*Research Article*

# Probabilistic (Quasi)metric Versions for a Stability Result of Baker

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By using the fixed point method, we obtain a version of a stability result of Baker in probabilistic metric and quasimetric spaces under triangular norms of Hadžić type. As an application, we prove a theorem regarding the stability of the additive Cauchy functional equation in random normed spaces.

## 1. Introduction

The use of the fixed point theory in the study of Ulam-Hyers stability was initiated by Baker in the paper [1]. Baker used the classical Banach fixed point theorem to prove the stability of the nonlinear functional equation

$$f(x) = \Phi\big(x, f(\eta(x))\big). \tag{1.1}$$

His result reads as follows.

**Theorem 1.1** (see [1, Theorem 2]). *Suppose $S$ is a nonempty set, $(X, d)$ is a complete metric space, $\eta : S \to S$, $\Phi : S \times X \to X$, $\lambda \in [0, 1)$, and*

$$d\big(\Phi(u, x), \Phi(u, y)\big) \leq \lambda d(x, y), \quad \forall u \in S, \ x, y \in X. \tag{1.2}$$

*Also, suppose that $f : S \rightarrow X$, $\delta > 0$, and*

$$d(f(u), \Phi(u, f(\eta(u)))) \leq \delta, \quad \forall u \in S. \tag{1.3}$$

*Then there exists a unique mapping $g : S \rightarrow X$ such that*

$$g(u) = \Phi(u, g(\eta(u))), \quad \forall u \in S,$$

$$d(f(u), g(u)) \leq \frac{\delta}{1 - \lambda}, \quad \forall u \in S. \tag{1.4}$$

Starting with the papers [2, 3], the fixed point method has become a fundamental tool in the study of Ulam-Hyers stability. In the probabilistic and fuzzy setting, this approach was first used in the papers [4, 5] for the case of random and fuzzy normed spaces endowed with the strongest triangular norm $T_M$. In fact, by identifying a suitable deterministic metric, the stability problem in such spaces was reduced to a fixed point theorem in generalized metric spaces. This idea was adopted by many authors, see for example, [6–11]. It is worth noting that, in applying this method, the fact that the triangular norm is $T_M$ is essential.

In this paper we study the stability of (1.1) when the unknown $f$ takes values in a probabilistic (quasi-) metric space endowed with a triangular norm of Hadžić type. To this end, we employ the fixed point theory in probabilistic metric spaces, rather than that in metric spaces.

## 2. Hyers-Ulam Stability of the Equation $f(x) = \Phi(x, f(\eta(x)))$ in Probabilistic Metric Spaces

In this section, we study the stability of the equation $f(x) = \Phi(x, f(\eta(x)))$, where the unknown function $f$ is a mapping from a nonempty set $S$ to a probabilistic metric space $(X, F, T)$, and $\Phi : S \times X \rightarrow X$ and $\eta : S \rightarrow S$ are given mappings.

We assume that the reader is familiar with the basic concepts of the theory of probabilistic metric spaces. As usual, $\Delta_+$ denotes the space of all functions $F : \mathbb{R} \rightarrow [0, 1]$, such that $F$ is left-continuous and nondecreasing on $\mathbb{R}$, $F(0) = 0$, and $D_+$ denotes the subspace of $\Delta_+$ consisting of functions $F$ with $\lim_{t \to \infty} F(t) = 1$. Here we adopt the terminology from [12], hence the probabilistic metric takes values in $\Delta_+$.

We recall some facts from the fixed point theory in probabilistic metric spaces.

*Definition 2.1.* A $t$-norm $T$ is said to be of $H$-type [13] if the family of its iterates $\{T^n\}_{n \in \mathbb{N}}$, given by $T^0(x) = 1$, and $T^n(x) = T(T^{n-1}(x), x)$ for all $n \geq 1$, is equicontinuous at $x = 1$.

A trivial example of a $t$-norm of $H$-type is the $t$-norm $T_M$, $T_M(a, b) = \text{Min}\{a, b\}$, but there exist $t$-norms of $H$-type different from Min [14].

The theorem below provides a characterization of continuous $t$-norms of $H$-type.

