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Research Article

Fuzzy Soft Multiset Theory

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In 1999 Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty. Alkhazaleh et al. in 2011 introduced the definition of a soft multiset as a generalization of Molodtsov's soft set. In this paper we give the definition of fuzzy soft multiset as a combination of soft multiset and fuzzy set and study its properties and operations. We give examples for these concepts. Basic properties of the operations are also given. An application of this theory in decision-making problems is shown.

1. Introduction

Most of the problems in engineering, medical science, economics, environments, and so forth have various uncertainties. Molodtsov [1] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. After Molodtsov's work, some different operations and application of soft sets were studied by Chen et al. [2] and Maji et al. [3, 4]. Furthermore, Maji et al. [5] presented the definition of fuzzy soft set and Roy and Maji [6] presented the applications of this notion to decision-making problems. Alkhazaleh et al. [7] generalized the concept of fuzzy soft set to possibility fuzzy soft set and they gave some applications of this concept in decision making and medical diagnosis. They also introduced the concept of fuzzy parameterized interval-valued fuzzy soft set [8], where the mapping is defined from the fuzzy set parameters to the interval-valued fuzzy subsets of the universal set and gave an application of this concept in decision making. In 2012 Alkhazaleh and Salleh [9] introduced the concept of generalised interval-valued fuzzy soft set and studied its properties and application. Alkhazaleh and Salleh [10] introduced the concept of soft expert sets where the user can know the opinion of all experts in one model and gave an application of this concept in decision-making problem. Salleh et al. in 2012 [11] introduced and studied the

concept of multiparameterized soft set as a generalization of Molodtsov's soft set. Alkhazaleh et al. [12] as a generalization of Molodtsov's soft set, presented the definition of a soft multiset and its basic operations such as complement, union, and intersection. Salleh and Alkhazaleh [13] studied the application of soft multiset in decision making problem. In 2011 Salleh gave a brief survey from soft sets to intuitionistic fuzzy soft sets [14]. In this paper we give the definition of a fuzzy soft multiset, a more general concept, which is a combination of fuzzy set and soft multiset and studied its properties. We also introduce its basic operations, namely, complement, union, and intersection, and their properties. An application of this theory in a decision-making problem is given.

2. Preliminaries

In this section, we recall some basic notions in soft set theory, soft multiset theory, and fuzzy soft set. Molodtsov defined soft set in the following way. Let U be a universe and E a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2.1 (see [1]). A pair (F, A) is called a soft set over U, where F is a mapping

$$F : A \longrightarrow P(U). \tag{2.1}$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 2.2 (see [5]). Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by

$$F : A \to I^U. \tag{2.2}$$

All the following definitions are due to Alkhazaleh and Salleh [12].

Definition 2.3. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a soft multiset over U, where F is a mapping given by $F : A \to U$.

In other words, a soft multiset over U is a parameterized family of subsets of U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft multiset (F, A). Based on the above definition, any change in the order of universes will produce a different soft multiset.

Definition 2.4. For any soft multiset (F, A), a pair $(e_{U_i,j}, F_{e_{U_i,j}})$ is called a U_i-soft multiset part $\forall e_{U_i,j} \in a_k$ and $F_{e_{U_i,j}} \subseteq F(A)$ is an approximate value set, where $a_k \in A, k = \{1, 2, \ldots, n\}$, $i \in \{1, 2, \ldots, m\}$, and $j \in \{1, 2, \ldots, r\}$.

Definition 2.5. For two soft multisets (F, A) and (G, B) over U, (F, A) is called a soft multisubset of (G, B) if

(1) $A \subseteq B$ *and*

(2) $\forall e_{U_i,j} \in a_k, (e_{U_i,j}, F_{e_{U_i,j}}) \subseteq (e_{U_i,j}, G_{e_{U_i,j}})$,

where $a_k \in A, k = \{1, 2, \ldots, n\}, i \in \{1, 2, \ldots, m\}$, and $j \in \{1, 2, \ldots, r\}$.

This relationship is denoted by $(F, A)\tilde{\subseteq}(G, B)$. In this case (G, B) is called a soft multisuperset of (F, A).

Definition 2.6. Two soft multisets (F, A) and (G, B) over U are said to be equal if (F, A) is a soft multisubset of (G, B) and (G, B) is a soft multisubset of (F, A).

Definition 2.7. Let $E = \prod_{i=1}^{m} E_{U_i}$, where E_{U_i} is a set of parameters. The NOT set of E denoted by $\rceil E$ is defined by

$$\rceil E = \prod_{i=1}^{m} \rceil E_{U_i}, \tag{2.3}$$

where $\rceil E_{U_i} = \{\rceil e_{U_i,j} = \text{not } e_{U_i,j}, \forall i, j\}$.

Definition 2.8. The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to U$ is a mapping given by $F^c(\alpha) = U - F(\rceil \alpha), \forall \alpha \in \rceil A$.

Definition 2.9. A soft multiset (F, A) over U is called a seminull soft multiset, denoted by $(F, A)_{\approx \Phi_i}$, if at least one of the soft multiset parts of (F, A) equals $\emptyset$.

Definition 2.10. A soft multiset (F, A) over U is called a null soft multiset, denoted by $(F, A)_{\Phi}$, if all the soft multiset parts of (F, A) equal $\emptyset$.

Definition 2.11. A soft multiset (F, A) over U is called a semiabsolute soft multiset, denoted by $(F, A)_{\approx A_i}$ if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i$ for at least one i, $a_k \in A$, $a_k \in A$, $k = \{1, 2, \ldots, n\}$, $i \in \{1, 2, \ldots, m\}$, and $j \in \{1, 2, \ldots, r\}$.

Definition 2.12. A soft multiset (F, A) over U is called an absolute soft multiset, denoted by $(F, A)_A$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i, \forall i$.

