Research Article

Neural Network Based Finite-Time Stabilization for Discrete-Time Markov Jump Nonlinear Systems with Time Delays

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This paper deals with the finite-time stabilization problem for discrete-time Markov jump nonlinear systems with time delays and norm-bounded exogenous disturbance. The nonlinearities in different jump modes are parameterized by neural networks. Subsequently, a linear difference inclusion state space representation for a class of neural networks is established. Based on this, sufficient conditions are derived in terms of linear matrix inequalities to guarantee stochastic finite-time boundedness and stochastic finite-time stabilization of the closed-loop system. A numerical example is illustrated to verify the efficiency of the proposed technique.

1. Introduction

Markov jump systems (MJSs) are an important class of stochastic dynamic systems, which are popular when modeling an abrupt change in the system structure and parameters, such as component failures or repairs, changing subsystem interconnections and environmental disturbance. This family of systems has great practical potential in a variety of fields, such as solar thermal central receivers systems, economic systems, communication systems, manufacturing systems, and networked control systems [1–4]. MJSs have been extensively studied since the pioneering work on quadratic control of MJSs [5], and many achievements have been made on Lyapunov stochastic stability and stabilization in the last three decades [6–18].

However, it is worth noting that the Lyapunov stochastically stable systems may not possess good or expected transient characteristics over a finite-time horizon. In many practical problems, it is of interest to investigate the stability of a system over a finite interval of time. For example, referring to aircraft control, it requests that, during the execution of a certain task, the state variables should not exceed some threshold under all admissible pilot inputs and in the presence of wind disturbances. Classical control theory does not directly address this requirement, because it focuses mainly on the asymptotic behavior of the system (over an infinite-time interval) and does not usually specify bounds on the trajectories. Therefore, it is necessary to limit the state in an acceptable region and consider finite-time stability (FTS) given by Dorato [19].

The concept of FTS has been further extended into finite-time boundedness (FTB) [20, 21], when system possesses bounded exogenous disturbance. A linear matrix inequality (LMI) framework has been established to distinguish FTS and Lyapunov asymptotical stability [22–24]. Compared with Lyapunov stochastically stable condition, FTS relaxes the condition by allowing that the Lyapunov-like function can increase at every sampling time instant. That is why FTS is so attractive and widely used in practical engineering.

As MJSs are considered, a number of results on stochastic FTS or stochastic FTB have been developed [25–28], and recently, the obtained results have been extended to
continuous-time MJSs with nonlinearities via fuzzy or neural network approach [24, 29, 30]. In order to make the stochastic systems more manageable and satisfy the requirements for finite-time behavior of a system in engineering fields, it motivates us to investigate the finite-time stability and stabilization problems for a class of MJSs. Furthermore, time delay is a common phenomenon and is inevitable in practice systems [31–33]. Due to the interaction among system dynamics, stochastic jumps, and time delays, the dynamics of MJSs with time delay become more complex than MJSs without time delay and time delay systems without jumps. So far, in comparison with the literatures available for continuous-time nonlinear MJSs with time delays, the corresponding FTS or FTB results for discrete-time nonlinear systems have been relatively few.

It is, therefore, the main purpose of this paper to shorten such a gap by investigating the finite-time stabilization problem for discrete-time nonlinear MJSs with time delays. With neural networks, the nonlinearities of MJSs are approximated firstly by linear difference inclusion under state-space representation. Then, a mode-dependent finite-time controller is developed to make the nonlinear MJSs stochastic finite-time stabilizable for all admissible approximation errors of the neural networks and the norm-bounded external disturbances. The controller gains could be derived by solving a set of LMIs. An attractive feature of the proposed scheme is that the coupling relationship between time delay and given finite-time horizon is explored by obtaining delay-independent conditions.

