Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 312479, 10 pages
http://dx.doi.org/10.1155/2013/312479

Research Article

Berinde-Type Generalized Contractions on Partial Metric Spaces

Hassen Aydi,[1] Sana Hadj Amor,[2] and Erdal Karapınar[3]

[1] *Department of Mathematics, College of Education of Jubail, Dammam University, Saudi Arabia*
[2] *Laboratoire Physique Mathmaétique, Fonctions Spéciales et Applications (MAPSFA) LR11ES35, Université de Sousse, Ecole Supérieure des Sciences et de Technologie de Hammam Sousse, Rue Lamine el Abbassi, 4011 Hammam Sousse, Tunisia*
[3] *Department of Mathematics, Atilim University, İncek, 06836 Ankara, Turkey*

Correspondence should be addressed to Erdal Karapınar; erdalkarapinar@yahoo.com

Received 21 October 2012; Accepted 4 December 2012

Academic Editor: Abdul Latif

We consider generalized Berinde-type contractions in the context of partial metric spaces. Such contractions are also known as generalized almost contractions in the literature. In this paper, we extend, generalize, and enrich the results in this direction. Some examples are presented to illustrate our results.

1. Introduction and Preliminaries

Matthews [1] introduced the notion of a partial metric space as a part of the study of denotational semantics of data for networks, showing that the contraction mapping principle [2] can be generalized to the partial metric context for applications in program verifications. Later, there have been several recent extensive researchs on (common) fixed points for different contractions on partial metric spaces, see [3–28].

First, we recall some basic concepts and notations.

Definition 1. A partial metric on a nonempty set X is a function $p : X \times X \to [0, +\infty)$ such that for all $x, y, z \in X$:

$(p1)$ $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

$(p2)$ $p(x, x) \leq p(x, y)$,

$(p3)$ $p(x, y) = p(y, x)$,

$(p4)$ $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

Example 2 (see [1]). Let $X = \mathbb{R}^+$ and p defined on X by $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a partial metric space.

Example 3 (see [20, 26]). Let (X, d) and (X, p) be a metric space and a partial metric space, respectively. Functions $\rho_i : X \times X \to \mathbb{R}^+$ $(i \in \{1, 2, 3\})$ given by

$$\begin{aligned} \rho_1(x, y) &= d(x, y) + p(x, y), \\ \rho_2(x, y) &= d(x, y) + \max\{u(x), u(y)\}, \\ \rho_3(x, y) &= d(x, y) + a, \end{aligned} \tag{1}$$

define partial metrics on X, where $u : X \to \mathbb{R}^+$ is an arbitrary function and $a \geq 0$.

Example 4 (see [1]). Let $X = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$ and define $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (X, p) is a partial metric space.

Example 5 (see [1]). Let $X = [0, 1] \cup [2, 3]$ and define $p : X \times X \to \mathbb{R}^+$ by

$$p(x, y) = \begin{cases} \max\{x, y\}, & \text{if } \{x, y\} \cap [2, 3] \neq \emptyset, \\ |x - y|, & \text{if } \{x, y\} \subset [0, 1]. \end{cases} \tag{2}$$

Then (X, p) is a partial metric space.

Remark 6. It is clear that, if $p(x, y) = 0$, then from $(p1)$ and $(p2)$, we get $x = y$. On the other hand, $p(x, y)$ may not be 0 even if $x = y$.

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x,\varepsilon), x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

If p is a partial metric on X, then the functions $d_p, d_m^p : X \times X \to \mathbb{R}_+$ given by

$$\begin{aligned} d_p(x,y) &= 2p(x,y) - p(x,x) - p(y,y), \\ d_m^p(x,y) &= \max\{p(x,y) - p(x,x), p(x,y) - p(y,y)\} \\ &= p(x,y) - \min\{p(x,x), p(y,y)\} \end{aligned} \tag{3}$$

are equivalent metrics on X.

Definition 7 (see [1]). Let (X,p) be a partial metric space.

(1) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is called a Cauchy sequence in (X,p) if $\lim_{n,m\to+\infty} p(x_n,x_m)$ exists and is finite.

(2) (X,p) is called complete if every Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$ converges with respect to τ_p to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to+\infty} p(x_n,x_m)$.

Lemma 8 (see [1]). *Let (X,p) be a partial metric space.*

(1) *$\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,d_p).*

(2) *A partial metric space (X,p) is complete if and only if the metric space (X,d_p) is complete. Furthermore, $\lim_{n\to+\infty} d_p(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to+\infty} p(x_n,x) = \lim_{n,m\to+\infty} p(x_n,x_m). \tag{4}$$

Lemma 9 (see [20]). *Let $\{x_n\}_{n\in\mathbb{N}}$ be a convergent sequence in a partial metric space X such that $x_n \to x$ and $x_n \to y$ with respect to τ_p. If*

$$\lim_{n\to+\infty} p(x_n,x_n) = p(x,x) = p(y,y), \tag{5}$$

then $x = y$.

Lemma 10 (see [20]). *Let $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$ be two sequences in a partial metric space X such that*

$$\begin{aligned} \lim_{n\to+\infty} p(x_n,x) &= \lim_{n\to+\infty} p(x_n,x_n) = p(x,x), \\ \lim_{n\to+\infty} p(y_n,y) &= \lim_{n\to+\infty} p(y_n,y_n) = p(y,y), \end{aligned} \tag{6}$$

then $\lim_{n\to+\infty} p(x_n,y_n) = p(x,y)$. In particular, $\lim_{n\to+\infty} p(x_n,z) = p(x,z)$ for every $z \in X$.

Lemma 11 (see [3]). *Let (X,p) be a partial metric space and $x_n \to z$, with respect to τ_p, with $p(z,z) = 0$. Then $\lim_{n\to+\infty} p(x_n,y) = p(z,y)$ for all $y \in X$.*

The concept of almost contractions was introduced by Berinde [29, 30] on metric spaces. Other results on almost contractions could be found in [31–34]. Recently, Altun and Acar [35] characterized this concept in the setting of partial metric space and proved some fixed point theorems using these concepts. Very recently, Turkoglu and Ozturk [27] established a fixed point theorem for four mappings satisfying an almost generalized contractive condition on partial metric spaces. In this paper, we generalize the results given in [27, 35] by presenting some fixed point results for self mappings involving some almost generalized contractions in the setting of partial metric spaces. Also, we give some illustrative examples making our results proper.