Definition 2.13. The union of two soft multisets (F, A) and (G, B) over U, denoted by $(F, A)\tilde{\cup}(G, B)$, is the soft multiset (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B, \\ G(\varepsilon), & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.4}$$

Definition 2.14. The intersection of two soft multisets (F, A) and (G, B) over U, denoted by $(F, A)\widetilde{\cap}(G, B)$, is the soft multiset (H, C) where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B, \\ G(\varepsilon), & \text{if } \varepsilon \in B - A, \\ F(\varepsilon) \cap G(\varepsilon), & \text{if } \varepsilon \in A \cap B. \end{cases} \tag{2.5}$$

3. Fuzzy Soft Multiset

In this section, we introduce the definition of a fuzzy soft multiset, and its basic operations such as complement, union, and intersection. We give examples for these concepts. Basic properties of the operations are also given.

Definition 3.1. Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a fuzzy soft multiset over U, where F is a mapping given by $F : A \to U$.

In other words, a fuzzy soft multiset over U is a parameterized family of fuzzy subsets of U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the fuzzy soft multiset (F, A). Based on the above definition, any change in the order of universes will produce a different fuzzy soft multiset.

Example 3.2. Suppose that there are three universes U_1, U_2, and U_3. Suppose that Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let us consider a fuzzy soft multiset (F, A) which describes "houses," "cars," and "hotels" that Mr. X is considering for accommodation purchase, transportation purchase, and a venue to hold a wedding celebration, respectively. Let $U_1 = \{h_1, h_2, h_3, h_4, h_5\}$, $U_2 = \{c_1, c_2, c_3, c_4\}$ and $U_3 = \{v_1, v_2, v_3\}$.

Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$\begin{aligned} E_{U_1} &= \{e_{U_1,1} = \text{expensive},\ e_{U_1,2} = \text{cheap},\ e_{U_1,3} = \text{wooden},\ e_{U_1,4} = \text{in green surroundings}\}, \\ E_{U_2} &= \{e_{U_2,1} = \text{expensive},\ e_{U_2,2} = \text{cheap},\ e_{U_2,3} = \text{sporty}\}, \\ E_{U_3} &= \{e_{U_3,1} = \text{expensive},\ e_{U_3,2} = \text{cheap},\ e_{U_3,3} = \text{in Kuala Lumpur},\ e_{U_3,4} = \text{majestic}\}. \end{aligned} \tag{3.1}$$

Let $U = \prod_{i=1}^{3} FS(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$, such that

$$\begin{aligned} A = \{&a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),\ a_3 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,1}), \\ &a_4 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,2}),\ a_5 = (e_{U_1,4}, e_{U_2,2}, e_{U_3,2}),\ a_6 = (e_{U_1,2}, e_{U_2,2}, e_{U_3,2})\}. \end{aligned} \tag{3.2}$$

Suppose that

$$\begin{aligned}
F(a_1) &= \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right),\\
F(a_2) &= \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.4}, \frac{c_2}{0.5}, \frac{c_3}{0.8}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.4}, \frac{v_2}{0.4}, \frac{v_3}{0.3}\right\}\right),\\
F(a_3) &= \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right),\\
F(a_4) &= \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0.2}, \frac{c_3}{0.7}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right),\\
F(a_5) &= \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.7}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right),\\
F(a_6) &= \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.6}\right\}\right).
\end{aligned} \tag{3.3}$$

Then we can view the fuzzy soft multiset (F, A) as consisting of the following collection of approximations:

$$\begin{aligned}
(F, A) = \Bigg\{ &\left(a_1, \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_2, \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.4}, \frac{c_2}{0.5}, \frac{c_3}{0.8}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.4}, \frac{v_2}{0.4}, \frac{v_3}{0.3}\right\}\right)\right),\\
&\left(a_3, \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right),\\
&\left(a_4, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0.2}, \frac{c_3}{0.7}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right),\\
&\left(a_5, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.7}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_6, \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}, \right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.6}\right\}\right)\right)\Bigg\}.
\end{aligned} \tag{3.4}$$

Each approximation has two parts: a predicate and an approximate value set.

We can logically explain the above example as follows: we know that $a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1})$ where $e_{U_1,1}$ = expensive house, $e_{U_2,1}$ = expensive car, and $e_{U_3,1}$ = expensive venue. Then,

$$F(a_1) = \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right). \tag{3.5}$$

We can see that the membership value for house h_1 is 0.2, so this house is not expensive for Mr. X; also we can see that the membership value for house h_3 is 0.8, this means that the house h_3 is expensive, and since the membership value for house h_5 is 0, then this house is absolutely not expensive. Now, since the first set is concerning expensive houses, then we can explain the second set as follows: the membership value for car c_1 is 0.8, so this car is expensive (this car maybe not expensive if the first set is concerning cheap houses), also we can see that the membership value for car c_3 is 0.4, this means that this car is not so expensive for him, and since the membership value for car c_4 is 0.6, then this car is quite expensive. Now, since the first set is concerning expensive houses and the second set is concerning expensive cars, then we can also explain the third set as follows: since the membership value for venue v_1 is 0.8, so this venue is expensive (this venue maybe not expensive if the first set is concerning cheap houses or/and the second set is concerning cheap cars), also we can see that the membership value for venue v_2 and v_3 is 0.7, this means that this venue is almost expensive. So depending on the previous explanation we can say the following.

If $\{h_1/0.2, h_2/0.4, h_3/0.8, h_4/0.5, h_5/0\}$ is the fuzzy set of expensive houses, then the fuzzy set of relatively expensive cars is $\{c_1/0.8, c_2/0.5, c_3/0.4, c_4/0.6\}$, and if $\{h_1/0.2, h_2/0.4, h_3/0.8, h_4/0.5, h_5/0\}$ is the fuzzy set of expensive houses and $\{c_1/0.8, c_2/0.5, c_3/0.4, c_4/0.6\}$ is the fuzzy set of relatively expensive cars, then the fuzzy set of relatively expensive hotels is $\{v_1/0.8, v_2/0.7, v_3/0.7\}$. It is clear that the relation in fuzzy soft multiset is a conditional relation.

