Research Article

Numerical Simulation of the FitzHugh-Nagumo Equations

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The variational iteration method and Adomian decomposition method are applied to solve the FitzHugh-Nagumo (FN) equations. The two algorithms are illustrated by studying an initial value problem. The obtained results show that only few terms are required to deduce approximated solutions which are found to be accurate and efficient.

1. Introduction

The pioneering work of Hodgkin and Huxley [1], and subsequent investigations, has established that good mathematical models for the conduction of nerve impulses along an axon can be given. These models take the form of a system of ordinary differential equations, coupled to a diffusion equation. Simpler models, which seem to describe the qualitative behavior, have been proposed by FitzHugh [2] and Nagumo [3]. This paper is devoted to the study of the FitzHugh-Nagumo (FN) system:

$$\begin{aligned}v_t(x, t) &= v_{xx}(x, t) - f(v(x, t)) - w(x, t), \\w_t(x, t) &= bv(x, t) - \gamma w(x, t),\end{aligned}\tag{1.1}$$

where b and γ are positive constants and $f(v(x, t))$ is nonlinear function. Existence and uniqueness for this system is given in 1978 by Rauch and Smoller [4], in which they showed

that small solutions $v(x, t)$ decay to 0 as $t \rightarrow \infty$ and large pulses produce a traveling wave. We consider the FN equations in the following form

$$\begin{aligned}v_t(x, t) &= v_{xx}(x, t) - f(v(x, t)) - w(x, t), \\w_t(x, t) &= bv(x, t),\end{aligned}\tag{1.2}$$

and the function $f(v(x, t))$ is given by McKean [5] such that:

$$f(v(x, t)) = v(x, t) - H(v(x, t) - a), \quad 0 \leq a \leq \frac{1}{2},\tag{1.3}$$

where H is the Heaviside step function

$$H(s) = \begin{cases} 0 & s < 0, \\ 1 & s \geq 0. \end{cases}\tag{1.4}$$

The exact solution of this system is given by:

$$v(x, t) = \begin{cases} a e^{\alpha_1 z}, & z \leq 0 \\ \left(a - \frac{1}{p'_1}\right) e^{\alpha_1 z} - \frac{1}{p'_2} e^{\alpha_2 z} - \frac{1}{p'_3} e^{\alpha_3 z}, & 0 \leq z \leq z_1 \\ \frac{e^{-\alpha_2 z_1} - 1}{p'_2} e^{\alpha_2 z} + \frac{e^{-\alpha_3 z_1} - 1}{p'_3} e^{\alpha_3 z}, & z_1 \leq z, \end{cases}\tag{1.5}$$

$$w(x, t) = v_{xx}(x, t) - v_t(x, t) - f(v(x, t)),\tag{1.6}$$

where $z = x + ct$, c is the speed of the traveling wave and α_i , $i = 1, 2, 3$ are the zeros of the polynomial

$$\begin{aligned}p(\alpha) &= \alpha^3 - c\alpha^2 - \alpha - \frac{b}{c}, \\p'_i &= p'(\alpha_i), \quad i = 1, 2, 3.\end{aligned}\tag{1.7}$$

A numerical scheme for FN equations [6] by collocation method and the ‘‘Hopscotch’’ finite difference scheme first proposed by Gordon [7], and further developed by Gourlay and McGuire [8, 9]. Other possible schemes which were considered are (i) finite difference schemes [10], (ii) Galerkin-type schemes [11], and (iii) collocation schemes with quadratic and cubic splines [6]. In this paper, we use the variational iteration and Adomian decomposition methods to find the numerical solutions of the FN equations which will be

useful in numerical studies. In our numerical study we consider the case $b = 0.1$ and $a = 0.3$, also

$$\begin{aligned}c &= 0.7122, \\ \alpha_1 &= 1.46192629534582, \\ \alpha_2 &= -0.1639653991443764, \\ \alpha_3 &= -0.5857608638090818, \\ z_1 &= 4.5976770121482735,\end{aligned}\tag{1.8}$$

with these parameters now we can use the exact travelling wave solution (1.5) to test the suggested numerical methods.

2. The Formalism

We introduce the main points of each of the two methods, where details can be found in [12–37].

