Research Article

λ -Statistical Convergence in Paranormed Space

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The concept of λ -statistical convergence for sequences of real numbers was introduced in Mursaleen (2000). In this paper, we prove decomposition theorem for λ -statistical convergence. We also define and study λ -statistical convergence, λ -statistically Cauchy, and strongly λ_p -summability in Paranormed Space.

1. Introduction

The notion of statistical convergence was first introduced by Fast [1]. In the recent years, statistical summability became one of the most active areas of research in summability theory, which was further generalized as lacunary statistical convergence [2], λ -statistical convergence [3], statistical A -summability [4], and statistical σ -convergence [5]. Maddox [6] studied this notion in locally convex Hausdorff topological spaces and Kolk [7] defined and studied this notion in Banach spaces while Çakalli [8] extended it to topological Hausdorff groups. The concept of statistical convergence is studied in probabilistic normed space and in intuitionistic fuzzy normed spaces in [9, 10]. Recently, the statistical convergence has been studied in Paranormed Space and locally solid Riesz spaces in [11, 12], respectively. Therefore, one can choose either some different setup to study these concepts or generalizing the existing concepts through different means. In this paper, we will study the concept of λ -statistical convergence, λ -statistical Cauchy, and strongly λ_p -summability in Paranormed Space.

A *paranorm* is a function $g : X \rightarrow \mathbb{R}$ defined on a linear space X such that for all $x, y, z \in X$

$$(P1) \quad g(x) = 0 \text{ if } x = \theta,$$

$$(P2) \quad g(-x) = g(x),$$

$$(P3) \quad g(x + y) \leq g(x) + g(y),$$

(P4) if (α_n) is a sequence of scalars with $\alpha_n \rightarrow \alpha_0$ ($n \rightarrow \infty$) and $x_n, a \in X$ with $x_n \rightarrow a$ ($n \rightarrow \infty$) in the sense that $g(x_n - a) \rightarrow 0$ ($n \rightarrow \infty$), then $\alpha_n x_n \rightarrow \alpha_0 a$ ($n \rightarrow \infty$), in the sense that $g(\alpha_n x_n - \alpha_0 a) \rightarrow 0$ ($n \rightarrow \infty$).

A paranorm g for which $g(x) = 0$ implies that $x = \theta$ is called a *total paranorm* on X , and the pair (X, g) is called a *total Paranormed Space*.

2. λ -Statistical Convergence

Let $\lambda = (\lambda_n)$ be a nondecreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{n+1} \leq \lambda_n + 1, \quad \lambda_1 = 0. \quad (1)$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) =: \frac{1}{\lambda_n} \sum_{j \in I_n} x_j, \quad (2)$$

where $I_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_j)$ is said to be (V, λ) -summable to a number L if

$$t_n(x) \rightarrow L \quad \text{as } n \rightarrow \infty. \quad (3)$$

Let K be a subset of the set of natural numbers $\mathbb{N}$. Then, the λ -density of K is defined as

$$\delta_\lambda(K) = \lim_n \frac{1}{\lambda_n} |\{n - \lambda_n + 1 \leq j \leq n : j \in K\}|. \quad (4)$$

The number sequence $x = (x_j)$ is said to be λ -statistically convergent to the number L (c.f. [3, 13, 14]) if $\delta_\lambda(K(\epsilon)) = 0$; that is, if for each $\epsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : |x_k - L| \geq \epsilon\}| = 0. \quad (5)$$

In this case we write $\text{st}_\lambda\text{-}\lim_k x_k = L$ and we denote the set of all λ -statistically convergent sequences by S_λ . In case $\lambda_n = n$, λ -density reduces to the natural density and λ -statistical convergence reduces to statistical convergence. This notion for double sequences has been studied in [15].

A sequence $x = (x_k)$ is said to be *strongly* λ_p -summable ($0 < p < \infty$) to the limit L [14] if

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} |x_k - L|^p = 0, \quad (6)$$

and we write it as $x_k \rightarrow L[V_\lambda]_p$. In this case L is called the $[V_\lambda]_p$ -limit of x .

The following relation was established in [14].