Notations in this paper are fairly standard. $R^n$ and $R^{n\times m}$ denote $n$-dimensional Euclidean space and the set of all the $n \times m$ real matrices, respectively; $A^T$ (or $x^T$) and $A^{-1}$ denote the transpose of the matrix $A$ (or the vector $x$) and the inverse of the matrix $A$, respectively. $\lambda_{\text{max}}(A)$ and $\lambda_{\text{min}}(A)$ denote, respectively, the maximal and minimal eigenvalues of a real matrix $A$. $\|A\|$ denotes the Euclidean norm of matrix $A$, $E[\cdot]$ denotes the mathematics statistical expectation of the stochastic process or vector, $I_{2n}(0, N)$ is the space of summable infinite sequence over $[0, N]$, $P > 0$ stands for a positive-definite matrix, $I$ is the unit matrix with appropriate dimensions, and “*” means the symmetric terms in a symmetric matrix.

2. System Description and Problem Formulation

We consider a nonlinear discrete-time MJSs, which can be described by the following mathematical model:

$$x_{k+1} = A(r_k) x_k + A_d(r_k) x_{k-d} + B(r_k) u_k + B_w(r_k) u_k + C(r_k) f(x_k, r_k),$$

$$f(x_0) = 0 \text{ but not assumed to be known a priori, and } u_k \in L^2[0, +\infty) \text{ is the exogenous disturbances satisfying}$$

$$\|u\|^2 = E \left[ \sum_{k=0}^{N} u_k^2 \right] < \delta^2.$$  \hspace{1cm} (2)

For each possible value of $r_k = i$, we denote

$$A(r_k) = A_i, \quad A_d(r_k) = A_{di}, \quad B(r_k) = B_i,$$

$$B_w(r_k) = B_{wi}, \quad C(r_k) = C_i, \quad f(x_k, r_k) = f_i(x_k),$$  \hspace{1cm} (3)

where $r_k$ is a discrete-state Markov chain taking values in $M = \{1, 2, \ldots, s\}$ with transition probabilities

$$\text{Prob} \{r_{k+1} = j \mid r_k = i\} = \pi_{ij}$$

where $\pi_{ij}$ is the transition probabilities from mode $i$ to mode $j$ that satisfies

$$\pi_{ij} \geq 0, \quad \sum_{j=1}^{m} \pi_{ij} = 1, \quad \forall i, j \in M.$$  \hspace{1cm} (5)

For each mode $i$, nonlinear function $f_i(x_k)$ is to be parameterized by neural networks. Such parameterization makes sense because any nonlinear function can be approximated arbitrarily well on a compact interval by a neural network. Let the $L$-layered perceptrons $N_i(x_k, W_{i1}, W_{i2}, \ldots, W_{iL})$ be suitably trained to approximate the nonlinear term $f_i(x_k)$, which is described in matrix-vector notation as

$$N_i(x_k, W_{i1}, W_{i2}, \ldots, W_{iL}) = \psi_{iL} \left[ W_{iL} \cdots \psi_{i2} \left[ W_{i2} \left[ \psi_{i1} \left[ W_{i1} x(t) \right] \right] \right] \right],$$

where all the weight matrices $W_r \in R^{n \times (n-1)}$, $r = 1, \ldots, L$, from the $(r-1)$th layer to the $(r+L)$th layer will be determined via back propagation (BP) procedure [24]; the activation function vector of $r$th layer is defined as $\psi_{ri}[\cdot] = \{\phi_{i1}(c_{i1}), \phi_{i2}(c_{i2}), \ldots, \phi_{in}(c_{in})\}^T$, where $n_r$ indicates the neurons of $r$th layer and let

$$\phi_{il}(c_{il}) = \lambda_{il} \left( 1 - e^{-c_{il}/\theta_l} \right), \quad q_{il}, \lambda_{il} > 0, \quad l = 1, 2, \ldots, n_r.$$  \hspace{1cm} (7)