**Proposition 2.2** (see [15]). (i) *Suppose that there exists a strictly increasing sequence $(b_n)_{n \in \mathbb{N}}$ in $[0, 1)$ such that $\lim_{n \to \infty} b_n = 1$ and $T(b_n, b_n) = b_n$. Then $T$ is of $H$-type.*

(ii) *Conversely, if $T$ is continuous and of $H$-type, then there exists a sequence as in* (i).

*Definition 2.3* (see [16]). Let $(X, F, T)$ be a probabilistic metric space. A mapping $f : X \to X$ is said to be a Sehgal contraction (or *B*-contraction) if the following relation holds:

$$F_{f(p)f(q)}(kt) \geq F_{pq}(t), \quad (p, q \in X, \ t > 0). \tag{2.1}$$

**Theorem 2.4** (see [17]). *Let $(X, F, T)$ be a complete probabilistic metric space with $T$ of Hadžić-type and $f : X \to X$ be a B-contraction. Then $f$ has a fixed point if and only if there is $p \in X$ such that $F_{pf(p)} \in D_+$. If $F_{pf(p)} \in D_+$, then $p^* := \lim_{n \to \infty} f^n(p)$ is the unique fixed point of $f$ in the set $Y = \{q \in X : F_{pq} \in D_+\}$.*

The following lemma completes Theorem 2.4 with an estimation relation, in the case $T = T_M$.

**Lemma 2.5** (see [18]). *Let $(X, F, T_M)$ be a complete probabilistic metric space and $f : X \to X$ be a $k - B$ contraction. Suppose that $F_{pf(p)} \in D_+$ and let $p^* = \lim_{n \to \infty} f^n(p)$. Then*

$$F_{pp^*}(t + 0) \geq F_{pf(p)}((1 - k)t), \quad \forall t > 0. \tag{2.2}$$

This lemma can be extended to the case of probabilistic metric spaces under a continuous *t*-norm of *H*-type.

**Lemma 2.6.** *Let $(X, F, T)$ be a complete probabilistic metric space, with $T$ a continuous t-norm of H-type and $(b_n)_n$ be a strictly increasing sequence of idempotents of $T$. Suppose $f : X \to X$ is a B-contraction with Lipschitz constant $k \in (0, 1)$. If there exists $p \in X$ such that $F_{pf(p)} \in D_+$, then $p^* = \lim_{n \to \infty} f^n(p)$ is the unique fixed point of $f$ in the set*

$$\{q \in X : F_{pq} \in D_+\}. \tag{2.3}$$

*Moreover, if $t > 0$ is so that $F_{pf(p)}((1 - k)t) \geq b_n$, then $F_{pp^*}(t + 0) \geq b_n$.*

*Proof.* We have to prove only the last part of the theorem. We show by induction on $m$ that $F_{pf(p)}((1 - k)s) \geq b_n$ implies $F_{pf^m(p)}(s) \geq b_n$, for all $m \geq 1$.

The case $m = 1$ is obvious. Now, suppose that $F_{pf^m(p)}(s) \geq b_n$. Then

$$
\begin{aligned}
F_{pf^{m+1}(p)}(s) &\geq T\big(F_{pf(p)}((1 - k)s), F_{f(p)f^{m+1}(p)}(ks)\big) \\
&\geq T\big(F_{pf(p)}((1 - k)s), F_{pf^m(p)}(s)\big) \\
&\geq T(b_n, b_n) = b_n.
\end{aligned}
\tag{2.4}
$$

Let $t > 0$ be such that $F_{pf(p)}((1 - k)t) \geq b_n$, and let $s > 0$. Then

$$F_{pp^*}(t + s) \geq T\big(F_{pf^m(p)}(t), F_{f^m(p)p^*}(s)\big) \geq T\big(b_n, F_{f^m(p)p^*}(s)\big), \tag{2.5}$$

for all $m \geq 1$. Since $(f^m(p))$ converges to $p^*$, $F_{f^m(p)p^*}(s)$ goes to 1 as $m$ tends to infinity, so

$$F_{pp^*}(t + s) \geq T(b_n, 1) = b_n. \tag{2.6}$$

By taking $s \to 0$ we obtain

$$F_{pp^*}(t+0) \geq b_n. \tag{2.7}$$

$\square$

In order to state our first stability result, we define an appropriate concept of approximate solution for the functional equation (1.1).