2.1. The Variational Iteration Method (VIM)

The VIM is the general Lagrange method, in which an extremely accurate approximation at some special point can be obtained, but not an analytical solution. To illustrate the basic idea of the VIM we consider the following general partial differential equation:

$$L_t u(x, t) + L_x u(x, t) + Nu(x, t) + g(t, x) = 0,\tag{2.1}$$

where L_t and L_x are linear operators of t and x respectively, and N is a nonlinear operator. According to the VIM, we can express the following correction functional in t -, and x -directions, respectively, as follows:

$$u_{n+1}(x, t) = u_n(x, t) + \int_{t_0}^t \lambda (L_s u_n(x, s) + (L_x + N)\tilde{u}_n(x, s) + g(x, s)) ds,\tag{2.2a}$$

$$u_{n+1}(x, t) = u_n(x, t) + \int_{x_0}^x \mu (L_s u_n(t, s) + (L_t + N)\tilde{u}_n(t, x) + g(t, s)) ds,\tag{2.2b}$$

where λ and μ are general Lagrange multipliers, which can be identified optimally via the variational theory, and $\tilde{u}_n(x, t)$ are restricted variations which mean that $\delta \tilde{u}_n(x, t) = 0$. By this

method, it is required first to determine Lagrange multipliers λ and μ that will be identified optimally. The successive approximations $u_{n+1}(x, t)$, $n \geq 0$ of the solution $u(x, t)$ will be readily obtained upon using the determined Lagrange multipliers and any selective function $u_0(x, t)$. Consequently, the solution is given by

$$u(x, t) = \lim_{n \rightarrow \infty} u_n(x, t). \quad (2.3)$$

The above analysis yields the following theorem.

Theorem 2.1. *The VIM solution of the partial differential equation (2.1) can be determined by (2.3) with the iterations (2.2a) or (2.2b).*

2.2. Adomian Decomposition Method (ADM)

Applying the inverse operator $L^{-1}(\cdot) = \int_0^t (\cdot) dt$ to both sides of (2.1) and using the initial condition, we get

$$\begin{aligned} u_0(x, t) &= u(x, 0), \\ u_{n+1}(x, t) &= \int_0^t (-L_x u_n(x, t) - A_n - g(x, t)) dt, \quad n \geq 0, \end{aligned} \quad (2.4)$$

where the nonlinear operator $N(u) = \sum_{n=0}^{\infty} A_n$ is the Adomian polynomial determined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left(\sum_{i=0}^{\infty} \lambda^i u_i(x, t) \right) \Big|_{\lambda=0}, \quad n = 0, 1, 2, \dots \quad (2.5)$$

We next decompose the unknown function $u(x, t)$ by a sum of components defined by the following decomposition series

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t). \quad (2.6)$$

The above analysis yields the following theorem

Theorem 2.2. *The ADM solution of the partial differential equation (2.1) can be determined by the series (2.6) with the iterations (2.4).*

3. Applications

We solve the FN equations using the two methods VIM and ADM.

3.1. The VIM for the FN Equations

Consider the FN equations in the form

$$\begin{aligned}v_t - v_{xx} + f(v) + w &= 0, \\w_t - bv &= 0.\end{aligned}\tag{3.1}$$

Then the VIM formulae take the forms

$$\begin{aligned}v_{n+1}(x, t) &= v_n(x, t) + \int_0^t \lambda (v_s(x, s) - \tilde{v}_{xx}(x, s) + f(\tilde{v}(x, s)) + \tilde{w}(x, s)) ds, \\w_{n+1}(x, t) &= w_n(x, t) + \int_0^t \mu (w_s(x, s) - b\tilde{v}(x, s)) ds,\end{aligned}\tag{3.2}$$

where $v_0(x, t) = v(x, 0)$, $w_0(x, t) = w(x, 0)$ and $n \geq 0$. This yields the stationary conditions

$$\lambda'(s) = 0, \quad \lambda + 1|_{s=t} = 0, \quad \mu'(s) = 0, \quad \mu + 1|_{s=t} = 0,\tag{3.3}$$

Hence, the Lagrange multipliers are

$$\lambda(s) = \mu(s) = -1.\tag{3.4}$$

Substituting these values of Lagrange multipliers into the functional correction (3.2) gives the iterations formulae

$$\begin{aligned}v_{n+1}(x, t) &= v_n(x, t) + \int_0^t \lambda (v_s(x, s) - v_{xx}(x, s) + f(v(x, s)) + w(x, s)) ds, \\w_{n+1}(x, t) &= w_n(x, t) + \int_0^t \mu (w_s(x, s) - bv(x, s)) ds.\end{aligned}\tag{3.5}$$

