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Research Article

On the Barnes' Type Related to Multiple Genocchi Polynomials on $\mathbb{Z}_p$

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Using fermionic p-adic invariant integral on $\mathbb{Z}_p$, we construct the Barnes' type multiple Genocchi numbers and polynomials. From those numbers and polynomials, we derive the twisted Barnes' type multiple Genocchi numbers and polynomials. Moreover, we will find the Barnes' type multiple Genocchi zeta function.

1. Introduction

Recently, theoretical physicists have devised ultrametric structures similar to tree-like structures arising in the study of physical systems because the fact is that the physical space may no longer be Archimedean seems plausible to some mathematical physicists at a very small distance. They have looked for construct-related models using p-adic numbers and p-adic analysis. So, p-adic numbers are used in mathematical physics (in particular string theory, field theory) as well as in other areas of natural sciences which complicated fractal behavior and hierarchical structures. Also, q-Volkenborn integral which is made by Kim is used in the functional equation of the q-zeta function, the q-Stirling numbers, and Mahler theory of integration with respect to the ring $\mathbb{Z}_p$ together with Iwasawas p-adic q-L function. So we study the q-Gnocchi numbers and polynomials in q-type of special generating functions (see [1, 2]).

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p, \mathbb{Q}_p, \mathbb{C}$, and $\mathbb{C}_p$ denote the ring of p-adic rational integers, the field of p-adic rational numbers, the complex number field, and the completion of the algebraic closure of $\mathbb{Q}_p$, respectively. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}^+ = \mathbb{N} \cup \{0\}$. Let v_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-v_p(p)} = 1/p$ (see[1–15]).

Let d be a fixed integer and let p be a fixed prime number. For any positive integer N, we set

$$X = X_d = \lim_{\overleftarrow{N}} \left(\frac{\mathbb{Z}}{dp^N \mathbb{Z}} \right), \qquad X_1 = \mathbb{Z}_p,$$
$$X^* = \bigcup_{\substack{0<a<dp \\ (a,p)=1}} (a + dp\mathbb{Z}_p), \tag{1.1}$$
$$a + dp^N \mathbb{Z}_p = \left\{ x \in X \mid x \equiv a \left(\operatorname{mod} dp^N \right) \right\},$$

where $a \in \mathbb{Z}$ lies in $0 \le a < dp^N$.

We say that $f : \mathbb{Z}_p \to \mathbb{C}_p$ is uniformly differentiable function at a point $a \in \mathbb{Z}_p$, and we write $f \in \mathrm{UD}(\mathbb{Z}_p)$ if the difference quotients $\Phi_f : \mathbb{Z}_p \times \mathbb{Z}_p \to \mathbb{C}_p$ such that

$$\Phi_f(x, y) = \frac{f(x) - f(y)}{x - y} \tag{1.2}$$

have a limit $f'(a)$ as $(x, y) \to (a, a)$.

As well-known definition, the Genocchi polynomials are defined by

$$F(t) = \frac{2t}{e^t + 1} = e^{Gt} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!},$$
$$F(t, x) = \frac{2t}{e^t + 1} e^{xt} = e^{G(x)t} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \tag{1.3}$$

with the usual convention of replacing $G^n(x)$ by $G_n(x)$. In the special case, $x = 0$, $G_n(0) = G_n$ are called the nth Genocchi numbers.

For $f \in \mathrm{UD}(\mathbb{Z}_p)$, Kim defined the q-deformed fermionic p-adic integral on Z_p,

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x) d\mu_{-q}(x) = \lim_{N \to \infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N - 1} f(x)(-q)^x. \tag{1.4}$$

Note that

$$I_{-1}(f) = \lim_{q \to 1} I_{-q}(f). \tag{1.5}$$

If we take $f(x) = te^{tx}$, by (1.4), we see that

$$\int_{\mathbb{Z}_p} te^{tx} d\mu_{-1}(x) = \frac{2t}{1 + e^t} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}. \tag{1.6}$$

By the same method, we note that

$$\int_{\mathbb{Z}_p} te^{t(x+y)} d\mu_{-1}(x) = \frac{2t}{1+e^t} e^{tx} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}. \tag{1.7}$$

And note that

$$\int_{\mathbb{Z}_p} f(x) d\mu_{-1}(x) = \int_{X} f(x) d\mu_{-1}(x). \tag{1.8}$$

