Research Article


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1. Introduction

Stagnation point flow is of great importance in the prediction of skin friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling, and thermal oil. In 1911, Hiemenz [1] revealed that stagnation point flow can be examined by the Navier-Stokes (NS) equations. He used the similarity of the solution to reduce number of variables by means of a coordinate transformation. Later Howann [2] discovered the stagnation point flow in case of axisymmetric situation. Recently, a number of researchers studied the stagnation point flow considering different fluids models, geometries, and assumptions that were proposed in the literature. The literature on the topic is quite extensive and hence cannot be described here in detail. However some most recent works of eminent researchers regarding the analytical/numerical solution of stagnation point for different geometries may be mentioned in [3–5]. Attia [6], Massoudi and Ramezan [7], and Garg [8] extended the stagnation point flow for heat transfer. The main aim of this paper is to extend the HPM [9–17] for solving non-Newtonian fluid flow and heat transfer analysis in the region of stagnation point flow towards a stretching/shrinking and axisymmetric shrinking sheet. Also the main motivation is to perform such analysis [3] (shrinking/axisymmetric shrinking sheet) for a non-Newtonian fluid in the presence of heat transfer. Heat transfer plays very important role in nuclear energy because nuclear chain reaction creates heat, and it is used to boil water, produce steam, and drive a steam turbine. The steady Navier-Stokes equations are reduced to the nonlinear ordinary differential equations by using similarity solutions. Graphical results implicitly reveal the complete reliability and efficiency of the suggested algorithm.
2. Governing Equations

The flow and heat characteristics are governed by the following equations [3]:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \]
(1)

\[ u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\rho} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \nu \frac{\partial^2 u}{\partial x \partial z}, \]
(2)

\[ \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial u^2}{\partial x \partial z} + \frac{\partial u}{\partial x} \frac{\partial v}{\partial z} = \kappa \frac{\partial^2 T}{\partial z^2}. \]
(3)

The similarity transformations for two-dimensional stagnation flow case are as follows [3]:

\[ \eta = \sqrt{\frac{\alpha}{y}} z, \quad u = a x f^l (\eta) + b c h (\eta), \quad v = 0, \]
(4)

\[ w = -\sqrt{\alpha v} f^l (\eta), \quad \theta = \frac{T - T_{\infty}}{T_0 - T_{\infty}}. \]

The steady Navier-Stokes equations yield a system of nonlinear ordinary differential equations in the form

\[ f''' + ff'' + f'' - 1 + \beta (2f'f'' + f'' - ff''') = 0, \]
\[ h'' + fh' - f'h + \beta (hf'' + f'h'' + f''h' - fh''') = 0, \]
(5)

\[ \theta'' + Pr f'\theta' = 0, \]

and corresponding boundary conditions take the form

\[ f (0) = 0, \quad f' (0) = \frac{b}{a} = \alpha, \quad \theta (0) = 1, \]
\[ f' (\infty) = 1, \quad h (0) = 1, \quad h (\infty) = 0, \quad \theta (\infty) = 0. \]
(6)

The similarity transformations for axisymmetric stagnation flow towards an axisymmetric shrinking surface are as follows [3]:

\[ \eta (x, y) = \sqrt{\frac{\alpha}{y}} z, \quad u = ax g' (\eta) + b c h (\eta), \]
\[ v = ay g' (\eta), \quad w = -2 \sqrt{\alpha v} g (\eta), \quad \theta = \frac{T - T_{\infty}}{T_0 - T_{\infty}}. \]
(7)

Upon making use of the above substitutions in (2) and (3), the resulting nonlinear system has the following form:

\[ g''' + 2gg'' - g'' + 1 + \beta (2g'g'' + g'' - 2gg'') = 0, \]
\[ l''' + 2gl' - g'l + \beta (lg''' + g'l'' + g''l' - 2gl''') = 0, \]
\[ \theta'' + 2Pr g'\theta' = 0, \]
\[ g (0) = 0, \quad g' (0) = \frac{b}{a} = \alpha, \quad \theta (0) = 1, \]
\[ g' (\infty) = 1, \quad l (0) = 1, \quad l (\infty) = 0, \quad \theta (\infty) = 0. \]
(8)

3. Analytical Solution

For the HPM [9] solution, we select

\[ f_0 (\eta) = (1 - \alpha) (e^\eta - 1) + \eta, \quad h_0 (\eta) = e^\eta, \]
(9)
\[ \theta_0 (\eta) = e^\eta, \]
(10)

as initial approximations of \( f, h, \) and \( \theta. \) We further choose the following auxiliary linear operators:

\[ L_1 = \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \eta^2}, \quad L_2 = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}, \quad L_3 = \frac{\partial^2}{\partial \eta^2} + \frac{\partial}{\partial \eta}. \]
(11)