2. Main Results

We start to this section by defining some sets of auxiliary functions. Let $\mathcal{F}$ denote all functions $f : [0,+\infty) \to [0,+\infty)$ such that $f(t) = 0$ if and only if $t = 0$. We denote by Ψ and Φ be subsets of $\mathcal{F}$ such that

$$\begin{aligned} \Psi &= \{\psi \in \mathcal{F} : \psi \text{ is continuous and nondecreasing}\}, \\ \Phi &= \{\phi \in \mathcal{F} : \phi \text{ is lower semicontinuous}\}. \end{aligned} \tag{7}$$

Let (X,p) a partial metric space. We consider the following expressions:

$$\begin{aligned} M(x,y) &= \max\left\{p(x,y), p(x,Tx), p(y,Ty), \frac{1}{2}[p(x,Ty) + p(y,Tx)]\right\}, \\ N(x,y) &= \min\{d_m^p(x,Tx), d_m^p(y,Ty), d_m^p(x,Ty), d_m^p(y,Tx)\}, \end{aligned} \tag{8}$$

for all $x, y \in X$.

Our first result is the following.

Theorem 12. *Let (X,p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\psi \in \Psi$, $\phi \in \Phi$ and $L \geq 0$ such that for all $x, y \in X$*

$$\psi(p(Tx,Ty)) \leq \psi(M(x,y)) - \phi(M(x,y)) + LN(x,y). \tag{9}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u,u) = 0$.

Proof. Let $x_0 \in X$. We construct a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X in a way that $x_n = Tx_{n-1}$ for all $n \geq 1$. Suppose that $p(x_{n_0}, x_{n_0+1}) = 0$ for some $n_0 \geq 0$. So we have $x_{n_0} = x_{n_0+1} = Tx_{n_0}$, that is, x_{n_0} is the fixed point of T.

From now on, assume that $p(x_n, x_{n+1}) > 0$ for all $n \geq 0$. By (9), we have

$$\begin{aligned} \psi(p(x_n,x_{n+1})) &\leq \psi(p(Tx_{n-1},Tx_n)) \\ &\leq \psi(M(x_{n-1},x_n)) - \phi(M(x_{n-1},x_n)) \\ &\quad + LN(x_{n-1},x_n), \end{aligned} \tag{10}$$

where

$$N(x_{n-1},x_n) = \min\{d_m^p(x_{n-1},x_n), d_m^p(x_n,x_{n+1}), d_m^p(x_{n-1},x_{n+1}), d_m^p(x_n,x_n)\}. \tag{11}$$

Since $d_m^p(x_n,x_n) = 0$, we get $N(x_{n-1},x_n) = 0$. Hence, it follows that

$$\psi(p(x_n,x_{n+1})) \le \psi(M(x_{n-1},x_n)) - \phi(M(x_{n-1},x_n)), \tag{12}$$

which yields that

$$\psi(p(x_n,x_{n+1})) \le \psi(M(x_{n-1},x_n)). \tag{13}$$

Since ψ is nondecreasing, then

$$p(x_n,x_{n+1}) \le M(x_{n-1},x_n), \tag{14}$$

where

$$\begin{aligned} M(x_{n-1},x_n) &= \max\left\{p(x_{n-1},x_n), p(x_{n-1},Tx_{n-1}), p(x_n,Tx_n), \frac{1}{2}[p(x_{n-1},Tx_n)+p(x_n,Tx_{n-1})]\right\} \\ &= \max\left\{p(x_{n-1},x_n), p(x_{n-1},x_n), p(x_n,x_{n+1}), \frac{1}{2}[p(x_{n-1},x_{n+1})+p(x_n,x_n)]\right\}. \end{aligned} \tag{15}$$

Due to $(p4)$, we have

$$p(x_{n-1},x_{n+1}) + p(x_n,x_n) \le p(x_{n-1},x_n) + p(x_n,x_{n+1}). \tag{16}$$

Hence, the expression (15) turns into

$$M(x_{n-1},x_n) = \max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\}. \tag{17}$$

If for some n,

$$\max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\} = p(x_n,x_{n+1}) \tag{18}$$

then by (12)

$$\psi(p(x_n,x_{n+1})) \le \psi(p(x_n,x_{n+1})) - \phi(p(x_n,x_{n+1})), \tag{19}$$

so $\phi(p(x_n,x_{n+1})) = 0$. By (ϕ_2), we get $p(x_n,x_{n+1}) = 0$, which is a contradiction with respect to $p(x_n,x_{n+1}) > 0$ for all $n \ge 0$. Thus

$$M(x_{n-1},x_n) = p(x_{n-1},x_n), \quad \text{for each } n \ge 1, \tag{20}$$

so from (14)

$$p(x_n,x_{n+1}) \le p(x_{n-1},x_n), \quad \text{for each } n \ge 1. \tag{21}$$

Thus, the sequence $\{p(x_n,x_{n+1})\}$ is non-increasing and so there exists $\delta \ge 0$ such that

$$\lim_{n\to+\infty} p(x_n,x_{n+1}) = \delta. \tag{22}$$

Suppose that $\delta > 0$. Taking $\limsup_{n\to+\infty}$ in inequality (12) and using (20), we get

$$\limsup_{n\to+\infty} \psi(p(x_n,x_{n+1})) \le \limsup_{n\to+\infty} \psi(p(x_{n-1},x_n)) - \liminf_{n\to+\infty} \phi(p(x_{n-1},x_n)). \tag{23}$$