Theorem 1. *If $0 < p < \infty$ and a sequence $x = (x_k)$ is strongly λ_p -summable to L , then it is λ -statistically convergent to L . If a bounded sequence is λ -statistically convergent to L , then it is strongly λ_p -summable to L .*

The following theorem is λ -statistical version of Connor's Decomposition Theorem [16].

Theorem 2. *If $x = (x_k)$ is strongly λ_p -summable or statistically λ -convergent to L , then there is a convergent sequence y and a λ -statistically null sequence z such that y is convergent to L , $x = y + z$ and*

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : z_k \neq 0\}| = 0. \quad (7)$$

Moreover, if x is bounded, then y and z both are bounded.

Proof. By Theorem 1, it follows that x is λ -statistically convergent to L if x is strongly λ_p -summable to L . Set $N_0 = 0$ and choose a strictly increasing sequence of positive integers $N_1 < N_2 < N_3 < \dots$ such that

$$\frac{1}{\lambda_n} |\{k \in I_n : |x_k - L| \geq j^{-1}\}| < j^{-1} \quad (8)$$

for $n > N_j$. Define y and z as follows.

If $N_0 < k < N_1$ set $z_k = 0$ and $y_k = x_k$. Let $j \geq 1$ and $N_j < k \leq N_{j+1}$. Now we set

$$y_k = \begin{cases} x_k, & z_k = 0, & \text{if } |x_k - L| < j^{-1}; \\ L, & z_k = x_k - L, & \text{if } |x_k - L| \geq j^{-1}. \end{cases} \quad (9)$$

Clearly, $x = y + z$ and y and z are bounded, if x is bounded. Also, we observe that for $k > N_j$, we have

$$\begin{aligned} |y_k - L| &< \epsilon & \text{since } |y_k - L| &= |x_k - L| < \epsilon \\ && \text{if } |x_k - L| < j^{-1}, \\ |y_k - L| &= |L - L| = 0 & \text{if } |x_k - L| > j^{-1}. \end{aligned} \quad (10)$$

Hence, $\lim_k y_k = L$, since ϵ was arbitrary.

Next we observe that

$$|\{k \in I_n : z_k \neq 0\}| \geq |\{k \in I_n : |z_k| \geq \epsilon\}| \quad (11)$$

for any natural number n and $\epsilon > 0$. Hence, $\lim_n (1/\lambda_n) |\{k \in I_n : z_k \neq 0\}| = 0$; that is, z is λ -statistically null.

We now show that if $\delta > 0$ and $j \in \mathbb{N}$ such that $j^{-1} < \delta$, then $|\{k \in I_n : z_k \neq 0\}| < \delta$ for all $n > N_j$. Recall from the construction that if $N_j < k \leq N_{j+1}$, then $z_k \neq 0$ only if $|x_k - L| > j^{-1}$. It follows that if $N_\ell < k \leq N_{\ell+1}$, then

$$\{k \in I_n : z_k \neq 0\} \subseteq \{k \in I_n : |x_k - L| > \ell^{-1}\}. \quad (12)$$

Consequently,

$$\begin{aligned} \frac{1}{\lambda_n} |\{k \in I_n : z_k \neq 0\}| &\leq \frac{1}{\lambda_n} |\{k \in I_n : |x_k - L| > \ell^{-1}\}| \\ &< \ell^{-1} < j^{-1} < \delta, \end{aligned} \quad (13)$$

if $N_\ell < n \leq N_{\ell+1}$ and $\ell > j$. That is,

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : z_k \neq 0\}| = 0. \quad (14)$$

This completes the proof of the theorem. $\square$

3. Application to Fourier Series

Let $f : \mathbb{T} \rightarrow \mathbb{C}$ be a Lebesgue integrable function on the torus $\mathbb{T} := [-\pi, \pi)$; that is, $f \in L^1(\mathbb{T})$. The Fourier series of f is defined by

$$f(x) \sim \sum_{j \in \mathbb{Z}} \hat{f}(j) e^{ijx}, \quad x \in \mathbb{T}, \quad (15)$$

where the Fourier coefficients $\hat{f}(j)$ are defined by

$$\hat{f}(j) := \frac{1}{2\pi} \int_{\mathbb{T}} f(t) e^{-ijt} dt, \quad j \in \mathbb{Z}. \quad (16)$$