*Definition 2.7.* A probabilistic uniform approximate solution of (1.1) is a function $f : S \to X$ with the property that

$$\lim_{t \to \infty} F_{f(u)\Phi(u,f(\eta(u)))}(t) = 1 \tag{2.8}$$

uniformly on $S$.

*Example 2.8.* Let $(X, d)$ be a metric space, and let $F : X \times X \to D_+$ be defined by

$$F_{xy}(t) = \frac{t}{t + d(x, y)} \quad (x, y \in X, \ t \geq 0). \tag{2.9}$$

Then $(X, F, T_M)$ is a probabilistic metric space (the induced probabilistic metric space). One can easily verify that $f$ is a probabilistic uniform approximate solution of (1.1) if and only if it satisfies relation (1.3), thus being an approximate solution in the sense of Theorem 1.1.

**Theorem 2.9.** *Let $S$ be a nonempty set, $(X, F, T)$ be a complete probabilistic metric space, with $T$ a continuous t-norm of $H$-type, and $(b_n)_n$ be a strictly increasing sequence of idempotents of $T$. Suppose $\Phi : S \times X \to X$ is a mapping for which there exists $k \in (0, 1)$ with*

$$F_{\Phi(u,x)\Phi(u,y)}(kt) \geq F_{xy}(t), \tag{2.10}$$

*for all $u \in S$, $x, y \in X$ and $t > 0$.*

   *If $f : S \to X$ is a probabilistic uniform approximate solution of (1.1), then there exists a function $a : S \to X$ which is an exact solution of (1.1), with the property that, if $t > 0$ is such that*

$$F_{f(u)\Phi(u,f(\eta(u)))}(t) > b_n, \quad \forall u \in S, \tag{2.11}$$

*then*

$$F_{f(u)a(u)}\left(\frac{t}{1-k} + 0\right) \geq b_n, \quad \forall u \in S. \tag{2.12}$$

*Proof.* Denote by $Y$ the set of all mappings $g : S \to X$, and let $J : Y \to Y$ be Baker's operator, given by $J(g)(u) = \Phi(u, g(\eta(u)))$ for all $g \in Y$, $u \in S$. We define the distribution function $\mathcal{F}_{gh}$ by

$$\mathcal{F}_{gh}(t) = \sup_{s<t} \inf_{u \in S} F_{g(u)h(u)}(s), \tag{2.13}$$

for all $g, h \in Y$.

The assumptions on the space $(X, F, T)$ ensure that $(Y, \mathcal{F}, T)$ is a complete probabilistic metric space. Also,

$$\mathcal{F}_{J(g)J(h)}(kt) = \sup_{s<kt} \inf_{u \in S} F_{J(g)(u)J(h)(u)}(s) = \sup_{s<t} \inf_{u \in S} F_{J(g)(u)J(h)(u)}(ks)$$

$$\geq \sup_{s<t} \inf_{u \in S} F_{g(\eta(u))h(\eta(u))}(s) \geq \mathcal{F}_{gh}(t), \tag{2.14}$$

that is, $J$ is a Sehgal contraction on $(Y, \mathcal{F}, T)$.

Moreover, the relation $\lim_{t \to \infty} F_{f(u)\Phi(u, f(\eta(u)))}(t) = 1$, uniformly on $X$ implies

$$\mathcal{F}_{fJ(f)} \in D_+. \tag{2.15}$$

Now we can apply Lemma 2.6 to obtain a fixed point of $J$, that is a mapping $a : S \to X$ which is a solution of (1.1), with $a(u) = \lim_{n \to \infty} J^n f(u)$ for all $u \in S$.

