \nu, n)\}$_{n=2}^\infty$ are nondecreasing sequences, then, for $z = re^{i\theta}$ and $0 < r < 1$, one has

$$
\int_0^{2\pi} |f(re^{i\theta})|^\eta d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\eta d\theta,
$$

(5.2)

where

$$
f_2(z)^\beta = z^\beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{\Phi(\alpha, \beta, k, \nu, 2)} z_{\beta+1},
$$

(5.3)

$$
\Phi(\alpha, \beta, k, \nu, n) = (\beta + n - 1) [(\beta + n - 1 + |\nu|)(1 - k) + (k - \alpha)] Q(n, \beta, \ell).
$$

Proof. Let $f$ of the form of (1.2) and

$$
f_2(z)^\beta = z^\beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{\Phi(\alpha, \beta, k, \nu, 2)} z_{\beta+1},
$$

(5.4)

then we must show that

$$
\int_0^{2\pi} \left| 1 - \sum_{n=2}^{\infty} \beta a_n z^{n-1} \right|^\eta d\theta \leq \int_0^{2\pi} \left| 1 - \beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{\Phi(\alpha, \beta, k, \nu, 2)} z \right|^\eta d\theta.
$$

(5.5)

By Lemma 5.2, it suffices to show that

$$
1 - \sum_{n=2}^{\infty} \beta a_n z^{n-1} < 1 - \beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{\Phi(\alpha, \beta, k, \nu, 2)} z.
$$

(5.6)

Setting

$$
1 - \sum_{n=2}^{\infty} \beta a_n z^{n-1} = 1 - \beta \frac{(k - \alpha) + (1 - k)(\beta + |\nu|)}{\Phi(\alpha, \beta, k, \nu, 2)} \omega(z),
$$

(5.7)
from (5.7) and (2.1) we obtain

\[|w(z)| = \left| \sum_{n=2}^{\infty} \frac{\Phi(\alpha, \beta, k, \nu, 2)}{(k-\alpha) + (1-k)(\beta + |\nu|)} a_n z^{n-1} \right| \leq |z| \sum_{n=2}^{\infty} \frac{\Phi(\alpha, \beta, k, \nu, n)}{(k-\alpha) + (1-k)(\beta + |\nu|)} a_n \]

(5.8)

\[\leq |z| < 1.\]

This completes the proof of Theorem 5.3.

References


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