The symmetric partial sums of the series in (15) are defined by

$$s_k(f; x) := \sum_{|j| \leq k} \hat{f}(j) e^{ijx}, \quad x \in \mathbb{T}, \quad k \in \mathbb{N}. \quad (17)$$

The conjugate series to the Fourier series in (15) is defined by [17, Vol. I, pp. 49]

$$\sum_{j \in \mathbb{Z}} (-i \operatorname{sgn} j) \hat{f}(j) e^{ijx}. \quad (18)$$

Clearly, it follows from (15) and (18) that

$$\begin{aligned} & \sum_{j \in \mathbb{Z}} \hat{f}(j) e^{ijx} + i \sum_{j \in \mathbb{Z}} (-i \operatorname{sgn} j) \hat{f}(j) e^{ijx} \\ &= 1 + 2 \sum_{j=1}^{\infty} \hat{f}(j) e^{ijx}, \end{aligned} \quad (19)$$

and the power series

$$1 + 2 \sum_{j=1}^{\infty} \hat{f}(j) e^{ijx}, \quad \text{where } z := re^{ix}, \quad 0 \leq r < 1, \quad (20)$$

is analytic on the open unit disk $|z| < 1$, due to the fact that

$$|\hat{f}(j)| \leq \frac{1}{2\pi} \int_{\pi} |f(t)| dt, \quad j \in \mathbb{Z}. \quad (21)$$

The conjugate function $\hat{f}$ of a function $f \in L^1(\mathbb{T})$ is defined by

$$\begin{aligned} \hat{f}(x) &:= -\lim_{\varepsilon \downarrow 0} \frac{1}{\pi} \int_{\varepsilon \leq |t| \leq \pi} \frac{f(x+t)}{2 \tan(t/2)} dt \\ &= \lim_{\varepsilon \downarrow 0} \frac{1}{\pi} \int_{\varepsilon}^{\pi} \frac{f(x-t) - f(x+t)}{2 \tan(t/2)} dt \end{aligned} \quad (22)$$

in the “principal value” sense and that $\hat{f}(x)$ exists at almost every $x \in \mathbb{T}$.

The following is λ -statistical version of [18] (c.f. [19, Theorem 2.1 (ii)]).

Theorem 3. *If $f \in L^1(\mathbb{T})$, then for any $p > 0$ its Fourier series is strongly λ_p -summable to $f(x)$ at almost every $x \in \mathbb{T}$. Furthermore, its conjugate series (18) is strongly λ_p -summable for any $p > 0$ to the conjugate function $\hat{f}(x)$ defined in (22) at almost every $x \in \mathbb{T}$.*

From Theorems 1 and 3, we easily get the following useful result.

Theorem 4. *If $f \in L^1(\mathbb{T})$, then its Fourier series is λ -statistically convergent to $f(x)$ at almost every $x \in \mathbb{T}$. Furthermore, its conjugate series (18) is λ -statistically convergent to the conjugate function $\hat{f}(x)$ defined in (22) at almost every $x \in \mathbb{T}$.*

4. λ -Statistical Convergence in Paranormed Space

Recently, statistical convergence, statistical Cauchy, and strongly Cesàro summability have been studied in Paranormed Space by Alotaibi and Alroqi [11].

In this paper, we define and study the notion of λ -summable, λ -statistical convergence, λ -statistical Cauchy, and strongly λ_p -summability in Paranormed Space.

Let (X, g) be a Paranormed Space.

A sequence $x = (x_k)$ is said to be *convergent* to the number ξ in (X, g) if, for every $\varepsilon > 0$, there exists a positive integer k_0 such that $g(x_k - \xi) < \varepsilon$ whenever $k \geq k_0$. In this case, we write $g\text{-}\lim x = \xi$, and ξ is called the *g-limit* of x .

We define the following.

Definition 5. A sequence $x = (x_k)$ is said to be λ -statistically convergent to the number ξ in (X, g) if, for each $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : g(x_k - \xi) \geq \varepsilon\}| = 0. \quad (23)$$

In this case we write $\operatorname{st}_{\lambda}(g)\text{-}\lim x = \xi$.