Next, let $t > 0$ be such that $F_{f(u)\Phi(u, f(\eta(u)))}(t) > b_n$ for all $u \in S$. Then, from the left continuity of $F$, it follows that $F_{f(u)\Phi(u, f(\eta(u)))}(s_0) > b_n (u \in S)$, for some $s_0 \in (0, t)$. Therefore $\inf_{u \in S} F_{f(u)\Phi(u, f(\eta(u)))}(s_0) \geq b_n$, so $\mathcal{F}_{fJ(f)}(t) \geq b_n$. By Lemma 2.6, $\mathcal{F}_{fa}(t/(1-k)+0) \geq b_n$, whence we conclude that the estimation (2.12) holds. $\qquad\square$

*Remark 2.10.* The result of Baker [1] can be obtained as a particular case of Theorem 2.9, by considering in this theorem the induced probabilistic metric space (see Example 2.8).

From Theorem 2.9 one can derive a stability result for the Cauchy additive functional equation

$$f(x + y) = f(x) + f(y) \tag{2.16}$$

in random normed spaces.

Recall (see [12]) that a random normed space ($RN$-space) is a triple $(X, \nu, T)$, where $X$ is a real linear space, $\nu$ is a mapping from $X$ to $D_+$, and $T$ is a $t$-norm, satisfying the following conditions ($\nu(x)$ will be denoted by $\nu_x$):

   (i) $\nu_x(t) = 1$ for all $t > 0$ iff $x = \theta$, the null vector of $X$;

   (ii) $\nu_{\alpha x}(t) = \nu_x(t/|\alpha|)$, for all $\alpha \in \mathbb{R}$, $\alpha \neq 0$, and all $x \in X$;

   (iii) $\nu_{x+y}(t + s) \geq T(\nu_x(t), \nu_y(s))$, for all $x, y \in X$ and all $t, s > 0$.

*Definition 2.11.* A probabilistic uniform approximate solution of (2.16) is a function $f : S \to X$ with the property that

$$\lim_{t \to \infty} \nu_{f(u+v)-f(u)-f(v)}(t) = 1 \tag{2.17}$$

uniformly on $S \times S$.

**Theorem 2.12.** *Let $S$ be a real linear space, $(X, \nu, T)$ be a complete RN-space with $T$—a continuous t-norm of H-type, and $(b_n)_n$ be a strictly increasing sequence of idempotents of $T$.*

*If $f : S \to X$ is a probabilistic uniform approximate solution of (2.16), then there exists a mapping $a : S \to X$ which is an exact solution of (2.16), with the property that, if $t > 0$ is such that*

$$\nu_{f(u)-f(2u)/2}(t) > b_n, \quad \forall u \in S, \tag{2.18}$$

*then*

$$\nu_{f(u)-a(u)}(2t + 0) \geq b_n, \quad \forall u \in S. \tag{2.19}$$

*Proof.* We apply Theorem 2.9 for $\Phi : S \times X \to X$, $\Phi(u, x) = x/2$, and $\eta : S \to S$, $\eta(u) = 2u$ in the probabilistic metric space $(X, F, T)$ with $F$ defined by

$$F_{xy}(t) = \nu_{x-y}(t) \tag{2.20}$$

for all $x, y \in X$, $t > 0$. Note that $F$ satisfies (2.10) for $k = 1/2$, since

$$F_{\Phi(u,x)\Phi(u,y)}\left(\frac{t}{2}\right) = F_{(x/2)(y/2)}\left(\frac{t}{2}\right) = \nu_{(1/2)(x-y)}\left(\frac{t}{2}\right) = \nu_{x-y}(t) = F_{xy}(t), \tag{2.21}$$

for all $u \in S$, $x, y \in X$ and $t > 0$.

It is easy to see that $f$ is a probabilistic uniform approximate solution of (1.1), so there exists an exact solution of (1.1), that is, a mapping $a : S \to X$ satisfying $a(u) = (1/2)a(2u)$ for all $u \in S$. The estimation (2.19) can be immediately derived from the corresponding one in Theorem 2.9.