The maximum and minimum derivatives of activation function $\phi_{il}$ are defined as follows:

$$s_{il}(m, \phi_{il}) = \begin{cases} \min_{c_{il}} \frac{\partial \phi_{il}(c_{il})}{\partial c_{il}}, & m = 0, \\ \max_{c_{il}} \frac{\partial \phi_{il}(c_{il})}{\partial c_{il}}, & m = 1. \end{cases}$$  \hspace{1cm} (8)

For $r$th layer of neural network, activation function $\phi_{il}$ can be rewritten as the following min-max form:

$$\phi_{il} = h_{il}(0) s_{il}(0, \phi_{il}) + h_{il}(1) s_{il}(1, \phi_{il}),$$

where $h_{il}(0)$ and $h_{il}(1)$ are the neuron states. 

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where \( h_i(m) \), \( m = 0, 1 \), are a set of positive real numbers associated with \( \phi_i \) satisfying \( h_i(m) > 0 \) and \( h_i(0) + h_i(1) = 1 \).

According to the approximation theorem, for given accuracy \( \rho_i > 0 \), there exist ideal constant weight matrices \( W_{i*}^* \) defined as

\[
(W_{i1*}, W_{i2*}, \ldots, W_{iL*}) = \arg \min_{(W_{i1*}, W_{i2*}, \ldots, W_{iL*})} \left\{ \max_{x_k \in D} \| f_i(x_k) - N_i(x_k, W_{i1*}, W_{i2*}, \ldots, W_{iL*}) \| \right\},
\]

(10)

where \( D \) is a compact set \( D \subseteq R^m \), such that

\[
\max_{x_k \in D} \| f_i(x_k) - N_i(x_k, W_{i1*}, W_{i2*}, \ldots, W_{iL*}) \| \leq \rho_i \| x_k \| .
\]

(11)

For each mode \( i \), denote a set of \( n_i \) dimensional index vectors of the \( r \)th layer as

\[
y_{i_r} = y_{i_r}(\sigma_i) = \{ \sigma_i \in R^{n_i} | \sigma_i \in \{0, 1\}, l = 1, \ldots, n_i \},
\]

(12)

where \( \sigma_i \) is used as a binary indicator. Obviously, the \( r \)th layer with \( n_i \) neurons has \( 2^{n_i} \) combinations of binary indicator with \( m = 0, 1 \), and the elements of index vectors for all \( L \) layers neural network have \( 2^{n_1} \times \cdots \times 2^{n_L} \) combinations in the set

\[
\Theta = y_{i_L} \oplus \cdots \oplus y_{i_2} \oplus y_{i_1}.
\]

(13)

By using (8) and adopting the compact representation [34], the multilayer neural network (6) can be expressed as follows:

\[
N(x, W_{i1*}, W_{i2*}, \ldots, W_{iL*}) = \psi_{iL} \begin{bmatrix} W_{iL*} \cdots W_{i2*} \cdots W_{i1*} \end{bmatrix} = \sum_{\sigma_i \in \Theta} \mu_{\sigma_i} A_{\sigma_i}(\sigma_i, \psi_i, W_i^*) x_k,
\]

where

\[
A_{\sigma_i} = \text{diag} \left[ s_{iL*}(\sigma_{iL}, \phi_{iL}) \right] W_{iL*} \cdots \text{diag} \left[ s_{i2*}(\sigma_{i2}, \phi_{i2}) \right] W_{i2*} \cdots \text{diag} \left[ s_{i1*}(\sigma_{i1}, \phi_{i1}) \right] W_{i1*},
\]

\[
\sum_{\sigma_i \in \Theta} \mu_{\sigma_i} = \sum_{m_{iL_1n_1} = 0}^{1} \cdots \sum_{m_{i1n_1} = 0}^{1} h_{iL_1n_1} \cdots h_{iL1} = 1.
\]

(15)

Thus by means of multilayer neural network, the nonlinear MJS (I) is translated into a group of LDIs with error bounds, in which the different inclusion is powered by stochastic Markov process; that is,

\[
x_{k+1} = \left[ \sum_{\sigma_i \in \Theta} \mu_{\sigma_i} A_{\sigma_i} + A_i \right] x_k + A_{di} x_{k-d} + B_i u_k + B_{di} w_k + C_i \Delta f_i(x_k),
\]