By continuity of ψ and lower semicontinuity of ϕ, we get $\psi(\delta) \le \psi(\delta) - \phi(\delta)$, so $\phi(\delta) = 0$, that is, $\delta = 0$, a contradiction. We conclude that

$$\lim_{n\to+\infty} p(x_n,x_{n+1}) = 0. \tag{24}$$

We will show that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in the partial metric space (X,p). From Lemma 8, we need to prove that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in the metric space (X,d_p). Suppose to the contrary that $\{x_n\}_{n\in\mathbb{N}}$ is not a Cauchy sequence in the metric space (X,d_p). Then, there is a $\varepsilon > 0$ such that for an integer k there exist integers $m(k) > n(k) > k$ such that

$$d_p(x_{n(k)},x_{m(k)}) > \varepsilon. \tag{25}$$

By definition of d_p, we have $d_p(x,y) \le 2p(x,y)$ for each $x,y \in X$, so (25) gives us

$$p(x_{n(k)},x_{m(k)}) > \frac{\varepsilon}{2}. \tag{26}$$

For every integer k, let $m(k)$ be the least positive integer exceeding $n(k)$ satisfying (26) then

$$p(x_{n(k)},x_{m(k)-1}) \le \frac{\varepsilon}{2}. \tag{27}$$

Now, using (26), (27), and the triangular inequality (which still holds for the partial metric p), we obtain

$$\begin{aligned} \frac{\varepsilon}{2} &< p(x_{n(k)},x_{m(k)}) \\ &\le p(x_{n(k)},x_{m(k)-1}) + p(x_{m(k)-1},x_{m(k)}) - p(x_{m(k)-1},x_{m(k)-1}) \\ &\le p(x_{n(k)},x_{m(k)-1}) + p(x_{m(k)-1},x_{m(k)}) \\ &\le \frac{\varepsilon}{2} + p(x_{m(k)-1},x_{m(k)}). \end{aligned} \tag{28}$$

Then by (24) it follows that

$$\lim_{k\to+\infty} p(x_{n(k)},x_{m(k)}) = \frac{\varepsilon}{2}. \tag{29}$$

Also, by the triangle inequality, we have

$$|p(x_{n(k)},x_{m(k)-1}) - p(x_{n(k)},x_{m(k)})| \le p(x_{m(k)-1},x_{m(k)}). \tag{30}$$

From (24) and (29) we get

$$\lim_{k\to+\infty} p\left(x_{n(k)}, x_{m(k)-1}\right) = \frac{\varepsilon}{2}. \tag{31}$$

Similarly, by triangle inequality

$$\begin{aligned} p\left(x_{n(k)}, x_{m(k)}\right) &\le p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)+1}, x_{m(k)}\right) \\ &\le p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)+1}, x_{m(k)-1}\right) \\ &\quad + p\left(x_{m(k)-1}, x_{m(k)}\right) \\ &\le 2p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)}, x_{m(k)-1}\right) \\ &\quad + p\left(x_{m(k)-1}, x_{m(k)}\right) \end{aligned} \tag{32}$$

and from (24), (29), and (31) we get

$$\lim_{k\to+\infty} p\left(x_{n(k)+1}, x_{m(k)}\right) = \frac{\varepsilon}{2}, \tag{33}$$

$$\lim_{k\to+\infty} p\left(x_{n(k)+1}, x_{m(k)-1}\right) = \frac{\varepsilon}{2}. \tag{34}$$

Having

$$\begin{aligned} &d_m^p\left(x_{n(k)}, x_{n(k)+1}\right) \\ &\quad = p\left(x_{n(k)}, x_{n(k)+1}\right) \\ &\qquad - \min\left\{p\left(x_{n(k)}, x_{n(k)}\right), p\left(x_{n(k)+1}, x_{n(k)+1}\right)\right\} \\ &\quad \le p\left(x_{n(k)}, x_{n(k)+1}\right), \end{aligned} \tag{35}$$

so referring to (24), we get

$$\lim_{k\to+\infty} d_m^p\left(x_{n(k)}, x_{n(k)+1}\right) = 0. \tag{36}$$

Moreover

$$\begin{aligned} &M\left(x_{n(k)}, x_{m(k)-1}\right) \\ &\quad = \max\left\{p\left(x_{n(k)}, x_{m(k)-1}\right), p\left(x_{n(k)}, Tx_{n(k)}\right),\right. \\ &\qquad p\left(x_{m(k)-1}, Tx_{m(k)-1}\right), \\ &\qquad \left.\frac{1}{2}\left[p\left(x_{n(k)}, Tx_{m(k)-1}\right) + p\left(x_{m(k)-1}, Tx_{n(k)}\right)\right]\right\} \\ &\quad = \max\left\{p\left(x_{n(k)}, x_{m(k)-1}\right), p\left(x_{n(k)}, x_{n(k)+1}\right),\right. \\ &\qquad p\left(x_{m(k)-1}, x_{m(k)}\right), \\ &\qquad \left.\frac{1}{2}\left[p\left(x_{n(k)}, x_{m(k)}\right) + p\left(x_{m(k)-1}, x_{n(k)+1}\right)\right]\right\}. \end{aligned} \tag{37}$$

Thus, from (24), (29), (31), and (34), we get

$$\lim_{k\to+\infty} M\left(x_{n(k)}, x_{m(k)-1}\right) = \max\left\{\frac{\varepsilon}{2}, 0, 0, \frac{\varepsilon}{2}\right\} = \frac{\varepsilon}{2}. \tag{38}$$

From (9), we have

$$\begin{aligned} &\psi\left(p\left(x_{n(k)+1}, x_{m(k)}\right)\right) \\ &\quad \le \psi\left(p\left(Tx_{n(k)}, Tx_{m(k)-1}\right)\right) \\ &\quad \le \psi\left(M\left(x_{n(k)}, x_{m(k)-1}\right)\right) \\ &\qquad - \phi\left(M\left(x_{n(k)}, x_{m(k)-1}\right)\right) + LN\left(x_{n(k)}, x_{m(k)-1}\right), \end{aligned} \tag{39}$$

where

$$\begin{aligned} &N\left(x_{n(k)}, x_{m(k)-1}\right) \\ &\quad = \min\left\{d_m^p\left(x_{n(k)}, Tx_{n(k)}\right), d_m^p\left(x_{m(k)-1}, Tx_{m(k)-1}\right),\right. \\ &\qquad \left. d_m^p\left(x_{n(k)}, Tx_{m(k)-1}\right), d_m^p\left(x_{m(k)-1}, Tx_{n(k)}\right)\right\} \\ &\quad = \min\left\{d_m^p\left(x_{n(k)}, x_{n(k)+1}\right), d_m^p\left(x_{m(k)-1}, x_{m(k)}\right),\right. \\ &\qquad \left. d_m^p\left(x_{n(k)}, x_{m(k)}\right), d_m^p\left(x_{m(k)-1}, x_{n(k)+1}\right)\right\}. \end{aligned} \tag{40}$$

By (36), we get

$$\lim_{k\to+\infty} N\left(x_{n(k)}, x_{m(k)-1}\right) = 0 \tag{41}$$

and referring to (33), (38) and letting $k \to +\infty$, we get

$$\psi\left(\frac{\varepsilon}{2}\right) \le \psi\left(\frac{\varepsilon}{2}\right) - \phi\left(\frac{\varepsilon}{2}\right), \tag{42}$$

so $\phi(\varepsilon/2) = 0$, which is a contradiction with respect to $\varepsilon > 0$. Thus we proved that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in the metric space (X, d_p).