It remains to show that $a$ is additive. This follows from the fact that $a(u) = \lim_{n \to \infty}(1/2^n)f(2^n u)$, for all $u \in S$, and $f$ is a probabilistic uniform approximate solution of (2.16). Namely, for all $t > 0$,

$$
\begin{aligned}
\mathcal{V}_{a(u+v)-a(u)-a(v)}(t) &\geq T\Bigg(\mathcal{V}_{a(u+v)-f(2^n(u+v))/2^n}\left(\frac{t}{4}\right), \mathcal{V}_{a(u)-f(2^n u)/2^n}\left(\frac{t}{4}\right), \\
&\qquad \mathcal{V}_{a(v)-f(2^n v)/2^n}\left(\frac{t}{4}\right), \mathcal{V}_{f(2^n(u+v))/2^n-f(2^n u)/2^n-f(2^n v)/2^n}\left(\frac{t}{4}\right)\Bigg) \\
&\geq T\Bigg(\mathcal{V}_{a(u+v)-f(2^n(u+v))/2^n}\left(\frac{t}{4}\right), \mathcal{V}_{a(u)-f(2^n u)/2^n}\left(\frac{t}{4}\right), \\
&\qquad \mathcal{V}_{a(v)-f(2^n v)/2^n}\left(\frac{t}{4}\right), \mathcal{V}_{f(2^n(u+v))-f(2^n u)-f(2^n v)}\left(\frac{2^n t}{4}\right)\Bigg) \xrightarrow{n \to \infty} 1,
\end{aligned}
\tag{2.22}
$$

implying $a(u + v) = a(u) + a(v)$ for all $u, v \in S$.  □

## 3. Hyers-Ulam Stability of the Equation $f(x) = \Phi(x, f(\eta(x)))$ in Probabilistic Quasimetric Spaces

The defining feature of quasimetric structures is the absence of symmetry. This allows one to consider different notions of convergence and completeness. We state the terminology and notations, following [19] (also see [20]).

*Definition 3.1.* A probabilistic quasimetric space is a triple $(X, P, T)$, where $X$ is a nonempty set, $T$ is a $t$-norm, and $P : X \times X \to \Delta_+$ is a mapping satisfying

(i) $P_{xy} = P_{yx} = \varepsilon_0$ if and only if $x = y$;

(ii) $P_{xy}(t + s) \geq T(P_{xz}(t), P_{zy}(s))$, for all $x, y, z \in X$, for all $t, s > 0$.

We note that if $P$ verifies the symmetry assumption $P_{xy} = P_{yx}$, for all $x, y \in X$, then $(X, P, T)$ is a probabilistic metric space.

If $(X, P, T)$ is a probabilistic quasimetric space, then the mapping $Q : X^2 \to \Delta_+$ defined by $Q_{xy} = P_{yx}$ for all $x, y \in X$ is called the conjugate probabilistic quasimetric of $P$.

*Definition 3.2.* Let $(X, P, T)$ be a probabilistic quasimetric space. A sequence $(x_n)_n$ in $X$ is said to be:

(i) right $K$-Cauchy (left $K$-Cauchy) if, for each $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists $k \in \mathcal{N}$ so that, for all $m \geq n \geq k$, $P_{x_n x_m}(\varepsilon) > 1 - \lambda$ $(Q_{x_n x_m}(\varepsilon) > 1 - \lambda$ resp.$)$;

(ii) $P$-convergent ($Q$-convergent) to $x \in X$ if, for each $\varepsilon > 0$ and $\lambda \in (0, 1)$, there exists $k \in \mathcal{N}$ so that $P_{xx_n}(\varepsilon) > 1 - \lambda$ $(Q_{xx_n}(\varepsilon) > 1 - \lambda)$, for all $n \geq k$.

*Definition 3.3.* Let $A \in \{\text{right } K, \text{left } K\}$ and $B \in \{P, Q\}$. The space $(X, P, T)$ is $(A-B)$ complete if every $A$-Cauchy sequence is $B$ convergent.

*Definition 3.4.* The probabilistic quasimetric space $(X, P, T)$ has the $L$-$US$ ($R$-$US$) property if every $P$-($Q$-) convergent sequence has a unique limit.

The following lemma is a quasimetric analogue of Lemma 2.6.