Definition 6. A sequence $x = (x_k)$ is said to be λ -statistically Cauchy sequence in (X, g) if for every $\varepsilon > 0$ there exists a number $N = N(\varepsilon)$ such that

$$\lim_n \frac{1}{\lambda_n} |\{j \in I_n : g(x_j - x_N) \geq \varepsilon\}| = 0. \quad (24)$$

Definition 7. A sequence $x = (x_k)$ is said to be *strongly λ_p -summable* ($0 < p < \infty$) to the limit ξ in (X, g) if

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} (g(x_k - \xi))^p = 0, \quad (25)$$

and we write it as $x_k \rightarrow \xi[V_{\lambda}, g]_p$. In this case ξ is called the $[V_{\lambda}, g]_p$ -limit of x .

Now we define another type of convergence in Paranormed Space.

Definition 8. A sequence (x_k) in a Paranormed Space (X, g) is said to $\operatorname{st}_{\lambda}^*(g)$ -convergent to $\xi \in X$ if there exists an index set $K = \{k_1 < k_2 < \dots < k_n < \dots\} \subseteq \mathbb{N}$, $n = 1, 2, \dots$, with $\delta_{\lambda}(K) = 1$ such that $g(x_{k_n} - \xi) \rightarrow 0$ ($n \rightarrow \infty$). In this case, we write $\xi = \operatorname{st}_{\lambda}^*(g)\text{-}\lim x$.

First we prove the following results on λ -statistical convergence in (X, g) .

Theorem 9. *If $g\text{-}\lim x = \xi$, then $\operatorname{st}_{\lambda}(g)\text{-}\lim x = \xi$ but converse need not be true in general.*

Proof. Let $g\text{-}\lim x = \xi$. Then, for every $\varepsilon > 0$, there is a positive integer N such that

$$g(x_n - \xi) < \varepsilon \quad (26)$$

for all $n \geq N$. Since the set $A(\varepsilon) := \{k \in \mathbb{N} : g(x_k - \xi) \geq \varepsilon\}$ is finite, $\delta_{\lambda}(A(\varepsilon)) = 0$. Hence, $\operatorname{st}_{\lambda}(g)\text{-}\lim x = \xi$.

The following example shows that the converse need not be true.

Example 10. Let $X = \ell(1/k) := \{x = (x_k) : \sum_k |x_k|^{1/(k+1)} < \infty\}$ with the paranorm $g(x) = (\sum_k |x_k|^{1/(k+1)})$. Define a sequence $x = (x_k)$ by

$$x_k := \begin{cases} k, & \text{if } n - [\lambda_n] + 1 \leq k \leq n, n \in \mathbb{N}; \\ 0, & \text{otherwise,} \end{cases} \quad (27)$$

and write

$$K(\varepsilon) := \{k \leq n : g(x_k) \geq \varepsilon\}, \quad 0 < \varepsilon < 1. \quad (28)$$

We see that

$$g(x_k) := \begin{cases} k^{1/(k+1)}, & \text{if } n - [\lambda_n] + 1 \leq k \leq n, n \in \mathbb{N}; \\ 0, & \text{otherwise,} \end{cases} \quad (29)$$

and hence

$$\lim_k g(x_k) := \begin{cases} 1, & \text{if } n - [\lambda_n] + 1 \leq k \leq n, n \in \mathbb{N}; \\ 0, & \text{otherwise.} \end{cases} \quad (30)$$

Therefore g -lim x does not exist. On the other hand $\delta_\lambda(K(\varepsilon)) = 0$; that is, $st_\lambda(g)$ -lim $x = 0$.

This completes the proof of the theorem. $\square$

We can easily prove the following results on λ -statistical convergence in (X, g) similar to those of [11].

Theorem 11. *If a sequence $x = (x_k)$ is λ -statistically convergent in (X, g) , then $st_\lambda(g)$ -limit is unique.*

Theorem 12. *Let $st_\lambda(g)$ -lim $x = \xi_1$ and $st_\lambda(g)$ -lim $y = \xi_2$. Then,*

- (i) $st_\lambda(g)$ -lim $(x \pm y) = \xi_1 \pm \xi_2$,
- (ii) $st_\lambda(g)$ -lim $\alpha x = \alpha \xi_1$, $\alpha \in \mathbb{R}$.