Definition 3.3. For any fuzzy soft multiset (F, A), a pair $(e_{U_i,j}, F_{e_{U_i,j}})$ is called a U_i-fuzzy soft multiset part $\forall e_{U_i,j} \in a_k$ and $F_{e_{U_i,j}} \subseteq F(A)$ is a fuzzy approximate value set, where $a_k \in A$, $k = \{1, 2, \ldots, n\}$, $i \in \{1, 2, \ldots, m\}$, and $j \in \{1, 2, \ldots, r\}$.

Example 3.4. Consider Example 3.2. Then

$$\begin{aligned}\left(e_{U_1,j}, F_{e_{U_1,j}}\right) = \Bigg\{ &\left(e_{U_1,1}, \left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}\right), \left(e_{U_1,1}, \left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}\right), \\ &\left(e_{U_1,2}, \left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}\right), \left(e_{U_1,4}, \left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}\right), \\ &\left(e_{U_1,4}, \left\{\frac{h_1}{0.9}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.2}, \frac{h_5}{0.7}\right\}\right), \left(e_{U_1,2}, \left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.8}, \frac{h_5}{0.7}\right\}\right)\Bigg\}\end{aligned} \tag{3.6}$$

is a U_1-fuzzy soft multiset part of (F, A).

Definition 3.5. For two fuzzy soft multisets (F, A) and (G, B) over U, (F, A) is called a fuzzy soft multisubset of (G, B) if

(a) $A \subseteq B$ and

(b) $\forall e_{U_i,j} \in a_k$, $(e_{U_i,j}, F_{e_{U_i,j}})$ is a fuzzy subset of $(e_{U_i,j}, G_{e_{U_i,j}})$,

where $a_k \in A, k = \{1, 2, \ldots, n\}, i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, r\}$.

This relationship is denoted by $(F,A)\tilde{\subseteq}(G,B)$. In this case (G,B) is called a fuzzy soft multisuperset of (F,A).

Definition 3.6. Two fuzzy soft multisets (F,A) and (G,B) over U are said to be equal if (F,A) is a fuzzy soft multisubset of (G,B) and (G,B) is a fuzzy soft multisubset of (F,A).

Example 3.7. Consider Example 3.2. Let

$$\begin{aligned} A = \{&a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),\ a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),\\ &a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\},\\ B = \{&b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),\ b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),\\ &b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),\ b_5 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),\ b_6 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\}. \end{aligned} \tag{3.7}$$

Clearly $A \subseteq B$. Let (F,A) and (G,B) be two fuzzy soft multisets over the same U such that

$$\begin{aligned} (F,A) = \Bigg\{&\left(a_1, \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right),\\ &\left(a_2, \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.8}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right),\\ &\left(a_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.7}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right),\\ &\left(a_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}, \frac{c_4}{0.2}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right)\Bigg\},\\ (G,B) = \Bigg\{&\left(b_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}, \frac{h_5}{0.2}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.6}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.9}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right),\\ &\left(b_2, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.9}, \frac{c_3}{0.9}, \frac{c_4}{0.8}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right),\\ &\left(b_3, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{1}, \frac{h_4}{0.8}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.7}, \frac{v_2}{0.6}, \frac{v_3}{0.5}\right\}\right)\right),\\ &\left(b_4, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}, \frac{h_5}{0.8}\right\}, \left\{\frac{c_1}{0.3}, \frac{c_2}{0.5}, \frac{c_3}{0.7}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right),\\ &\left(b_5, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.9}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),\\ &\left(b_6, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.7}, \frac{c_3}{0.6}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.9}, \frac{v_3}{0.9}\right\}\right)\right)\Bigg\}. \end{aligned} \tag{3.8}$$

Therefore, $(F,A)\tilde{\subseteq}(G,B)$.

Definition 3.8. The complement of a fuzzy soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to U$ is a mapping given by $F^c(\alpha) = c\ (F(\alpha)), \forall \alpha \in A$ where c is any fuzzy complement.

Example 3.9. Consider Example 3.2. By using the basic fuzzy complement which is $c(x) = 1 - x$, we have

$$\begin{aligned}
(F, A)^c = \{&(a_1, (F(a_1))), (a_2, (F(a_2))), (a_3, (F(a_3))), (a_4, (F(a_4)))\\
&(a_5, (F(a_5))), (a_6, (F(a_6)))\}\\
= \Bigg\{&\left(a_1, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.2}, \frac{h_4}{0.5}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.2}, \frac{c_2}{0.5}, \frac{c_3}{0.6}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.2}, \frac{v_2}{0.3}, \frac{v_3}{0.3}\right\}\right)\right),\\
&\left(a_2, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.2}, \frac{h_4}{0.5}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.6}, \frac{c_2}{0.5}, \frac{c_3}{0.2}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.6}, \frac{v_2}{0.6}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_3, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.3}, \frac{h_3}{0.9}, \frac{h_4}{0.2}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{0.2}, \frac{c_2}{0.4}, \frac{c_3}{0.7}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),\\
&\left(a_4, \left(\left\{\frac{h_1}{0.1}, \frac{h_2}{0.5}, \frac{h_3}{0.5}, \frac{h_4}{0.8}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.8}, \frac{c_3}{0.3}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.2}, \frac{v_2}{0.3}, \frac{v_3}{0.1}\right\}\right)\right),\\
&\left(a_5, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0.2}, \frac{h_3}{0.3}, \frac{h_4}{1}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{0.3}, \frac{c_2}{0.2}, \frac{c_3}{0.5}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.3}\right\}\right)\right),\\
&\left(a_6, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.3}, \frac{h_3}{0.9}, \frac{h_4}{0.2}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{0.2}, \frac{c_2}{0.4}, \frac{c_3}{0.7}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.2}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right)\Bigg\}.
\end{aligned} \tag{3.9}$$

Definition 3.10. A fuzzy soft multiset (F, A) over U is called a seminull fuzzy soft multiset, denoted by $(F, A)_{\approx\Phi_i}$, if at least one of a fuzzy soft multiset parts of (F, A) equals $\emptyset$.

