Letter to the Editor

Periodic Solution of the Hematopoiesis Equation

Ji-Huan He

National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, 199 Ren-Ai Road, Suzhou 215123, China

Correspondence should be addressed to Ji-Huan He; hejihuan@suda.edu.cn

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Wu and Liu (2012) presented some results for the existence and uniqueness of the periodic solutions for the hematopoiesis model. This paper gives a simple approach to find an approximate period of the model.

Wu and Liu studied the following hematopoiesis model [1]:

$$x'(t) = -ax(t) + \frac{\beta\theta^n}{\theta^n + x^n(t-\tau)}, \quad (1)$$

where x denotes the density of mature cells in blood circulation. The physical meaning of other parameters is referred to [1].

Equation (1) admits periodic solutions as revealed in [1]. Hereby we suggest a simple approach to the search for an approximate period of (1) using a simple amplitude-frequency formulation [2–5]. To this end, we rewrite (1) in the form

$$\begin{aligned} x'(t)\theta^n + x'(t)x^n(t-\tau) + ax(t)\theta^n \\ + ax(t)x^n(t-\tau) - \beta\theta^n = 0. \end{aligned} \quad (2)$$

Assume that the periodic solution can be expressed in the form

$$x(t) = A \cos \omega t. \quad (3)$$

Submitting (3) into (2) results in the following residual:

$$\begin{aligned} R(\omega, t) = & -A\omega\theta^n \sin \omega t \\ & - A^{1+n} \omega \sin \omega t \cos^n \omega(t-\tau) + a\theta^n A \cos \omega t \\ & + aA^{1+n} \cos \omega t \cos^n \omega(t-\tau) - \beta\theta^n. \end{aligned} \quad (4)$$

In order to use the amplitude-frequency formulation [2–5], we choose two trial frequencies and locate them at $t = \pi/(4\omega)$.

Setting $\omega_1 = 1$, $\omega_1 t = \pi/4$, and $\omega_1 = 2$, $\omega_2 t = \pi/4$, respectively, we have

$$\begin{aligned} R_1 = & -\frac{\sqrt{2}}{2}A\theta^n - \frac{\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4} - \tau\right) \\ & + \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4} - \tau\right) - \beta\theta^n, \\ R_2 = & -\sqrt{2}A\theta^n - \sqrt{2}A^{1+n}\cos^n\left(\frac{\pi}{4} - 2\tau\right) \\ & + \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4} - 2\tau\right) - \beta\theta^n. \end{aligned} \quad (5)$$

The frequency can be then obtained approximately in the form [2–5]

$$\begin{aligned} \omega^2 = & \frac{R_1\omega_1^2 - R_2\omega_2^2}{R_1 - R_2} = \frac{R_1 - 4R_2}{R_1 - R_2} \\ = & \left(\frac{7\sqrt{2}}{2}A\theta^n + \frac{7\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4} - 2\tau\right) - \frac{3\sqrt{2}}{2}a\theta^n A \right. \\ & \left. - \frac{3\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4} - 2\tau\right) + 3\beta\theta^n \right) \end{aligned}$$

$$\times \left(\left(\sqrt{2} - \frac{\sqrt{2}}{2} \right) A \theta^n - \left(\sqrt{2} - \frac{\sqrt{2}}{2} \right) A^{1+n} \cos^n \left(\frac{\pi}{4} - \tau \right) \right)^{-1}. \quad (6)$$

This formulation has been widely used to solve periodic solutions of various nonlinear oscillators [6–13], and it is often called as He's frequency formulation, He's amplitude-frequency formulation, or He's frequency-amplitude formulation. In case $\omega^2 < 0$, no period solution is admitted. A similar criterion is given for a nonlinear equation arising in electrospinning process [14].

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