In view of the basic idea of the HPM [9], (5) is expressed as

\[ (1 - p) L_1 (f - f_0) + p \left( f''' + ff'' - f'' + 1 + \beta (2f'f'' + f'' - ff''') \right) = 0, \]
\[ (1 - p) L_2 (h - h_0) + p \left( h'' + fh' - f'h + \beta (hf'' + f'h'' + f''h' - fh''') \right) = 0, \]
\[ (1 - p) L_3 (\theta - \theta_0) + p \left( \theta'' + Pr f'\theta' \right) = 0, \]
(12)

\[ f = f_0 + pf_1 + p^2 f_2 + \cdots, \]
\[ h = h_0 + ph_1 + p^2 h_2 + \cdots, \]
\[ \theta = \theta_0 + p\theta_1 + p^2 \theta_2 + \cdots. \]
(13)

Assuming \( L_1 f = 0, \quad L_2 h = 0, \quad L_3 \theta = 0 \) and substituting \( f, h, \) and \( \theta \) from (13) into (12) and some simplification and rearrangement based on powers of \( p \)-terms, we have

\[ p^1 : L_1 f_1 + f_0 'f_0'' - f_0 'f_0'' + 1 \
+ \beta (2f_0 'f_0'' + f_0 '' - f_0 f_0 ''') = 0, \]
\[ f_1 (0) = f_1 ' (0) = f_1 ' (\infty) = 0, \]
\[ : \]
\[ p^j : L_1 f_j - L_1 f_{j-1} + f_0 'f_0'' + \sum_{k=0}^{j-1} f_k f_k '' + \sum_{k=0}^{j-1} f_k f_k '' + \sum_{k=0}^{j-1} f_k f_k '' + \sum_{k=0}^{j-1} f_k f_k '' = 0, \]
\[ f_j ' (0) = f_j ' (\infty) = 0, \quad j \geq 2; \]
Figure 1: Effect of $\alpha$ on $f'$ for two-dimensional case.

Figure 2: Effect of $\beta$ on $h$ for two-dimensional case.

Figure 3: Effect of Pr on $\theta$ for two-dimensional case.

Figure 4: Effect of $\alpha$ on $g'$ for axisymmetric case.

Figure 5: Effect of $\beta$ on $l$ for axisymmetric case.

Figure 6: Effect of Pr on $\theta$ for axisymmetric case.
\[ p^1 : L_3 h''_j + h'''_j + f_0 h'_0 - f_j h_0 + \beta \left( h_0 f'''_0 + f'_0 h''_0 + f''_0 h'_0 - f_0 h''_0 \right) = 0, \]
\[ h_1 (0) = h_1 (\infty) = 0, \]
\[ \vdots \]
\[ p^i : L_2 h_j - L_2 h_{j-1} + h''_{j-1} + \sum_{k=0}^{j-1} f_k h'_{j-1-k} - \sum_{k=0}^{j-1} f'k h_{j-1-k} + \beta \left( \sum_{k=0}^{j-1} f_k h''_{j-1-k} + \sum_{k=0}^{j-1} f''_k h'_{j-1-k} \right) = 0, \]
\[ h_j (0) = h_j (\infty) = 0, \quad j \geq 2; \]
\[ \theta_1 (0) = \theta_1 (\infty) = 0, \]
\[ \vdots \]
\[ p^i : L_3 \theta_j - L_3 \theta'_{j-1} + \theta''_{j-1} + \operatorname{Pr} \sum_{k=0}^{j-1} f_k \theta'_{j-1-k} = 0, \]
\[ \theta_j (0) = \theta_j (\infty) = 0, \quad j \geq 2. \quad (14) \]

On solving (14) in any software like Mathematica, Maple or MATLAB we can get any order of approximation.

Adopting the same procedure for axisymmetric stagnation flow towards an axisymmetric shrinking surface (8), we can get the required solution for (8)
\[ g = g_0 + g_1 + g_2 + \cdots, \]
\[ l = l_0 + l_1 + l_2 + \cdots, \]
\[ \theta = \theta_0 + \theta_1 + \theta_2 + \cdots. \quad (15) \]

4. Concluding Remarks

In this paper, we have studied non-Newtonian Stagnation point flow in the presence of heat transfer by using HPM. The HPM is used in a direct way without using linearization, discretization, or restrictive assumption. The variations of various emerging parameters on the velocities \((f'', h, g', l)\) and temperature field \((\theta)\) are discussed through Figures 1, 2, 3, 4, 5, and 6. The main results of the present analysis are as follows:

(i) for two-dimensional case, the velocity \(f'\) decreases for shrinking parameter \(\alpha\) while for axisymmetric shrinking surface, the velocity \(g'\) shows opposite behavior for \(\alpha\);

(ii) for two dimensional case and axisymmetric shrinking surface, the velocity profiles \(h\) and \(l\) increase with increasing value of \(\beta\);

(iii) the effects of Prandtl number \(\text{Pr}\) are same on the temperature field for both cases.

Notations

| \(\rho\) | Density of fluid |
| \(\nu\) | Kinematic viscosity |
| \(\alpha_{ij}\) | Second grade parameter |
| \(T\) | Temperature |
| \(\alpha\) | Stretching and shrinking parameter |
| \(k\) | Thermal conductivity |
| \(\beta\) | Dimensionless second grade parameter |
| \(\theta\) | Dimensionless temperature profile |
| \(u\) | Velocity component in x direction |
| \(v\) | Velocity component in y direction |
| \(w\) | Velocity component in z direction |
| \(\eta\) | Independent dimensionless parameter |

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References


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