Example 3.11. Consider Example 3.2. Let us consider a fuzzy soft multiset (F, A) which describes "stone houses," "cars," and "hotels" with

$$A = \{a_1 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}),\ a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3})\}. \tag{3.10}$$

Then a seminull fuzzy soft multiset $(F, A)_{\approx\Phi_1}$ is given as

$$\begin{aligned}
(F, A)_{\approx\Phi_1} = \Bigg\{&\left(a_1, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_2, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.8}, \frac{c_3}{0.6}, \frac{c_4}{0.8}\right\}, \left\{\frac{v_1}{0.6}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right),\\
&\left(a_3, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right)\Bigg\}.
\end{aligned} \tag{3.11}$$

Definition 3.12. A fuzzy soft multiset (F, A) over U is called a null fuzzy soft multiset, denoted by $(F, A)_{\Phi}$, if all the fuzzy soft multiset parts of (F, A) equal $\emptyset$.

Example 3.13. Consider Example 3.2. Let us consider a fuzzy soft multiset (F, A) which describes "stone houses," "very cheap classic cars," and "hotels in Kajang" with

$$A = \{a_1 = (e_{U_1,3}, e_{U_2,2}, e_{U_3,3}),\ a_2 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,3})\}. \tag{3.12}$$

Then a null fuzzy soft multiset $(F, A)_{\Phi}$ is given as

$$\begin{aligned}(F, A)_{\Phi} = \Bigg\{ &\left(a_1, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}, \frac{c_4}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right), \\ &\left(a_2, \left(\left\{\frac{h_1}{0}, \frac{h_2}{0}, \frac{h_3}{0}, \frac{h_4}{0}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}, \frac{c_4}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right)\Bigg\}.\end{aligned} \tag{3.13}$$

Definition 3.14. A fuzzy soft multiset (F, A) over U is called a semi-absolute fuzzy soft multiset, denoted by $(F, A)_{\approx U_i}$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i$ for at least one i, $a_k \in A$, $k = \{1, 2, \ldots, n\}$, $i \in \{1, 2, \ldots, m\}$, and $j \in \{1, 2, \ldots, r\}$.

Example 3.15. Consider Example 3.2. Let us consider a fuzzy soft multiset (F, A) which describes "wooden houses," "cars," and "hotels" with

$$A = \{a_1 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}),\ a_3 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,3})\}. \tag{3.14}$$

Then a semi-absolute fuzzy soft multiset $(F, A)_{\approx U_1}$ is given as

$$\begin{aligned}(F, A)_{\approx U_1} = \Bigg\{ &\left(a_1, \left(\left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right), \\ &\left(a_2, \left(\left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.8}, \frac{c_3}{0.6}, \frac{c_4}{0.8}\right\}, \left\{\frac{v_1}{0.6}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right), \\ &\left(a_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right)\Bigg\}.\end{aligned} \tag{3.15}$$

Definition 3.16. A fuzzy soft multiset (F, A) over U is called an absolute fuzzy soft multiset, denoted by $(F, A)_U$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i$, $\forall i$.

Example 3.17. Consider Example 3.2. Let us consider a fuzzy soft multiset (F, A) which describes "wooden houses," "expensive classic cars," and "hotels in Kuala Lumpur" with

$$A = \{a_1 = \{e_{U_1,3}, e_{U_2,1}, e_{U_3,3}\},\ a_2 = \{e_{U_1,3}, e_{U_2,3}, e_{U_3,3}\}\}. \tag{3.16}$$

Then an absolute fuzzy soft multiset $(F, A)_U$ is given as

$$(F,A)_U = \left\{ \left(a_1, \left(\left\{ \frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}, \frac{h_5}{1} \right\}, \left\{ \frac{c_1}{1}, \frac{c_2}{1}, \frac{c_3}{1}, \frac{c_4}{1} \right\}, \left\{ \frac{v_1}{1}, \frac{v_2}{1}, \frac{v_3}{1} \right\} \right) \right), \right.$$
$$\left. \left(a_2, \left(\left\{ \frac{h_1}{1}, \frac{h_2}{1}, \frac{h_3}{1}, \frac{h_4}{1}, \frac{h_5}{1} \right\}, \left\{ \frac{c_1}{1}, \frac{c_2}{1}, \frac{c_3}{1}, \frac{c_4}{1} \right\}, \left\{ \frac{v_1}{1}, \frac{v_2}{1}, \frac{v_3}{1} \right\} \right) \right) \right\}. \tag{3.17}$$

Proposition 3.18. *If (F, A) is a fuzzy soft multiset over U, then*

(a) $((F,A)^c)^c = (F,A)$,

(b) $(F,A)^c_{\approx\Phi_i} = (F,A)_{\approx U_i}$,

(c) $(F,A)^c_{\Phi} = (F,A)_U$,

(d) $(F,A)^c_{\approx U_i} = (F,A)_{\approx\Phi_i}$,

(e) $(F,A)^c_U = (F,A)_{\Phi}$.

Proof. The proof is straightforward. □

4. Union and Intersection

In this section we define the operation of union and intersection and give some examples by using the basic fuzzy union and intersection.

Definition 4.1. The union of two fuzzy soft multisets (F, A) and (G, B) over U, denoted by $(F, A)\tilde{\cup}(G, B)$, is the fuzzy soft multiset (H, C), where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ \bigcup(F(\varepsilon), G(\varepsilon)) & \text{if } \varepsilon \in A \cap B, \end{cases} \tag{4.1}$$

where $\bigcup(F(\varepsilon), G(\varepsilon)) = s(F_{\varepsilon_{U_i,j}}, G_{\varepsilon_{U_i,j}})$, $\forall i \in \{1, 2, \ldots, m\}$ with s as an s-norm.