Theorem 13. *Let (X, g) be a complete Paranormed Space. Then a sequence $x = (x_k)$ of points in (X, g) is λ -statistically convergent if and only if it is λ -statistically Cauchy.*

Theorem 14. (a) *If $0 < p < \infty$ and $x_k \rightarrow \xi[V_\lambda, g]_p$, then $x = (x_k)$ is λ -statistically convergent to ξ in (X, g) .*

(b) *If $x = (x_k)$ is bounded and λ -statistically convergent to ξ in (X, g) , then $x_k \rightarrow \xi[V_\lambda, g]_p$.*

Theorem 15. *Let (X, g) be a complete Paranormed Space. Then a sequence $x = (x_k)$ of points in (X, g) is λ -statistically convergent if and only if it is λ -statistically Cauchy.*

Note that the proof of Theorem 2.4 [11] is incorrect and the correct proof is given in the following theorem which is generalization of Theorem 2.4 [11]. Another form of this result is given in [20] for ideal convergence.

Theorem 16. *A sequence $x = (x_k)$ in (X, g) is λ -statistically convergent to ξ if and only if it is $st_\lambda^*(g)$ -convergent to ξ .*

Proof. Suppose that $x = (x_k)$ is λ -statistically convergent to ξ ; that is, $st_\lambda(g)$ -lim $x = \xi$. Now, write for $r = 1, 2, \dots$

$$K_r := \left\{ n \in \mathbb{N} : g(x_{k_n} - \xi) \geq \frac{1}{r} \right\}, \quad (31)$$

$$M_r := \left\{ n \in \mathbb{N} : g(x_{k_n} - \xi) < \frac{1}{r} \right\} \quad (r = 1, 2, \dots).$$

Then $\delta_\lambda(K_r) = 0$,

$$M_1 \supset M_2 \supset \dots \supset M_i \supset M_{i+1} \supset \dots, \quad (32)$$

$$\delta_\lambda(M_r) = 1, \quad r = 1, 2, \dots \quad (33)$$

Now we have to show that, for $n \in M_r$, (x_{k_n}) is g -convergent to ξ . On contrary suppose that (x_{k_n}) is not g -convergent to ξ . Therefore, there is $\varepsilon > 0$ such that $g(x_{k_n} - \xi) \geq \varepsilon$ for infinitely many terms. Let $M_\varepsilon := \{n \in \mathbb{N} : g(x_{k_n} - \xi) < \varepsilon\}$ and $\varepsilon > 1/r$, $r \in \mathbb{N}$.

Then

$$\delta_\lambda(M_\varepsilon) = 0, \quad (34)$$

and by (32), $M_r \subset M_\varepsilon$. Hence $\delta_\lambda(M_r) = 0$, which contradicts (33) and we get that (x_{k_n}) is g -convergent to ξ . Hence, x is $st_\lambda^*(g)$ -convergent to ξ .

Conversely, suppose that x is $st_\lambda^*(g)$ -convergent to ξ . Then there exists a set $K = \{k_1 < k_2 < k_3 < \dots < k_n < \dots\}$ with $\delta_\lambda(K) = 1$ such that g -lim $_{n \rightarrow \infty} x_{k_n} = \xi$. Therefore, there is a positive integer N such that $g(x_{k_n} - \xi) < \varepsilon$ for $n \geq N$. Put $K_\varepsilon := \{n \in \mathbb{N} : g(x_n - \xi) \geq \varepsilon\}$ and $K' := \{k_{N+1}, k_{N+2}, \dots\}$. Then $\delta_\lambda(K') = 1$ and $K_\varepsilon \subseteq \mathbb{N} - K'$ which implies that $\delta_\lambda(K_\varepsilon) = 0$. Hence $x = (x_k)$ is λ -statistically convergent to ξ ; that is $st_\lambda(g)$ -lim $x = \xi$.

This completes the proof of the theorem. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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