Since (X, p) is complete, then from Lemma 8, (X, d_p) is a complete metric space. Therefore, the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to some $u \in X$ in (X, d_p), that is,

$$\lim_{n\to+\infty} d_p\left(x_n, u\right) = 0. \tag{43}$$

Again, from Lemma 8,

$$p(u, u) = \lim_{n\to+\infty} p\left(x_n, u\right) = \lim_{n\to+\infty} p\left(x_n, x_n\right). \tag{44}$$

On the other hand, thanks to (24) and the condition $(p2)$ from Definition 1,

$$\lim_{n\to+\infty} p\left(x_n, x_n\right) = 0, \tag{45}$$

so it follows that

$$p(u, u) = \lim_{n\to+\infty} p\left(x_n, u\right) = \lim_{n\to+\infty} p\left(x_n, x_n\right) = 0. \tag{46}$$

Now, we show that $p(u, Tu) = 0$. Assume this is not true, then from (9) we obtain

$$\begin{aligned} \psi\left(p\left(x_{n+1}, Tu\right)\right) &= \psi\left(p\left(Tx_n, Tu\right)\right) \\ &\le \psi\left(M\left(x_n, u\right)\right) - \phi\left(M\left(x_n, u\right)\right) \\ &\quad + L\min\left\{d_m^p\left(x_n, Tx_n\right), d_m^p\left(u, Tu\right),\right. \\ &\qquad \left. d_m^p\left(u, Tx_n\right), d_m^p\left(x_n, Tu\right)\right\}, \end{aligned} \tag{47}$$

where

$$\begin{aligned} M(x_n,u) &= \max\left\{p(x_n,u), p(x_n,Tx_n), p(u,Tu), \frac{1}{2}\left[p(x_n,Tu)+p(u,Tx_n)\right]\right\} \\ &= \max\left\{p(x_n,u), p(x_n,x_{n+1}), p(u,Tu), \frac{1}{2}\left[p(x_n,Tu)+p(u,x_{n+1})\right]\right\}. \end{aligned} \tag{48}$$

Thanks to (46), it is obvious that $\lim_{n\to+\infty} p(x_n,Tu) = p(u,Tu)$. Therefore, using (24) and again (46), we deduce that

$$\begin{aligned} \lim_{n\to+\infty} M(x_n,u) &= \max\left\{0,0,p(u,Tu),\frac{1}{2}p(u,Tu)\right\} \\ &= p(u,Tu). \end{aligned} \tag{49}$$

Also

$$\lim_{n\to+\infty} N(x_n,u) = 0 \tag{50}$$

because (24) and (45) give $\lim_{n\to+\infty} d_m^p(x_n,Tx_n) = 0$. Now, taking the upper limit as $n \to +\infty$, we obtain using the properties of ψ and ϕ

$$\psi(p(u,Tu)) \le \psi(p(u,Tu)) - \phi(p(u,Tu)), \tag{51}$$

so $\phi(p(u,Tu)) = 0$, that is, $p(u,Tu) = 0$, so $Tu = u$. We conclude that T has a fixed point $u \in X$ and $p(u,u) = 0$.

Now if $v \ne u$ (so $p(u,v) \ne 0$) is another fixed point of T (with $p(v,v) = 0$), then by (46),

$$\begin{aligned} N(u,v) &= \min\{d_m^p(u,Tu), d_m^p(v,Tv), d_m^p(u,Tv), d_m^p(v,Tu)\} \\ &= \min\{d_m^p(u,u), d_m^p(v,v), d_m^p(u,v), d_m^p(v,u)\} \\ &= 0, \\ M(u,v) &= \max\left\{p(u,v), p(u,Tu), p(v,Tv), \frac{1}{2}\left[p(u,Tv)+p(v,Tu)\right]\right\} \\ &= \max\left\{p(u,v), p(u,u), p(v,v), \frac{1}{2}\left[p(u,v)+p(v,u)\right]\right\} \\ &= \max\left\{p(u,v), 0, 0, \frac{1}{2}\left[p(u,v)+p(v,u)\right]\right\} \\ &= p(u,v). \end{aligned} \tag{52}$$

Hence, using (9) we obtain

$$\begin{aligned} \psi(p(u,v)) &= \psi(p(Tu,Tv)) \\ &\le \psi(M(u,v)) - \phi(M(u,v)) + LN(v,u) \\ &= \psi(p(u,v)) - \phi(p(u,v)), \end{aligned} \tag{53}$$

that is, $p(u,v) = 0$, which is a contradiction. The proof of Theorem 12 is completed. □

As a consequence of Theorem 12, we may state the following corollaries.

First, taking $L = 0$ in Theorem 12, we have the following.

Corollary 13. *Let (X,p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that for all $x, y \in X$*

$$\psi(p(Tx,Ty)) \le \psi(M(x,y)) - \psi(M(x,y)). \tag{54}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u,u) = 0$.

Corollary 14. *Let (X,p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $k \in [0,1)$ and $L \ge 0$ such that for all $x, y \in X$*

$$\begin{aligned} p(Tx,Ty) &\le kM(x,y) \\ &\quad + L\min\{d_m^p(x,Tx), d_m^p(y,Ty), d_m^p(x,Ty), d_m^p(y,Tx)\}. \end{aligned} \tag{55}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u,u) = 0$.