Example 4.2. Consider Example 3.2. Let

$$\begin{aligned} A = \{ & a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),\ a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}), \\ & a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1}) \}, \\ B = \{ & b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}),\ b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}),\ b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), \\ & b_4 = (e_{U_1,5}, e_{U_2,4}, e_{U_3,2}),\ b_5 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}),\ b_6 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2}) \}. \end{aligned} \tag{4.2}$$

Suppose (F, A) and (G, B) are two fuzzy soft multisets over the same U such that

$$\begin{aligned}
(F, A) = \Bigg\{ & \left(a_1, \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}, \frac{h_5}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}, \frac{c_4}{0.6}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right), \\
& \left(a_2, \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.8}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right), \\
& \left(a_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.7}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right), \\
& \left(a_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}, \frac{c_4}{0.2}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right)\Bigg\}, \\
(G, B) = \Bigg\{ & \left(b_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}, \frac{h_5}{0.2}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.6}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.9}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right), \\
& \left(b_2, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.9}, \frac{c_3}{0.9}, \frac{c_4}{0.8}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right), \\
& \left(b_3, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{1}, \frac{h_4}{0.8}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.7}, \frac{v_2}{0.6}, \frac{v_3}{0.5}\right\}\right)\right), \\
& \left(b_4, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}, \frac{h_5}{0.8}\right\}, \left\{\frac{c_1}{0.3}, \frac{c_2}{0.5}, \frac{c_3}{0.7}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right), \\
& \left(b_5, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.9}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right), \\
& \left(b_6, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.7}, \frac{c_3}{0.6}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.9}, \frac{v_3}{0.9}\right\}\right)\right)\Bigg\}.
\end{aligned} \tag{4.3}$$

By using the basic fuzzy union (maximum) we have

$$\begin{aligned}
(F, A)\tilde{\cup}(G, B) &= (H, D) \\
&= \Bigg\{ \left(d_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}, \frac{h_5}{0.2}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.6}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.9}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right), \\
& \left(d_2, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{1}, \frac{h_4}{0.8}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.7}, \frac{v_2}{0.6}, \frac{v_3}{0.5}\right\}\right)\right), \\
& \left(d_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.7}, \frac{c_2}{0.8}, \frac{c_3}{0.5}, \frac{c_4}{0.4}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right), \\
& \left(d_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}, \frac{h_5}{1}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}, \frac{c_4}{0.2}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right), \\
& \left(d_5, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}, \frac{h_5}{0.3}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.9}, \frac{c_3}{0.9}, \frac{c_4}{0.8}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right),
\end{aligned}$$

$$\left(d_6, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}, \frac{h_5}{0.8}\right\}, \left\{\frac{c_1}{0.3}, \frac{c_2}{0.5}, \frac{c_3}{0.7}, \frac{c_4}{0.7}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right),$$
$$\left(d_7, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}, \frac{h_5}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.9}, \frac{c_3}{0.5}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),$$
$$\left(d_8, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}, \frac{h_5}{0.6}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.7}, \frac{c_3}{0.6}, \frac{c_4}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.9}, \frac{v_3}{0.9}\right\}\right)\right)\Bigg\}, \tag{4.4}$$

where

$$D = \{d_1 = a_1 = b_1,\ d_2 = a_2 = b_3,\ d_3 = a_3,\ d_4 = a_4,\ d_5 = b_2,\ d_6 = b_4,\ d_7 = b_5,\ d_8 = b_6\}. \tag{4.5}$$

Proposition 4.3. *If (F, A), (G, B), and (H, C) are three fuzzy soft multisets over U, then*

(a) $(F, A) \tilde{\cup} ((G, B) \tilde{\cup} (H, C)) = ((F, A) \tilde{\cup} (G, B)) \tilde{\cup} (H, C)$,

(b) $(F, A) \tilde{\cup} (F, A) = (F, A)$,

(c) $(F, A) \tilde{\cup} (G, A)_{\approx\Phi_i} = (R, A)$, *where R is defined by* (4.1)

(d) $(F, A) \tilde{\cup} (G, A)_{\Phi} = (F, A)$,

(e) $(F, A) \tilde{\cup} (G, B)_{\approx\Phi_i} = (R, D)$, *where $D = A \cup B$ and R is defined by* (4.1)

(f) $(F, A) \tilde{\cup} (G, B)_{\Phi} = \begin{cases} (F,A) & \text{if } A=B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A \cup B$,*

(g) $(F, A) \tilde{\cup} (G, A)_{\approx U_i} = (R, A)_{\approx U_i}$,

(h) $(F, A) \tilde{\cup} (G, A)_{U} = (G, A)_{U}$,

(i) $(F, A) \tilde{\cup} (G, B)_{\approx U_i} = \begin{cases} (R,D)_{\approx U_i} & \text{if } A=B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A \cup B$,*

(j) $(F, A) \tilde{\cup} (G, B)_{U} = \begin{cases} (G,B)_{U} & \text{if } A=B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A \cup B$.*

Proof. The proof is straightforward. □

Definition 4.4. The intersection of two fuzzy soft multisets (F, A) and (G, B) over U, denoted by $(F, A) \tilde{\cap} (G, B)$, is the fuzzy soft multiset (H, C), where $C = A \cup B$, and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B, \\ G(\varepsilon) & \text{if } \varepsilon \in B - A, \\ \bigcap(F(\varepsilon), G(\varepsilon)) & \text{if } \varepsilon \in A \cap B, \end{cases} \tag{4.6}$$

where $\bigcap(F(\varepsilon), G(\varepsilon)) = t(F_{\varepsilon_{U_i,j}}, G_{\varepsilon_{U_i,j}})$, $\forall i \in \{1, 2, 3, \ldots, m\}$ with t as a t-norm.