Barnes' type multiple zeta function $\zeta_N(s, w \mid a_1, \ldots, a_N)$ depends on the parameters $a_1, \ldots, a_N$ that will be assumed to be positive. It is defined by

$$\zeta_N(s, w \mid a_1, \ldots, a_N) = \sum_{m_1, \ldots, m_N = 0}^{\infty} (w + m_1 a_1 + \cdots + m_N a_N)^{-s}, \tag{1.9}$$

for $\mathrm{Re}(s) > N$, $\mathrm{Re}(w) > 0$. Barnes showed that ζ_N had a meromorphic continuation in s (with simple pole only at $s = 1, \ldots, N$) and defined his multiple gamma function $\Gamma_N(w)$ in terms of the s-derivative at $s = 0$, which may be recalled here as follows:

$$\Psi_n(w \mid a_1, \ldots, a_N) = \partial_s \zeta_N(s, w \mid a_1 \ldots, a_N) \mid s = 0. \tag{1.10}$$

2. Barnes' Type Multiple Genocchi Polynomials

In this section, we assume that $w_1, \ldots, w_r \in \mathbb{Z}_p$. For $x \in \mathbb{Q}^+$, $r \in \mathbb{N}$, we define Barnes' type multiple Genocchi polynomials as follows:

$$\begin{aligned} &\underbrace{\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p}}_{r\text{-times}} t^r e^{(x + w_1 y_1 + \cdots + w_r y_r)t} d\mu_{-1}(y_1) \cdots d\mu_{-1}(y_r) \\ &\quad = \frac{2^r t^r}{(1+e^{tw_1})(1+e^{tw_2}) \cdots (1+e^{tw_r})} e^{tx} \\ &\quad = \sum_{n=0}^{\infty} G_n^{(r)}(x \mid w_1, \ldots, w_r) \frac{t^n}{n!}. \end{aligned} \tag{2.1}$$

In the special case, $x = 0$, $G_n^{(r)}(w_1, \ldots, w_r) = G_n^{(r)}(0 \mid w_1, \ldots, w_r)$ are called the nth Barnes' type multiple Genocchi numbers.

Thus, we have

$$G_0^{(r)}(x \mid w_1,\dots,w_r) = G_1^{(r)}(x \mid w_1,\dots,w_r) = \cdots = G_{r-1}^{(r)}(x \mid w_1,\dots,w_r) = 0,$$
$$\frac{G_{n+r}^{(r)}(x \mid w_1,\dots,w_r)}{\binom{n+r}{r} r!} = \underbrace{\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p}}_{r\text{-times}} (x + w_1 y_1 + \cdots + w_r y_r)^n d\mu_{-1}(y_1) \cdots d\mu_{-1}(y_r). \tag{2.2}$$

For $r \geq 1$, $n > 0$, we can readily prove the following (2.3) from (2.2):

$$G_{n+r}^{(r)}(x \mid w_1,\dots,w_r) = \sum_{l=0}^{n} \binom{n+r}{l+r} x^{n-l} G_{l+r}^{(r)}(w_1,\dots,w_r), \tag{2.3}$$

where $\binom{n+r}{l+r}$ is a binomial coefficient.

Theorem 2.1 (Property of distribution of $G_n^{(r)}(x \mid w_1,\dots,w_r)$). *For $r \geq 1$, $n > 0$ and $m \in \mathbb{N}$ with $m \equiv 1 \pmod 2$,*

$$G_n^{(r)}(x \mid w_1,\dots,w_r) = \frac{m^n}{m^r} \sum_{n_1,\dots,n_r=0}^{m-1} (-1)^{\sum_{i=1}^r n_i} G_n^{(r)}\left(\frac{x + \sum_{i=1}^r w_i n_i}{m} \mid w_1,\dots,w_r\right). \tag{2.4}$$

Let χ be Dirichlet's character with conductor $d \in \mathbb{N}$ with $d \equiv 1 \pmod 2$:

$$\begin{aligned} F^{(r)}(t, x \mid w_1,\dots,w_r) &= \frac{2^r t^r}{\prod_{i=1}^r (1 + e^{t w_i})} e^{tx} \\ &= \sum_{n=0}^{\infty} G_n^{(r)}(x \mid w_1,\dots,w_r) \frac{t^n}{n!}. \end{aligned} \tag{2.5}$$