We start with initial approximations as follows

$$v_0(x, t) = \begin{cases} a e^{\alpha_1 x}, & x \leq 0, \\ \left(a - \frac{1}{p'_1}\right) e^{\alpha_1 x} - \frac{1}{p'_2} e^{\alpha_2 x} - \frac{1}{p'_3} e^{\alpha_3 x}, & 0 \leq x \leq z_1, \\ \frac{e^{-\alpha_2 z_1} - 1}{p'_2} e^{\alpha_2 x} + \frac{e^{-\alpha_3 z_1} - 1}{p'_3} e^{\alpha_3 x}, & z_1 \leq x, \end{cases}\tag{3.6}$$

$$w_0(x, t) = (v_0(x, t))_{xx} - (v_0(x, t))_t - f(v_0(x, t)),$$

and then the first iterations are

$$v_1(x, t) = \begin{cases} v_{11}, & z \leq 0, \\ v_{12}, & 0 \leq z \leq z_1, \\ v_{13} & z_1 \leq z, \end{cases}$$

$$v_{11} = 0.312355e^{1.46193x}(0.960445 + t),$$

$$v_{12} = e^{-0.749726x} \left(e^{0.585761x} (1.45816 - 0.170279t) + e^{2.21165x} (-0.000361868 - 0.000376771t) \right. \\ \left. + e^{0.163965x} (-1.1578 + 0.483011t) \right),$$

$$v_{13} = e^{-0.749726x} \left(e^{0.163965x} (15.9522 - 6.65492t) + e^{0.585761x} (-1.64071 + 0.191596t) \right),$$

$$w_1(x, t) = \begin{cases} w_{11}, & z \leq 0 \\ w_{12}, & 0 \leq z \leq z_1 \\ w_{13} & z_1 \leq z \end{cases}$$

$$w_{11} = 0.03 e^{1.46193x}(0.960445 + t),$$

$$w_{12} = e^{-0.749726x} \left(e^{0.749726x} + e^{0.16396x} (0.277531 - 0.11578t) \right. \\ \left. + e^{2.21165x} (-0.000034755 - 0.0000361868t) + e^{0.585761x} (-1.24868 + 0.145816t) \right),$$

$$w_{13} = e^{-0.749726x} \left(e^{0.585761x} (1.405 - 0.164071t) + e^{0.163965x} (-3.82383 + 1.59522t) \right), \quad (3.7)$$

and so on.

The VIM produces the solutions $v(x, t)$, $w(x, t)$ as follows

$$v(x, t) = \lim_{n \rightarrow \infty} v_n(x, t), \quad w(x, t) = \lim_{n \rightarrow \infty} w_n(x, t), \quad (3.8)$$

where $v_n(x, t)$, $w_n(x, t)$, will be determined in a recursive manner.

3.2. The ADM for the FN Equations

Consider the FN equations in the following form:

$$Lv = v_{xx} - f(v) - w, \quad (3.9)$$

$$Lw = bv, \quad (3.10)$$

where $L(\cdot) = \partial(\cdot)/\partial t$. Operating by $L^{-1}(\cdot) = \int_0^t (\cdot) dt$ on both sides of (3.9), we get

$$\begin{aligned} v(x, t) &= v(x, 0) + \int_0^t (v_{xx}(x, t) - f(v(x, t)) - w(x, t)) dt, \\ w(x, t) &= w(x, 0) + \int_0^t (bv(x, t)) dt. \end{aligned} \quad (3.11)$$

The ADM assumes that the unknown functions $v(x, t)$ and $w(x, t)$ can be expressed by an infinite series in the forms

$$\begin{aligned} v(x, t) &= \sum_{n=0}^{\infty} v_n(x, t), \\ w(x, t) &= \sum_{n=0}^{\infty} w_n(x, t), \end{aligned} \quad (3.12)$$

where $v_n(x, t)$, and $w_n(x, t)$ can be determined by using the recurrence relations:

$$\begin{aligned} v_{n+1}(x, t) &= \int_0^t (v_{nxx}(x, t) - f(v_n(x, t)) - w_n(x, t)) dt, \\ w_{n+1}(x, t) &= \int_0^t (bv_n(x, t)) dt, \quad n = 0, 1, \dots, \end{aligned} \quad (3.13)$$

where

$$f(v_n(x, t)) = \begin{cases} v_n(x, t), & v_n(x, t) < a, \\ v_n(x, t) - 1 & v_n(x, t) \geq a \end{cases} \quad (3.14)$$

such that

$$\begin{aligned} v_0(x, t) &= v(x, 0), \\ w_0(x, t) &= w(x, 0). \end{aligned} \quad (3.15)$$

Then the first iterations are

$$v_1(x, t) = \begin{cases} 0.312355e^{1.46193}, & z \leq 0 \\ t + (-1 + 0.483011e^{-0.585761x} - 0.170279e^{-0.163965x} \\ \quad - 0.000376771e^{1.46193x})t, & 0 \leq z \leq z_1 \\ (-6.65492e^{-0.585761x} + 0.191596e^{-0.163965x})t, & z_1 \leq z \end{cases} \quad (3.16a)$$

$$w_1(x, t) = \begin{cases} 0.03 e^{1.46193x}, & z \leq 0 \\ 0.1(-1.1578 e^{-0.585761x} + 1.45816e^{-0.163965x} - 0.000361868e^{1.46193x})t, & 0 \leq z \leq z_1, \\ 0.1 (15.9522 e^{-0.585761x} - 1.64071 e^{-0.163965x})t, & z_1 \leq z \end{cases} \quad (3.16b)$$

and so on.