Proof. It follows by taking $\psi(t) = t$ and $\phi(t) = (1-k)(t)$ in Theorem 12. □

Denote by Λ the set of functions $\lambda : [0,+\infty) \to [0,+\infty)$ satisfying the following hypotheses:

(1) λ is a Lebesgue-integrable mapping on each compact subset of $[0,+\infty)$,

(2) for every $\epsilon > 0$, we have $\int_0^\epsilon \lambda(s)ds > 0$.

We have the following result.

Corollary 15. *Let (X,p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\alpha, \beta \in \Lambda$ and $L \ge 0$ such that for all $x, y \in X$*

$$\begin{aligned} \int_0^{p(Tx,Ty)} \alpha(s)\,ds &\le \int_0^{p(Tx,Ty)} \alpha(s)\,ds - \int_0^{M(x,y)} \beta(s)\,ds \\ &\quad + L\min\{d_m^p(x,Tx), d_m^p(y,Ty), d_m^p(x,Ty), d_m^p(y,Tx)\}. \end{aligned} \tag{56}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u,u) = 0$.

Proof. It follows from Theorem 12 by taking

$$\begin{aligned} \psi(t) &= \int_0^t \alpha(s)\,ds, \\ \phi(t) &= \int_0^t \beta(s)\,ds. \end{aligned} \tag{57}$$

□

Taking $L = 0$ in Corollary 15, we obtain the following result.

Corollary 16. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\alpha, \beta \in \Lambda$ such that for all $x, y \in X$*

$$\int_0^{p(Tx,Ty)} \alpha(s)\,ds \le \int_0^{p(Tx,Ty)} \alpha(s)\,ds - \int_0^{M(x,y)} \beta(s)\,ds. \tag{58}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Now, let $\mathcal{F}$ be the set of functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(φ_1) φ is nondecreasing

(φ_2) $\sum_{n=0}^{+\infty} \varphi^n(t)$ converges for all $t > 0$.

Note that if $\varphi \in \mathcal{F}$, φ is said a (C)-comparison function. It is easily proved that if φ is a (C)-comparison function, then $\varphi(t) < t$ for any $t > 0$. Our second main result is as follows.

Theorem 17. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a mapping such that there exist $\varphi \in \mathcal{F}$ and $L \ge 0$ such that for all $x, y \in X$*

$$\begin{aligned} p(Tx, Ty) \le{}& \varphi(M(x, y)) \\ &+ L \min\{d_m^p(x, Tx), d_m^p(y, Ty), \\ &\quad d_m^p(x, Ty), d_m^p(y, Tx)\}. \end{aligned} \tag{59}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Proof. Let $x_0 \in X$. Let $\{x_n\}_{n\in\mathbb{N}}$ in X such that $x_n = Tx_{n-1}$ for all $n \ge 1$.

If for some $n \in \mathbb{N}$, $p(x_n, x_{n+1}) = 0$, the proof is completed. Assume that $p(x_n, x_{n+1}) \ne 0$ for all $n \ge 0$.

From (59)

$$\begin{aligned} p(x_n, x_{n+1}) &= p(Tx_{n-1}, Tx_n) \\ &\le \varphi(M(x_{n-1}, x_n)) \\ &= L \min\{d_m^p(x_{n-1}, Tx_{n-1}), d_m^p(x_n, Tx_n), \\ &\qquad d_m^p(x_{n-1}, Tx_n), d_m^p(x_n, Tx_{n-1})\}. \end{aligned} \tag{60}$$

As explained in the proof of Theorem 12, we may get

$$\begin{aligned} &\min\{d_m^p(x_{n-1}, Tx_{n-1}), d_m^p(x_n, Tx_n), \\ &\quad d_m^p(x_{n-1}, Tx_n), d_m^p(x_n, Tx_{n-1})\} = 0, \\ &M(x_{n-1}, x_n) = \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}. \end{aligned} \tag{61}$$

Therefore

$$p(x_n, x_{n+1}) \le \varphi(\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}). \tag{62}$$

If for some $n \ge 1$, we have $p(x_{n-1}, x_n) \le p(x_n, x_{n+1})$. So from (62), we obtain that

$$p(x_n, x_{n+1}) \le \varphi(p(x_n, x_{n+1})) < p(x_n, x_{n+1}), \tag{63}$$

a contradiction. Thus, for all $n \ge 1$, we have

$$\begin{aligned} M(x_{n-1}, x_n) &= \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} \\ &= p(x_{n-1}, x_n). \end{aligned} \tag{64}$$

Using (62) and (64), we get that

$$p(x_n, x_{n+1}) \le \varphi(p(x_{n-1}, x_n)) \quad \forall n \ge 1. \tag{65}$$

By induction, we get

$$p(x_n, x_{n+1}) \le \varphi^n(p(x_0, x_1)) \tag{66}$$

for all $n \ge 0$. By triangle inequality, we have for $m > n$

$$\begin{aligned} p(x_n, x_m) &\le \sum_{k=n}^{k=m-1} p(x_k, x_{k+1}) - \sum_{k=n+1}^{k=m-1} p(x_k, x_k) \\ &\le \sum_{k=n}^{k=m-1} p(x_k, x_{k+1}) \\ &\le \sum_{k=n}^{k=+\infty} p(x_k, x_{k+1}) \\ &\le \sum_{k=n}^{k=+\infty} \varphi^k(p(x_0, x_1)). \end{aligned} \tag{67}$$

Keeping in mind that φ is a (C)-comparison function, then $\lim_{n\to+\infty} \sum_{k=n}^{k=+\infty} \varphi^k(p(x_0, x_1)) = 0$ and so $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in (X, p) with $\lim_{n,m\to+\infty} p(x_n, x_m) = 0$. Since (X, p) is complete then $\{x_n\}_{n\in\mathbb{N}}$ converges, with respect to τ_p, to a point $u \in X$ such that

$$p(u, u) = \lim_{n\to+\infty} p(x_n, u) = \lim_{n,m\to+\infty} p(x_n, x_m) = 0. \tag{68}$$