By (2.1) *and* (2.5), *we see that*

$$\begin{aligned} &\underbrace{\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p}}_{r\text{-times}} t^r e^{(x + w_1 y_1 + \cdots + w_r y_r)t} d\mu_{-1}(y_1) \cdots d\mu_{-1}(y_r) \\ &\quad = 2^r t^r \sum_{m_1,\dots,m_r=0}^{\infty} (-1)^{m_1+\cdots+m_r} e^{(x + w_1 m_1 + \cdots + w_r m_r)t}. \end{aligned} \tag{2.6}$$

From (2.2) *and* (2.6), *we have*

$$\begin{aligned}\frac{G_{n+r}^{(r)}(x \mid w_1,\dots,w_r)}{\binom{n+r}{r}r!} &= \underbrace{\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}}_{r\text{-times}}(x+w_1y_1+\cdots+w_ry_r)^n d\mu_{-1}(y_1)\cdots d\mu_{-1}(y_r)\\ &= 2^r\sum_{m_1,\dots,m_r=0}^{\infty}(-1)^{m_1+\cdots+m_r}(x+w_1m_1+\cdots+w_rm_r)^n.\end{aligned} \tag{2.7}$$

By (2.5), *we see that*

$$\begin{aligned}F_{\chi}^{(r)}(t,x \mid w_1,\dots,w_r) &= \frac{2^rt^r\left(\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{w_1at}\right)\cdots\left(\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{w_rat}\right)}{\left(1+e^{w_1dt}\right)\cdots\left(1+e^{w_rdt}\right)}e^{tx}\\ &= 2^rt^r\sum_{m_1,\dots,m_r=0}^{\infty}(-1)^{m_1+\cdots+m_r}\left(\prod_{i=1}^{r}\chi(m_i)\right)e^{(x+w_1m_1+\cdots+w_rm_r)t}.\end{aligned} \tag{2.8}$$

We define the generalized Barnes' type multiple Genocchi polynomials attached to χ *as follows: for* $m \geq 0$ *and* $w_i \in \mathbb{Z}_p$ $(i = 1,2,\dots,n)$,

$$\begin{aligned}&G_{0,\chi}^{(r)}(x \mid w_1,\dots,w_r) = G_{1,\chi}^{(r)}(x \mid w_1,\dots,w_r) = \cdots = G_{r-1,\chi}^{(r)}(x \mid w_1,\dots,w_r) = 0,\\ &\frac{G_{n+r}^{(r)}(x \mid w_1,\dots,w_r)}{\binom{n+r}{r}r!} = \underbrace{\int_X\cdots\int_X}_{r\text{-times}}\left(\prod_{i=1}^{r}\chi(x_i)\right)\left(x+\sum_{i=1}^{r}w_iy_i\right)^n d\mu_{-1}(y_1)\cdots d\mu_{-1}(y_r).\end{aligned} \tag{2.9}$$

For simple calculation of fermionic p-adic invariant on $\mathbb{Z}_p$, we note the following theorem.

Theorem 2.2 (Property of distribution of $G_{n,\chi}^{(r)}(x \mid w_1,\dots,w_r)$).

$$G_{n,\chi}^{(r)}(x \mid w_1,\dots,w_r) = \frac{d^n}{d^r}\sum_{n_1,\dots,n_r=0}^{d-1}(-1)^{\sum_{i=1}^{r}n_i}G_{n,\chi}^{(r)}\left(\frac{x+\sum_{i=1}^{r}w_in_i}{d} \mid w_1,\dots,w_r\right). \tag{2.10}$$

Theorem 2.2 is distribution relation for the generalized Barnaes' type multiple Genocchi polynomials attached to χ. In the special case, $x = 0$, $G_{n,\chi}^{(r)}(w_1,\dots,w_r) = G_{n,\chi}^{(r)}(0 \mid w_1,\dots,w_r)$ are called the generalized Barnes' type multiple Genocchi numbers attached to χ.