**Lemma 3.5.** *Let $(X, P, T)$ be a (right $K-Q$)-complete probabilistic quasimetric space with the R-US property, where $T$ is a continuous t-norm of H-type. Let $(b_n)_n$ be a strictly increasing sequence of idempotents of $T$.*

*Suppose $f : X \to X$ is a Sehgal contraction with Lipschitz constant $k \in (0, 1)$, and $p$ is an element of $X$ such that $P_{pf(p)} \in D_+$. Then $p^* := \lim_{n \to \infty} f^n(p)$ is a fixed point of $f$ and if $t > 0$ is so that $P_{pf(p)}((1 - k)t) \geq b_n$, then $P_{pp^*}(t + 0) \geq b_n$.*

*Proof.* We proceed in the classical manner to show that the sequence of iterates $(f^n(p))_n$ is right $K$-Cauchy, therefore it is $Q$-convergent to $p^* \in X$. The fact that $p^*$ is a fixed point of f is a consequence of the *R-US* property of the space $X$. Next, as in the proof of **Lemma 2.6** we show by induction on $m$ that $P_{pf(p)}((1 - k)s) \geq b_n$ implies $P_{pf^m(p)}(s) \geq b_n$, for all $m \geq 1$.

Let $t > 0$ be such that $P_{pf(p)}((1 - k)t) \geq b_n$, and let $s > 0$. Then

$$P_{pp^*}(t + s) \geq T\left(P_{pf^m(p)}(t), P_{f^m(p)p^*}(s)\right) \geq T\left(b_n, P_{f^m(p)p^*}(s)\right), \tag{3.1}$$

for all $m \geq 1$. Since $(f^m(p))$ is $Q$-convergent to $p^*$, $P_{f^m(p)p^*}(s)$ goes to 1 as $m$ tends to infinity, so

$$P_{pp^*}(t + s) \geq T(b_n, 1) = b_n. \tag{3.2}$$

By taking $s \to 0$ we obtain

$$P_{pp^*}(t + 0) \geq b_n. \tag{3.3}$$

$\square$

The probabilistic quasimetric version of Baker's theorem can be stated as follows.

**Theorem 3.6.** *Let $S$ be a nonempty set, $(X, P, T)$ be a (right $K-Q$)-complete probabilistic quasimetric space with the R-US property, with $T$ a continuous t-norm of H-type, and $(b_n)_n$ be a strictly increasing sequence of idempotents of $T$. Suppose $\Phi : S \times X \to X$ is a mapping for which there exists $k \in (0, 1)$ with*

$$P_{\Phi(u,x)\Phi(u,y)}(kt) \geq P_{xy}(t), \tag{3.4}$$

*for all $u \in S$, $x, y \in X$ and $t > 0$.*

*If $f : S \to X$ is a probabilistic uniform approximate solution of (1.1), then there exists a function $a : S \to X$ which is an exact solution of (1.1), with the property that, if $t > 0$ is such that*

$$P_{f(u)\Phi(u,f(\eta(u)))}(t) > b_n, \quad \forall u \in S, \tag{3.5}$$

*then*

$$P_{f(u)a(u)}\left(\frac{t}{1 - k} + 0\right) \geq b_n, \quad \forall u \in S. \tag{3.6}$$

*Proof.* We only sketch the proof, as it is very similar to that of Theorem 2.9.

As in the mentioned proof, denote by $Y$ the set of all mappings $g : S \to X$, and define the distribution function $F_{gh}$ by

$$F_{gh}(t) = \sup_{s<t} \inf_{u \in S} P_{g(u)h(u)}(s), \tag{3.7}$$

for all $g, h \in Y$ and Baker's operator $J : Y \to Y$, $J(g)(u) = \Phi(u, g(\eta(u)))$ for all $g \in Y$, $u \in S$.

The assumptions on the space $(X, P, T)$ ensure that $(Y, F, T)$ is a (right $K-Q$)-complete probabilistic quasimetric space with the $R$-$US$ property and that $J$ is a Sehgal contraction on $(Y, F, T)$, and the relation $\lim_{t \to \infty} P_{f(u)\Phi(u,f(\eta(u)))}(t) = 1$, uniformly on $X$ implies

$$F_{fJ(f)} \in D_+. \tag{3.8}$$

We can now apply Lemma 3.5 to obtain a mapping $a : S \to X$ which is a solution of (1.1), with $a(u) = \lim_{n \to \infty} J^n f(u)$ for all $u \in S$.

The estimation (3.6) follows by using the left continuity of $P$, as in the proof of Theorem 2.9. $\qquad\square$

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