Now we claim that $p(u, Tu) = 0$. Suppose the contrary, then $p(u, Tu) > 0$. By (59), we have

$$\begin{aligned} p(u, Tu) &\le p(u, x_{n+1}) + p(Tx_n, Tu) \\ &\le p(u, x_{n+1}) + \varphi(M(x_n, u)) \\ &\quad + L \min\{d_m^p(x_n, Tx_n), d_m^p(u, Tu), \\ &\qquad d_m^p(u, Tx_n), d_m^p(x_n, Tu)\}, \end{aligned} \tag{69}$$

where

$$\begin{aligned} M(x_n, u) &= \max\left\{p(x_n, u), p(x_n, Tx_n), p(u, Tu), \right. \\ &\qquad \left. \frac{1}{2}[p(x_n, Tu) + p(u, Tx_n)]\right\} \\ &= \max\left\{p(x_n, u), p(x_n, x_{n+1}), p(u, Tu), \right. \\ &\qquad \left. \frac{1}{2}[p(x_n, Tu) + p(u, x_{n+1})]\right\}. \end{aligned} \tag{70}$$

By (68), we have

$$\lim_{n\to+\infty} \min\{d_m^p(x_n, Tx_n), d_m^p(u, Tu), d_m^p(u, Tx_n), d_m^p(x_n, Tu)\} = 0, \quad (71)$$
$$\lim_{n\to+\infty} M(x_n, u) = p(u, Tu).$$

Therefore

$$p(u, Tu) \le \varphi(p(u, Tu)) < p(u, Tu), \quad (72)$$

which is a contradiction. That is $p(u, Tu) = 0$. Thus, we obtained that u is a fixed point for T and $p(u, u) = 0$.

Now if $v \ne u$ (so $p(u, v) \ne 0$) is another fixed point of T, then by (68),

$$\begin{aligned}
&\min\{d_m^p(u, Tu), d_m^p(v, Tv), d_m^p(u, Tv), d_m^p(v, Tu)\} \\
&\quad = \min\{d_m^p(u, u), d_m^p(v, v), d_m^p(u, v), d_m^p(v, u)\} \\
&\quad = 0, \\
&M(u, v) = \max\left\{p(u, v), p(u, Tu), p(v, Tv), \frac{1}{2}[p(u, Tv) + p(v, Tu)]\right\} \\
&\quad = \max\left\{p(u, v), p(u, u), p(v, v), \frac{1}{2}[p(u, v) + p(v, u)]\right\} \\
&\quad = p(u, v).
\end{aligned} \quad (73)$$

Hence, using (59) we obtain

$$\begin{aligned}
p(u, v) &= p(Tu, Tv) \\
&\le \varphi(M(u, v)) \\
&\quad + L\min\{d_m^p(u, Tu), d_m^p(v, Tv), d_m^p(u, Tv), d_m^p(v, Tu)\} \\
&\le \varphi(p(u, v)) \\
&< p(u, v)
\end{aligned} \quad (74)$$

which is a contradiction. Thus $u = v$ and the proof of Theorem 17 is completed. □

Taking $L = 0$ in Theorem 17, we have the following.

Corollary 18. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a mapping such that there exists $\varphi \in \mathcal{F}$ such that for all $x, y \in X$*

$$p(Tx, Ty) \le \varphi(M(x, y)). \quad (75)$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Taking $\varphi(t) = ht$ where $0 \le h < 1$ in Corollary 18, we obtain the Ćirić fixed point theorem [36] in the setting of metric spaces (by considering $p = d$ is a metric).

Corollary 19. *Let (X, d) be a complete metric space. Let $T : X \to X$ be a mapping such that there exists $h \in [0, 1)$ such that for all $x, y \in X$*

$$d(Tx, Ty) \le h\max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2}[d(x, Ty) + d(y, Tx)]\right\}. \quad (76)$$

Then T has a unique fixed point.

Remark 20. Corollary 14 generalizes Theorem 10 (with $f = g = T = S$) of Turkoglu and Ozturk [27]. Corollary 18 improves Theorem 1 of Altun et al. [4] by assuming that φ is not continuous.

3. Examples

We give in this section some examples making effective our obtained results.

Example 21. Let $X = [0, 1]$ and $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a complete partial metric space. Consider $T : X \to X$ defined by

$$Tx = \frac{x^2}{1 + x}. \quad (77)$$

Take $\psi(t) = t$ and $\phi(t) = t/(1 + t)$ for all $t \ge 0$. Note that $\psi \in \Psi$ and $\phi \in \Phi$. Take $x \le y$, then

$$\begin{aligned}
&\psi(p(Tx, Ty)) \\
&\quad = \frac{y^2}{1 + y} = y - \frac{y}{1 + y} \\
&\quad = \psi(M(x, y)) - \phi(M(x, y)) \\
&\quad\quad (\text{since } M(x, y) = y) \\
&\quad \le \psi(M(x, y)) - \phi(M(x, y)) \\
&\quad\quad + L\min\{d_m^p(x, Tx), d_m^p(y, Ty), d_m^p(x, Ty), d_m^p(y, Tx)\}
\end{aligned} \quad (78)$$

for all $L \ge 0$. Thus, (9) holds. Applying Theorem 12, T has a unique fixed point, which is $u = 0$.

Example 22. Let $X = \{0, 1, 2, 3, 4\}$ and $p(x, y) = \max\{x, y\}$. Let $T : X \to X$ be defined as follows:

$$T0 = 0, \qquad T1 = T3 = 2, \qquad T4 = T2 = 1. \quad (79)$$

By simple calculation, we get that

$$\begin{aligned} p(T2,T2) &= p(T4,T4) = p(T2,T0) \\ &= p(T4,T0) = p(T2,T4) = 1, \\ &\quad p(T0,T0) = 0, \\ p(T1,T1) &= p(T3,T3) = p(T1,T0) \\ &= p(T3,T0) = p(T1,T3) = 2, \\ p(T4,T1) &= p(T4,T3) = p(T2,T1) \\ &= p(T2,T3) = 2. \end{aligned} \tag{80}$$

Hence, we derive that

$$\begin{aligned} M(T1,T4) &= M(T2,T4) = M(T3,T4) \\ &= M(T4,T4) = M(T0,T4) = 4, \\ M(T1,T3) &= M(T2,T3) \\ &= M(T0,T3) = M(T3,T3) = 3, \\ &\quad M(T0,T0) = 0, \\ M(T1,T2) &= M(T2,T0) = M(T1,T1) \\ &= M(T0,T1) = M(T2,T2) = 2, \\ N(T1,T4) &= N(T2,T1) = N(T1,T0) = N(T2,T0) \\ &= N(T3,T0) = N(T4,T0) = N(T0,T0) = 0, \\ N(T1,T1) &= N(T1,T3) = N(T2,T3) \\ &= N(T3,T3) = N(T2,T2) = N(T2,T4) = 1, \\ N(T4,T3) &= 2, \qquad N(T4,T4) = 3. \end{aligned} \tag{81}$$

For $\psi(t) = t/3$, $\phi(t) = t/6$ and $L \geq 1/5$ all conditions of Theorem 12 are satisfied. Notice that 0 is the unique fixed point of T.