By (2.8) and (2.9), we get

$$\frac{G_{n+r,\chi}^{(r)}(x \mid w_1,\dots,w_r)}{\binom{n+r}{r}r!} = 2^r\sum_{m_1,\dots,m_r=0}^{\infty}(-1)^{\sum_{i=1}^{r}m_i}\left(\prod_{i=1}^{r}\chi(m_i)\right)\left(x+\sum_{i=1}^{r}w_im_i\right)^n. \tag{2.11}$$

3. Twisted Barnes' Type Multiple Genocchi Polynomials

Let $T_p = \lim_{N\to\infty} C_{p^N} = \cup_{N\geq 0} C_{p^N}$, where $C_{p^N} = \{\zeta \mid \zeta^{p^N} = 1\}$ is the cyclic group of order p^N. For $\varepsilon \in T_p$, we denote by $\phi_\varepsilon : \mathbb{Z}_p \to \mathbb{C}_p$ the locally constant function $x \mapsto \phi_\varepsilon(x) = \varepsilon^x$. If we take $f(x) = \phi_\varepsilon(x)e^{tx}$, then we get

$$\int_{\mathbb{Z}_p} t\phi_\varepsilon(x)e^{tx}d\mu_{-1}(x) = \frac{2t}{1+\varepsilon e^t}. \tag{3.1}$$

By (3.1), we easily see that

$$\int_X t\phi_\varepsilon(x)\chi(x)e^{tx}d\mu_{-1}(x) = \frac{2t\sum_{a=0}^{d-1}(-1)^a\varepsilon^a\chi(a)e^{at}}{1+\varepsilon^d e^{dt}}. \tag{3.2}$$

From (3.1) and (3.2), we define the twisted Genocchi numbers and the generalized twisted Genocchi numbers attached to χ as follows:

$$\frac{2t}{1+\varepsilon e^t} = \sum_{n=0}^{\infty} G_{n,\varepsilon}\frac{t^n}{n!}, \qquad \frac{2t\sum_{a=0}^{d-1}(-1)^a\varepsilon^a\chi(a)e^{at}}{1+\varepsilon^d e^{dt}} = \sum_{n=0}^{\infty} G_{n,\chi,\varepsilon}\frac{t^n}{n!}. \tag{3.3}$$

In (3.1), (3.2), and (3.3), we get

$$\int_{\mathbb{Z}_p} \phi_\varepsilon(x)x^n d\mu_{-1}(x) = \frac{G_{n+1,\varepsilon}}{n+1}, \qquad \int_X \phi_\varepsilon(x)\chi(x)x^n d\mu_{-1}(x) = \frac{G_{n+1,\chi,\varepsilon}}{n+1}. \tag{3.4}$$

By using fermionic multivariate p-adic invariant integral on $\mathbb{Z}_p$, we define the twisted Barnes' type multiple Genocchi polynomials as follows:

$$\begin{aligned}
&\underbrace{\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}}_{r\text{-times}} t^r\left(\prod_{i=1}^{r}\phi_\varepsilon(y_i)\right)e^{(x+w_1y_1+\cdots+w_ry_r)t}d\mu_{-1}(y_1)\cdots d\mu_{-1}(y_r) \\
&\quad = \frac{2^r t^r}{(1+\varepsilon e^{w_1t})\cdots(1+\varepsilon e^{w_rt})}e^{tx} \\
&\quad = \sum_{n=0}^{\infty} G_{n,\varepsilon}^{(r)}(x \mid w_1,\ldots,w_r)\frac{t^n}{n!}, \quad \varepsilon \in T_p.
\end{aligned} \tag{3.5}$$

$G_{n,\varepsilon}^{(r)}(x \mid w_1,\ldots,w_r)$ are called the twisted Barnes' type multiple Genocchi polynomials. In the special case, $x = 0$, $G_{n,\varepsilon}^{(r)}(w_1,\ldots,w_r) = G_{n,\varepsilon}^{(r)}(0 \mid w_1,\ldots,w_r)$ are called the twisted Barnes' type multiple Genocchi numbers.

From (3.5), we can derive the following:

$$\begin{aligned}
&\underbrace{\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}}_{r\text{-times}} t^r\left(\prod_{i=1}^{r}\phi_\varepsilon(y_i)\right)(x+w_1y_1+\cdots+w_ry_r)^n d\mu_{-1}(y_1)\cdots d\mu_{-1}(y_r)\\
&\quad=\frac{G_{n+r,\varepsilon}^{(r)}(x\mid w_1,\ldots,w_r)}{\binom{n+r}{r}r!}.
\end{aligned}\tag{3.6}$$