Example 4.5. Consider Example 4.2. By using the basic fuzzy intersection (minimum) we have

$$
\begin{aligned}
(F,A)\,\widetilde{\cap}(G,B) &= (H,D) \\
&= \left\{\left(d_1,\left(\left\{\frac{h_1}{0.2},\frac{h_2}{0.4},\frac{h_3}{0.8},\frac{h_4}{0.5},\frac{h_5}{0}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.5},\frac{c_3}{0.4},\frac{c_4}{0.6}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.7},\frac{v_3}{0.7}\right\}\right)\right),\right. \\
&\quad \left(d_2,\left(\left\{\frac{h_1}{0.7},\frac{h_2}{0.7},\frac{h_3}{1},\frac{h_4}{0.8},\frac{h_5}{0.3}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.6},\frac{c_3}{0.3},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.4},\frac{v_3}{0.2}\right\}\right)\right), \\
&\quad \left(d_3,\left(\left\{\frac{h_1}{1},\frac{h_2}{0.8},\frac{h_3}{0.7},\frac{h_4}{0},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.7},\frac{c_2}{0.8},\frac{c_3}{0.5},\frac{c_4}{0.4}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.5},\frac{v_3}{0.7}\right\}\right)\right), \\
&\quad \left(d_4,\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.6},\frac{h_3}{0.1},\frac{h_4}{0.5},\frac{h_5}{1}\right\},\left\{\frac{c_1}{0.5},\frac{c_2}{0.3},\frac{c_3}{0.1},\frac{c_4}{0.2}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.3},\frac{v_3}{0.4}\right\}\right)\right), \\
&\quad \left(d_5,\left(\left\{\frac{h_1}{0.4},\frac{h_2}{0.6},\frac{h_3}{0.8},\frac{h_4}{0.7},\frac{h_5}{0.3}\right\},\left\{\frac{c_1}{1},\frac{c_2}{0.9},\frac{c_3}{0.9},\frac{c_4}{0.8}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.5},\frac{v_3}{0.4}\right\}\right)\right), \\
&\quad \left(d_6,\left(\left\{\frac{h_1}{0.9},\frac{h_2}{0.6},\frac{h_3}{0.7},\frac{h_4}{0.5},\frac{h_5}{0.8}\right\},\left\{\frac{c_1}{0.3},\frac{c_2}{0.5},\frac{c_3}{0.7},\frac{c_4}{0.7}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.8},\frac{v_3}{1}\right\}\right)\right), \\
&\quad \left(d_7,\left(\left\{\frac{h_1}{1},\frac{h_2}{0.9},\frac{h_3}{0.8},\frac{h_4}{0.4},\frac{h_5}{0.7}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.9},\frac{c_3}{0.5},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.5},\frac{v_2}{0.6},\frac{v_3}{0.8}\right\}\right)\right), \\
&\quad \left.\left(c_8,\left(\left\{\frac{h_1}{0.8},\frac{h_2}{0.7},\frac{h_3}{1},\frac{h_4}{0.9},\frac{h_5}{0.6}\right\},\left\{\frac{c_1}{0.8},\frac{c_2}{0.7},\frac{c_3}{0.6},\frac{c_4}{0.5}\right\},\left\{\frac{v_1}{0.8},\frac{v_2}{0.9},\frac{v_3}{0.9}\right\}\right)\right)\right\},
\end{aligned}
\tag{4.7}
$$

where

$$
D = \{d_1 = a_1 = b_1,\ d_2 = a_2 = b_3,\ d_3 = a_3,\ d_4 = a_4,\ d_5 = b_2,\ d_6 = b_4,\ d_7 = b_5,\ d_8 = b_6\}. \tag{4.8}
$$

Proposition 4.6. *If (F,A), (G,B), and (H,C) are three fuzzy soft multisets over U, then*

(a) $(F,A)\,\widetilde{\cap}\,((G,B)\widetilde{\cap}\,(H,C)) = ((F,A)\widetilde{\cap}\,(G,B))\widetilde{\cap}\,(H,C)$,

(b) $(F,A)\,\widetilde{\cap}\,(F,A) = (F,A)$,

(c) $(F,A)\,\widetilde{\cap}\,(G,A)_{\approx\Phi_i} = (R,A)_{\approx\Phi_i}$, *where R is defined by* (4.6)

(d) $(F,A)\,\widetilde{\cap}\,(G,A)_{\Phi_i} = (R,A)_{\Phi_i}$, *where R is defined by* (4.6)

(e) $(F,A)\,\widetilde{\cap}\,(G,B)_{\approx\Phi_i} = \begin{cases} (R,D)_{\approx\Phi_i} & \text{if } A\subseteq B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A\cup B$,*

(f) $(F,A)\,\widetilde{\cap}\,(G,B)_{\Phi} = \begin{cases} (R,D)_{\Phi} & \text{if } A\subseteq B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A\cup B$,*

(g) $(F,A)\,\widetilde{\cap}\,(G,A)_{\approx U_i} = (R,D)$, *where $D = A\cup B$ and R is defined by* (4.6)

(h) $(F,A)\,\widetilde{\cap}\,(G,A)_{U} = (F,A)$,

(i) $(F,A)\,\widetilde{\cap}\,(G,B)_{\approx U_i} = (R,D)$, *where $D = A\cup B$ and R is defined by* (4.6)

(j) $(F,A)\,\widetilde{\cap}\,(G,B)_{U} = \begin{cases} (F,A) & \text{if } A\supseteq B, \\ (R,D) & \text{otherwise,} \end{cases}$ *where $D = A\cup B$ and R is defined by* (4.6)

Proof. The proof is straightforward. □

5. Fuzzy Soft Set-Based Decision Making

We begin this section with a novel algorithm designed for solving fuzzy soft set-based decision-making problems, which was presented in [6].

5.1. Roy and Maji's Original Algorithm Using Scores

Roy and Maji [6] used the following algorithm to solve a decision-making problem.

(a) Input the fuzzy soft sets (F, A), (G, B), and (H, C).

(b) Input the parameter set P as observed by the observer.

(c) Compute the corresponding resultant fuzzy soft set (S, P) from the fuzzy soft sets (F, A), (G, B), and (H, C) and place it in tabular form.

(d) Construct the comparison table of the fuzzy soft set (S, P) and compute r_i and t_i for $o_i, \forall i$.

(e) Compute the score of $o_i, \forall i$.

(f) The decision is S_k if $S_k = \max_i S_i$.

(g) If k has more than one value, then any one of o_k may be chosen.

5.2. A Fuzzy Soft Multiset Theoretic Approach to Decision-Making Problem

In this section we suggest the following algorithm to solve fuzzy soft multisets-based decision-making problem, which is a generalization of the algorithm given by Salleh and Alkhazaleh in [13]. We note here that we will use the abbreviation RMA for Roy and Maji's a Algorithm.