Example 23. Let $X = [0,2]$ and $p : X \times X \to \mathbb{R}^+$ be defined by $p(x,y) = \max\{x,y\}$. Define $T : X \to X$ by

$$T(x) = \begin{cases} \dfrac{x^2}{x+1}, & \text{if } x \in [0,1[, \\ 0, & \text{if } x \in [1,2[, \\ \dfrac{4}{3} & \text{if } x = 2 \end{cases} \tag{82}$$

and let $\varphi : [0,+\infty[\to [0,+\infty[$ defined by

$$\varphi(t) = \frac{t^2}{t+1}. \tag{83}$$

By induction, we have $\varphi^n(t) \leq t(t/(1+t))^n$ for all $n \geq 1$, so it is clear that φ is a (C)-comparison function. Now we show that (59) is satisfied for all $x, y \in X$. It suffices to prove it for $x \leq y$. Consider the following six cases.

Case 1. Let $x, y \in [0,1[$, then

$$\begin{aligned} p(Tx,Ty) &= \frac{y^2}{y+1} \\ &= \varphi(p(x,y)) \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{84}$$

Case 2. Let $x, y \in [1,2[$, then

$$\begin{aligned} p(Tx,Ty) &= p(0,0) = 0 \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{85}$$

Case 3. Let $x = y = 2$, then

$$\begin{aligned} p(Tx,Ty) &= p\left(\frac{4}{3},\frac{4}{3}\right) = \frac{4}{3} \\ &= \varphi(2) \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{86}$$

Case 4. Let $x \in [0,1[$ and $y \in [1,2[$ then

$$\begin{aligned} p(Tx,Ty) &= p\left(\frac{x^2}{x+1},0\right) = \frac{x^2}{x+1} \\ &\leq \frac{y^2}{y+1} \\ &= \varphi(p(x,y)) \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{87}$$

Case 5. Let $x \in [0,1[$ and $y = 2$, then

$$\begin{aligned} p(Tx,Ty) &= p\left(\frac{x^2}{x+1},\frac{4}{3}\right) = \frac{4}{3} \\ &= \varphi(2) \\ &= \varphi(p(x,y)) \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{88}$$

Case 6. Let $x \in [1,2[$ and $y = 2$ then

$$\begin{aligned} p(Tx,Ty) &= p\left(0,\frac{4}{3}\right) = \frac{4}{3} \\ &= \varphi(2) \\ &= \varphi(p(x,y)) \\ &\leq \varphi(M(x,y)). \end{aligned} \tag{89}$$

Since, for all $x, y \in X$

$$L \min\{d_m^p(x,Tx), d_m^p(y,Ty), d_m^p(x,Ty), d_m^p(y,Tx)\} \geq 0 \tag{90}$$

then (59) is verified. Applying Theorem 17, T has a unique fixed point, which is $u = 0$.

All presented theorems involve generalized almost contractive mappings which have a unique fixed point. But, one of the main features of Berinde contractions is the fact that they do possess more that one fixed point. In this direction, Altun and Acar [35] proved the following result.

Theorem 24. *Let (X, p) a complete partial metric space. Given $T : X \to X$ satisfying*

$$\begin{aligned} &\text{there exist } k \in [0,1), \quad L \geq 0 \\ &\text{such that } p(Tx, Ty) \leq kp(x, y) + Ld_m^p(x, Ty), \end{aligned} \tag{91}$$

for all $x, y \in X$. Then, T has a fixed point.

The following example illustrates Theorem 24 where we have two fixed points.

Example 25. Let $X = \{0, 1, 2\}$. A partial metric $p : X \times X \to \mathbb{R}^+$ is defined by

$$\begin{gathered} p(0,0) = p(1,1) = 0, \qquad p(2,2) = \frac{1}{4}, \\ p(0,1) = p(1,0) = \frac{1}{3}, \\ p(0,2) = p(2,0) = \frac{11}{24}, \\ p(1,2) = p(2,1) = \frac{1}{2}. \end{gathered} \tag{92}$$

Define the mapping $T : X \to X$ by

$$T0 = T2 = 0, \qquad T1 = 1. \tag{93}$$

It is easy to show that (91) is satisfied. Applying Theorem 24, T has a fixed point. Note that T has two fixed points which are 0 and 1.

References

[1] S. G. Matthews, "Partial metric topology," in *Papers on General Topology and Applications*, vol. 728 of *Annals of the New York Academy of Sciences*, pp. 183–197, New York Academy of Sciences, New York, NY, USA, 1994.

[2] S. Banach, "Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales," *Fundamenta Mathematicae*, vol. 3, pp. 133–181, 1922.

[3] T. Abedeljawad, E. Karapınar, and K. Taş, "Existence and uniqueness of common fixed point on partial metric spaces," *Applied Mathematics Letters*, vol. 24, pp. 1894–1899, 2011.

[4] I. Altun, F. Sola, and H. Simsek, "Generalized contractions on partial metric spaces," *Topology and Its Applications*, vol. 157, no. 18, pp. 2778–2785, 2010.

[5] I. Altun and A. Erduran, "Fixed point theorems for monotone mappings on partial metric spaces," *Fixed Point Theory and Applications*, Article ID 508730, 10 pages, 2011.