Let χ be the primitive Drichlet character with conductor $d\in\mathbb{N}$ with $d\equiv 1 \pmod 2$. Then we define the generalized twisted Barnes' type multiple Genocchi numbers attached to χ:

$$\begin{aligned}
&\underbrace{\int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}}_{r\text{-times}} t^r\left(\prod_{i=1}^{r}\phi_\varepsilon(x_i)\right)\left(\prod_{i=1}^{r}\chi(x_i)\right)e^{(w_1x_1+\cdots+w_rx_r)t}d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_r)\\
&\quad=\frac{2^rt^r\left(\sum_{l=0}^{d-1}(-1)^l\varepsilon^l\chi(l)e^{w_1lt}\right)\cdots\left(\sum_{l=0}^{d-1}(-1)^l\varepsilon^l\chi(l)e^{w_rlt}\right)}{\left(1+\varepsilon^de^{w_1dt}\right)\cdots\left(1+\varepsilon^de^{w_rdt}\right)}\\
&\quad=\sum_{n=0}^{\infty}G_{n,\varepsilon,\chi}^{(r)}(w_1,\ldots,w_r)\frac{t^n}{n!}=F_{\varepsilon,\chi}^{(r)}(t\mid w_1,\ldots,w_r).
\end{aligned}\tag{3.7}$$

From (3.7), we see that

$$F_{\varepsilon,\chi}^{(r)}(t\mid w_1,\ldots,w_r)=2^rt^r\sum_{m_1,\ldots,m_r=0}^{\infty}(-1)^{\sum_{i=1}^{r}m_i\varepsilon\sum_{i=1}^{r}m_i}\left(\prod_{i=1}^{r}\chi(m_i)\right)e^{t(\sum_{i=1}^{r}w_im_i)}.\tag{3.8}$$

By using (3.7), we easily see that

$$\begin{aligned}
&\frac{G_{n,\varepsilon,\chi}^{(r)}(w_1,\ldots,w_r)}{\binom{n+r}{r}r!}\\
&\quad=\underbrace{\int_{\mathrm{X}}\cdots\int_{\mathrm{X}}}_{r\text{-times}}\left(\prod_{i=1}^{r}\phi_\varepsilon(x_i)\right)\left(\prod_{i=1}^{r}\chi(x_i)\right)(w_1x_1+\cdots+w_rx_r)^n d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_r).
\end{aligned}\tag{3.9}$$

Theorem 3.1 (Property of distribution of $G_{n,\varepsilon,\chi}^{(r)}(x \mid w_1,\ldots,w_r)$).

$$G_{n,\varepsilon,\chi}^{(r)}(x \mid w_1,\ldots,w_r) = \frac{d^n}{d^r}\sum_{n_1,\ldots,n_r=0}^{d-1}(-1)^{\sum_{i=1}^r n_i\varepsilon\sum_{i=1}^r n_i}\left(\prod_{i=1}^r \chi(n_i)\right)G_{n,\varepsilon^d}^{(r)}\left(\frac{x+\sum_{i=1}^r w_i n_i}{d} \mid w_1,\ldots,w_r\right). \tag{3.10}$$

4. Remark

Let $w_1,\ldots,w_r$ be taken nonnegative in complex plane. We consider the Barnes' type multiple Genocchi zeta function $\zeta(s,x \mid w_1,\ldots,w_r)$ as follows:

$$\zeta(s,x \mid w_1,\ldots,w_r) = 2^r\sum_{m_1,\ldots,m_r=0}^{\infty}\frac{(-1)^{m_1+\cdots+m_r}}{(x+w_1m_1+\cdots+w_rm_r)^s}, \tag{4.1}$$

where $s\in\mathbb{C}$, and $\mathrm{Re}(x)>0$.

By (4.1) and (2.7), we get

$$\zeta(-n,x \mid w_1,\ldots,w_r) = \frac{G_{n+r}^{(r)}(x \mid w_1,\ldots,w_r)}{\binom{n+r}{r}r!}, \tag{4.2}$$

where $n\in\mathbb{Z}^+$.

Define

$$L_\chi(s,x \mid w_1,\ldots,w_r) = 2^r\sum_{m_1,\ldots,m_r=0}^{\infty}\frac{(-1)^{m_1+\cdots+m_r}\left(\prod_{i=1}^r\chi(m_i)\right)}{(x+w_1m_1+\cdots+w_rm_r)^s}, \tag{4.3}$$

where $s\in\mathbb{C}$, and $\mathrm{Re}(x)>0$.

From (4.3), we note that

$$L_\chi(-n,x \mid w_1,\ldots,w_r) = \frac{G_{n+r,\chi}^{(r)}(x \mid w_1,\ldots,w_r)}{\binom{n+r}{r}r!}, \quad n\in\mathbb{Z}^+. \tag{4.4}$$

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