(a) Input the fuzzy soft multiset (H, C) which is introduced by making any operations between (F, A) and (G, B).

(b) Apply RMA to the first fuzzy soft multiset part in (H, C) to get the decision S_{k_1}.

(c) Redefine the fuzzy soft multiset (H, C) by keeping all values in each row where S_{k_1} is maximum and replacing the values in the other rows by zero, to get $(H, C)_1$.

(d) Apply RMA to the second fuzzy soft multiset part in $(H, C)_1$ to get the decision S_{k_2}.

(e) Redefine the fuzzy soft multiset $(H, C)_1$ by keeping the first and second parts and apply the method in step (c) to the third part.

(f) Apply RMA to the third fuzzy soft multiset part in $(H, C)_2$ to get the decision S_{k_3}.

(g) The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

5.3. Application in a Decision-Making Problem

Let $U_1 = \{h_1, h_2, h_3, h_4\}$, $U_2 = \{c_1, c_2, c_3\}$, and $U_3 = \{v_1, v_2, v_3\}$ be the sets of "houses," "cars," and "hotels", respectively. Let $\{E_{U_1}, E_{U_2}, E_{U_3}\}$ be a collection of sets of decision parameters related to the above universes, where

$$\begin{aligned}
E_{U_1} &= \{e_{U_1,1} = \text{expensive},\ e_{U_1,2} = \text{cheap},\ e_{U_1,3} = \text{wooden}, e_{U_1,4} = \text{in green surroundings}\},\\
E_{U_2} &= \{e_{U_2,1} = \text{expensive},\ e_{U_2,2} = \text{cheap},\ e_{U_2,3} = \text{sporty}\},\\
E_{U_3} &= \{e_{U_3,1} = \text{expensive},\ e_{U_3,2} = \text{cheap},\ e_{U_3,3} = \text{in KualaLumpur}, e_{U_3,4} = \text{majestic}\}.
\end{aligned} \tag{5.1}$$

Let

$$\begin{aligned}
A = \{&a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_3 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,3}),\\
&a_4 = (e_{U_1,3}, e_{U_2,1}, e_{U_3,1})\},\\
B = \{&b_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), b_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), b_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}),\\
&b_4 = (e_{U_1,4}, e_{U_2,3}, e_{U_3,2}), b_5 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,2}), b_6 = (e_{U_1,1}, e_{U_2,3}, e_{U_3,2})\}.
\end{aligned} \tag{5.2}$$

Suppose Mr. X wants to choose objects from the sets of given objects with respect to the sets of choice parameters. Let there be two observations (F, A) and (G, B) by two experts Y_1 and Y_2, respectively. Let

$$\begin{aligned}
(F, A) = \Bigg\{&\left(a_1, \left(\left\{\frac{h_1}{0.2}, \frac{h_2}{0.4}, \frac{h_3}{0.8}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.5}, \frac{c_3}{0.4}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.7}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_2, \left(\left\{\frac{h_1}{0.7}, \frac{h_2}{0.7}, \frac{h_3}{0.1}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.4}, \frac{v_3}{0.2}\right\}\right)\right),\\
&\left(a_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right),\\
&\left(a_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right)\Bigg\},\\
(G, B) = \Bigg\{&\left(b_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.6}\right\}, \left\{\frac{v_1}{0.9}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right),\\
&\left(b_2, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.9}, \frac{c_3}{0.9}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right),\\
&\left(b_3, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{0.3}, \frac{h_4}{0.8}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.7}, \frac{v_2}{0.6}, \frac{v_3}{0.5}\right\}\right)\right),\\
&\left(b_4, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right),
\end{aligned}$$

$$\left(b_5, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),$$
$$\left(b_6, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.9}, \frac{v_3}{0.9}\right\}\right)\right)\Bigg\}. \tag{5.3}$$

By using the basic fuzzy union we have

$$\begin{aligned}
(F, A)\,\tilde{\cup}\,(G, B) &= (H, D) \\
&= \left\{\left(d_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.6}\right\}, \left\{\frac{v_1}{0.9}, \frac{v_2}{0.7}, \frac{v_3}{0.9}\right\}\right)\right),\right. \\
&\quad \left(d_2, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{0.3}, \frac{h_4}{0.8}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.7}, \frac{v_2}{0.6}, \frac{v_3}{0.5}\right\}\right)\right), \\
&\quad \left(d_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right), \\
&\quad \left(d_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right), \\
&\quad \left(d_5, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{1}, \frac{c_2}{0.9}, \frac{c_3}{0.9}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.5}, \frac{v_3}{0.4}\right\}\right)\right), \\
&\quad \left(d_6, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right), \\
&\quad \left(d_7, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right), \\
&\quad \left.\left(d_8, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.9}, \frac{v_3}{0.9}\right\}\right)\right)\right\}.
\end{aligned} \tag{5.4}$$

Now we apply RMA to the first fuzzy soft multiset part in (H, D) to take the decision from the availability set U_1. The tabular representation of the first resultant fuzzy soft multiset part will be as in Table 1.

The comparison table for the first resultant fuzzy soft multiset part will be as in Table 2.

Next we compute the row-sum, column-sum, and the score for each h_i as shown in Table 3.

From Table 3, it is clear that the maximum score is 6, scored by h_1.

Table 1: Tabular representation: U_1-fuzzy soft multiset part of (H, D).

U_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$	$d_{1,5}$	$d_{1,6}$	$d_{1,7}$	$d_{1,8}$
h_1	0.3	0.8	1	0.8	0.4	0.9	1	0.8
h_2	0.4	0.9	0.8	0.6	0.6	0.6	0.9	0.7
h_3	0.9	0.3	0.7	0.1	0.7	0.7	0.8	1
h_4	0.7	0.8	0	0.5	0.7	0.5	0.4	0.9

Table 2: Comparison table: U_1-fuzzy soft multiset part of (H, D).