[6] H. Aydi, "Some coupled fixed point results on partial metric spaces," *International Journal of Mathematics and Mathematical Sciences*, Article ID 647091, 11 pages, 2011.

[7] H. Aydi, "Some fixed point results in ordered partial metric spaces," *Journal of Nonlinear Science and Its Applications*, vol. 4, no. 3, pp. 210–217, 2011.

[8] H. Aydi, "Fixed point results for weakly contractive mappings in ordered partial metric spaces," *Journal of Advanced Mathematical Studies*, vol. 4, no. 2, pp. 1–12, 2011.

[9] H. Aydi, "Fixed point theorems for generalized weakly contractive condition in ordered partial metric spaces," *Journal of Nonlinear Analysis and Optimization*, vol. 2, no. 2, pp. 269–284, 2011.

[10] H. Aydi, "Common fixed point results for mappings satisfying (ψ, ϕ)-weak contractions in ordered partial metric spaces," *International Journal of Mathematics and Statistics*, no. 2, pp. 53–64, 2012.

[11] H. Aydi, "Coupled fixed point results in ordered partial metric spaces," *Selçuk Journal of Applied Mathematics*, vol. 13, no. 1, pp. 23–33, 2012.

[12] L. Ćirić, B. Samet, H. Aydi, and C. Vetro, "Common fixed points of generalized contractions on partial metric spaces and an application," *Applied Mathematics and Computation*, vol. 218, no. 6, pp. 2398–2406, 2011.

[13] C. Di Bari, M. Milojević, S. Radenović, and P. Vetro, "Common fixed points for self-mappings on partial metric spaces," *Fixed Point Theory and Applications*, vol. 2012, article 140, 2012.

[14] D. Ilić, V. Pavlović, and V. Rakočević, "Some new extensions of Banach's contraction principle to partial metric space," *Applied Mathematics Letters*, vol. 24, no. 8, pp. 1326–1330, 2011.

[15] T. Abdeljawad, E. Karapınar, and K. Taş, "Existence and uniqueness of a common fixed point on partial metric spaces," *Applied Mathematics Letters*, vol. 24, no. 11, pp. 1900–1904, 2011.

[16] E. Karapınar, "A note on common fixed point theorems in partial metric spaces," *Miskolc Mathematical Notes*, vol. 12, no. 2, pp. 185–191, 2011.

[17] E. Karapınar, "Some fixed point theorems on the class of comparable partial metric spaces," *Applied General Topology*, vol. 12, no. 2, pp. 187–192, 2011.

[18] E. Karapınar, "Generalizations of Caristi Kirk's theorem on partial metric spaces," *Fixed Point Theory and Applications*, vol. 4, 7 pages, 2011.

[19] E. Karapınar, "Ćirić types nonunique fixed point theorems on partial metric spaces," *Journal of Nonlinear Science and Its Applications*, vol. 5, no. 2, pp. 74–83, 2012.

[20] E. Karapınar, N. Shobkolaei, S. Sedghi, and S. M. Vaezpour, "A common fixed point theorem for cyclic operators on partial metric spaces," *Filomat*, vol. 26, pp. 407–414, 2012.

[21] E. Karapınar, "Weak ϕ-contraction on partial metric spaces," *Journal of Computational Analysis and Applications*, vol. 14, no. 2, pp. 206–210, 2012.

[22] S. Romaguera, "Fixed point theorems for generalized contractions on partial metric spaces," *Topology and Its Applications*, vol. 159, no. 1, pp. 194–199, 2012.

[23] S. Romaguera, "Matkowski's type theorems for generalized contractions on (ordered) partial metric spaces," *Applied General Topology*, vol. 12, no. 2, pp. 213–220, 2011.

[24] S. Romaguera and M. Schellekens, "Partial metric monoids and semivaluation spaces," *Topology and Its Applications*, vol. 153, no. 5-6, pp. 948–962, 2005.

[25] S. Romaguera and O. Valero, "A quantitative computational model for complete partial metric spaces via formal balls," *Mathematical Structures in Computer Science*, vol. 19, no. 3, pp. 541–563, 2009.

[26] N. Shobkolaei, S. M. Vaezpour, and S. Sedghi, "A common fixed point theorem on ordered partial metric spaces," *Journal of Basic and Applied Scientific Research*, vol. 1, pp. 3433–3439, 2011.

[27] A. D. T. Turkoglu and V. Ozturk, "Common fixed point results for four mappings on partial metric spaces," *Abstract and Applied Analysis*, vol. 2012, Article ID 190862, 11 pages, 2012.

[28] F. Vetro and S. Radenović, "Nonlinear ψ-quasi-contractions of Ćirić-type in partial metric spaces," *Applied Mathematics and Computation*, vol. 219, no. 4, pp. 1594–1600, 2012.

[29] V. Berinde, "Approximating fixed points of weak contractions using the Picard iteration," *Nonlinear Analysis Forum*, vol. 9, no. 1, pp. 43–53, 2004.

[30] V. Berinde, "General constructive fixed point theorems for Ćirić-type almost contractions in metric spaces," *Carpathian Journal of Mathematics*, vol. 24, no. 2, pp. 10–19, 2008.

[31] M. Abbas and D. Ilić, "Common fixed points of generalized almost nonexpansive mappings," *Filomat*, vol. 24, no. 3, pp. 11–18, 2010.

[32] V. Berinde, "Some remarks on a fixed point theorem for Ćirić-type almost contractions," *Carpathian Journal of Mathematics*, vol. 25, no. 2, pp. 157–162, 2009.

[33] V. Berinde, "Approximating common fixed points of noncommuting almost contractions in metric spaces," *Fixed Point Theory*, vol. 11, no. 2, pp. 179–188, 2010.

[34] V. Berinde, "Common fixed points of noncommuting almost contractions in cone metric spaces," *Mathematical Communications*, vol. 15, no. 1, pp. 229–241, 2010.

[35] I. Altun and O. Acar, "Fixed point theorems for weak contractions in the sense of Berinde on partial metric spaces," *Topology and Its Applications*, vol. 159, no. 10-11, pp. 2642–2648, 2012.

[36] L. Ćirić, "A generalization of Banach's contraction principle," *Proceedings of the American Mathematical Society*, vol. 45, pp. 267–273, 1974.