The ADM yields the solutions $v(x, t)$, $w(x, t)$ as

$$v(x, t) = \sum_{n=0}^{\infty} v_n(x, t), \quad w(x, t) = \sum_{n=0}^{\infty} w_n(x, t), \quad (3.17)$$

where $v_n(x, t)$, $w_n(x, t)$, will be determined in a recursive manner.

4. A Test Problem for the FN Equations

We discuss the solutions of the FN equations using the two considered VIM and ADM methods.

4.1. The VIM

Solve the FN equations (1.2) using the VIM with finite iterations at time $T = 5$. A comparison between the computed solutions and the exact solutions at different values of x are given in Table 1. We note that the VIM solutions converge to the exact solutions specially when n is increased. We show in Figure 1 the behavior of the VIM solutions of FN equations at time $T = 5$. If the exact solutions are plotted on Figure 1 we will find that the VIM and exact solutions curves are indistinguishable.

4.2. The ADM

Consider the same problems and use the ADM with the same initial conditions and use the technique discussed in Section 2. A comparison between the exact solutions and ADM solutions are shown in Table 2 and it seems that the errors are very small. We show in Figure 2 the numerical solutions of the FN equations.

The results listed in Table 3 are representing the maximum errors at different times of VIM and ADM which shows that the VIM is better than ADM in the solutions of FN equations.

Figure 1: The approximated solutions for $v(x,t)$, $w(x,t)$ at time $T = 5$.

Table 1: Comparison between the exact and approximate (VIM) solutions for the FN equations at time $T = 5$.

x	v_{VIM}	v_{exact}	w_{VIM}	w_{exact}
-7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
-3.561	0.3	0.3	0.0288134	0.0288134
-0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	-0.25639	-0.25639	0.382524	0.382524
8.439	-0.215232	-0.215232	0.193024	0.193024
16.439	-0.0616494	-0.0616494	16.439	16.439
22.439	-0.023095	-0.023095	0.0197795	0.0197795
48.439	-0.000325199	-0.000325199	0.000278481	0.000278481

Now we show a comparison between our schemes and other methods as shown in Table 4.

It is clear that the suggested methods for solving FN equation are the best methods than all other methods. Also all other methods give the solution as a discrete solution but our methods give the solution as a function x and t .

Figure 2: The approximation solutions $v(x, t)$, $w(x, t)$.

Table 2: Comparison between the exact solutions and approximation solutions (ADM) for FN equations at time = 5.

x	v_{ADM}	v_{exact}	w_{ADM}	w_{exact}
-7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
-3.561	0.3	0.3	0.0288134	0.0288134
-0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	-0.25639	-0.25639	0.382524	0.382524
8.439	-0.215232	-0.215232	0.193024	0.193024
16.439	-0.0616494	-0.0616494	16.439	16.439
22.439	-0.023095	-0.023095	0.0197795	0.0197795
48.439	-0.000325199	-0.000325199	0.000278481	0.000278481

Table 3: The maximum errors of our suggested methods VIM and ADM.

Time	VIM		ADM	
	Max. errors for $v(x, t)$	Max. errors for $w(x, t)$	Max. errors for $v(x, t)$	Max. errors for $w(x, t)$
2.0	$3.66374E-15$	$4.02456E-16$	$3.71925E-15$	$4.71845E-16$
4.0	$1.14429E-9$	$1.09902E-10$	$1.15225E-9$	$1.10668E-10$
6.0	$3.37523E-7$	$3.24191E-8$	$3.37523E-7$	$3.24191E-8$

Table 4: Comparison between VIM, ADM, and other methods by maximum errors.

Method	$T = 1.60$	$T = 10.0$
Finite difference		
C-N	$0.848E - 2$	0.189
Hopscotch [9]	$0.557E - 2$	0.0506
Collocation method		
Quadratic [6]	$0.758E - 2$	0.138
Cubic [6]	$0.589E - 2$	0.12
VIM	$3.33067E - 16$	0.000316341
ADM	$4.44089E - 16$	0.000316341

5. Conclusion

In this paper the solutions for the FN equations using VIM and ADM methods have been generated. All numerical results obtained using few terms of the VIM and ADM show very good agreement with the exact solutions. Comparing our results with those of previous several methods shows that the considered techniques are more reliable, powerful, and promising mathematical tools. We believe that the accuracy of the VIM and ADM recommend it to be much wider applicability and also we find that the VIM more accurate than ADM.

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