U_1	h_1	h_2	h_3	h_4
h_1	8	5	5	5
h_2	3	8	4	5
h_3	3	4	8	6
h_4	3	3	3	8

Table 3: Score table: U_1–fuzzy soft multiset part of (H, D).

	Row-sum (r_i)	Column-sum (t_i)	Score (S_i)
h_1	23	17	6
h_2	20	20	0
h_3	21	20	1
h_4	17	24	–7

Now we redefine the fuzzy soft multiset (H, D) by keeping all values in each row where h_1 is maximum and replacing the values in the other rows by zero:

$$\begin{aligned}(H,D)_1 = \Bigg\{ &\left(d_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_2, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{0.3}, \frac{h_4}{0.8}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right),\\ &\left(d_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right),\\ &\left(d_5, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_6, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right),\\ &\left(d_7, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),\\ &\left(d_8, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right)\Bigg\}.\end{aligned} \tag{5.5}$$

Table 4: Tabular representation: U_2-fuzzy soft multiset part of $(H,D)_1$.

U_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$	$d_{1,5}$	$d_{1,6}$	$d_{1,7}$	$d_{1,8}$
c_1	0	0	0.8	0.5	0	0.8	0.8	0
c_2	0	0	0.6	0.3	0	0.8	0.8	0
c_3	0	0	0.3	0.1	0	0.5	0.5	0

Table 5: Comparison table: U_2-fuzzy soft multiset part of of $(H,D)_1$.

U_2	c_1	c_2	c_3
c_1	8	8	8
c_2	6	8	8
c_3	4	4	8

Now we apply RMA to the second fuzzy soft multiset part in $(H,D)_1$ to take the decision from the availability set U_2. The tabular representation of the second resultant fuzzy soft multiset part of $(H,D)_1$ will be as in Table 4.

The comparison table for the second resultant fuzzy soft multiset part of $(H,D)_1$ is as in Table 5.

Next we compute the row-sum, column-sum, and the score for each c_i is shown in Table 6.

From Table 6, it is clear that the maximum score is 6, scored by c_1.

Now we redefine the fuzzy soft multiset $(H,D)_1$ by keeping all values in each row where c_1 is maximum and replacing the values in the other rows by zero:

$$\begin{aligned}(H,D)_2 = \Bigg\{ &\left(d_1, \left(\left\{\frac{h_1}{0.3}, \frac{h_2}{0.4}, \frac{h_3}{0.9}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_2, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.9}, \frac{h_3}{0.3}, \frac{h_4}{0.8}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_3, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.8}, \frac{h_3}{0.7}, \frac{h_4}{0}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.6}, \frac{c_3}{0.3}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.5}, \frac{v_3}{0.7}\right\}\right)\right),\\ &\left(d_4, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.6}, \frac{h_3}{0.1}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.5}, \frac{c_2}{0.3}, \frac{c_3}{0.1}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.3}, \frac{v_3}{0.4}\right\}\right)\right),\\ &\left(d_5, \left(\left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.8}, \frac{h_4}{0.7}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right),\\ &\left(d_6, \left(\left\{\frac{h_1}{0.9}, \frac{h_2}{0.6}, \frac{h_3}{0.7}, \frac{h_4}{0.5}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.8}, \frac{v_2}{0.8}, \frac{v_3}{1}\right\}\right)\right),\\ &\left(d_7, \left(\left\{\frac{h_1}{1}, \frac{h_2}{0.9}, \frac{h_3}{0.8}, \frac{h_4}{0.4}\right\}, \left\{\frac{c_1}{0.8}, \frac{c_2}{0.8}, \frac{c_3}{0.5}\right\}, \left\{\frac{v_1}{0.5}, \frac{v_2}{0.6}, \frac{v_3}{0.8}\right\}\right)\right),\\ &\left(d_8, \left(\left\{\frac{h_1}{0.8}, \frac{h_2}{0.7}, \frac{h_3}{1}, \frac{h_4}{0.9}\right\}, \left\{\frac{c_1}{0}, \frac{c_2}{0}, \frac{c_3}{0}\right\}, \left\{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}\right\}\right)\right)\Bigg\}.\end{aligned} \tag{5.6}$$

Table 6: Score table: U_2-fuzzy soft multiset part of $(H, D)_1$.

	Row-sum (r_i)	Column-sum (t_i)	Score (S_i)
c_1	24	18	6
c_2	22	20	2
c_3	16	24	−8

Table 7: Tabular representation: U_3-fuzzy soft multiset part of $(H, D)_2$.

U_3	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$	$d_{1,5}$	$d_{1,6}$	$d_{1,7}$	$d_{1,8}$
v_1	0	0	0.5	0.5	0	0.8	0.5	0
v_2	0	0	0.5	0.3	0	0.8	0.6	0
v_3	0	0	0.7	0.4	0	1	0.8	0

Table 8: Comparison table: U_3-fuzzy soft multiset part of of $(H, D)_2$.

U_3	v_1	v_2	v_3
v_1	8	7	5
v_2	7	8	4
v_3	7	8	8

Table 9: Score table: U_3-fuzzy soft multiset part of $(H, D)_2$.

	Row-sum (r_i)	Column-sum (t_i)	Score (S_i)
v_1	20	22	−2
v_2	19	23	−2
v_3	23	17	6

Now we apply RMA to the third fuzzy soft multiset part in $(H, D)_2$ to take the decision from the availability set U_3. The tabular representation of the third resultant fuzzy soft multiset part of $(H, D)_2$ is as in Table 7.

The comparison table for the second resultant fuzzy soft multiset part of $(H, D)_2$ is as in Table 8.

Next we compute the row-sum, column-sum, and the score for each v_i as shown in Table 9.

From Table 9, it is clear that the maximum score is 6, scored by v_3.

Then from the above results the decision for Mr. X is (h_1, c_1, v_3).

6. Conclusion

In this paper we have introduced the concept of fuzzy soft multiset and studied some of its properties. The operations complement, union, and intersection have been defined on the fuzzy soft multisets. An application of this theory is given in solving a decision-making problem.

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