(16)

\[
x_f = \phi_f, \quad f \in \{-d, \ldots, 0\}, \quad r(0) = r_0.
\]

3. Main Results

Based on the LDI model (16) of networks, we consider the following discrete-time state feedback control law for nonlinear stochastic MJS (1):

\[
u_k = K_x x_k.
\]

(18)
The resulting closed-loop system can be obtained as follows:

\[ x_{k+1} = \overline{A}_i x_k + A_d (r_k) x_{k-d} + B_w (r_k) w_k + \Delta f_i (x_k), \]

\[ x_f = \phi_f, \quad f \in \{-d, \ldots, 0\}, \quad r(0) = r_0, \]  

(19)

where

\[ \overline{A}_i = \sum_{\sigma \in \Theta} \mu_\sigma A_\sigma + A_i + B_i K_i. \]  

(20)

The aim of this paper is to find some sufficient conditions which guarantee stochastic finite-time boundedness and stochastic finite-time stabilization of the closed-loop system (19). The general idea of finite-time control can be formalized through the following definitions over a finite-time interval for some given initial conditions.

**Definition 2** (stochastic finite-time stability). A discrete-time nonlinear MJS (1) (setting \( u_k = 0 \) and \( w_k = 0 \)) is said to be, stochastic finite-time stability (FTS) with respect to given \((c_1, c_2, G, N)\), where \( c_2 > c_1 \) and \( G > 0 \), if \( E[x_T^2 x_k x_k] < c_2^2, \) for some given initial conditions. \( x_k \) is a stochastic process such that \( x_k \) is a positive-definite matrix, \( x_k \) is a bounded matrix, and \( x_k \) is a positive-definite matrix.

**Definition 3** (stochastic finite-time boundedness). A discrete-time nonlinear MJS (1) (setting \( u_k = 0 \)) is said to be of stochastic finite-time boundedness (FTB) with respect to \((c_1, c_2, G, N, \delta)\) with \( c_2 > c_1 \) and \( G > 0 \), if \( E[x_T^2 x_k x_k] < c_2^2, \) for some given initial conditions. \( x_k \) is a stochastic process such that \( x_k \) is a positive-definite matrix, \( x_k \) is a bounded matrix, and \( x_k \) is a positive-definite matrix.

Before proceeding further, we introduce the following lemmas which will be needed for the derivation of our main results.

**Lemma 4.** The closed-loop system (19) is stochastic STB with respect to the given \((c_1, c_2, G, N, \delta)\) and scalar \( \alpha \geq 0 \), if there exist mode-dependent positive-definite matrices \( P_i \) and symmetric positive-definite matrices \( Q \) and \( S \) such that

\[
\begin{bmatrix}
\bar{X}_i P_i - (1 + \alpha) P + Q & \bar{X}_i P_i A_i & \bar{X}_i P_i B_i & \bar{X}_i P_i C_i \\
A_i^T P_i A_i - Q & A_i^T P_i A_i - Q & A_i^T P_i C_i \\
B_i^T P_i B_i & B_i^T P_i A_i & B_i^T P_i B_i \\
C_i^T P_i A_i & C_i^T P_i A_i & C_i^T P_i C_i
\end{bmatrix} < 0,
\]

\[ \lambda_{\max} \left( \bar{P}_i \right) \right) + c_1^2 d \lambda_{\max} \left( \bar{Q} \right) + \delta \lambda_{\max} \left( S \right) + c_2^2 \rho_i \lambda_{\max} \left( \bar{R} \right) < 0, \]

(21)

\[ c_1^2 \max_{i \in M} \left\{ \lambda_{\max} \left( \bar{P}_i \right) \right\} + c_2^2 d \lambda_{\max} \left( \bar{Q} \right) + \delta \lambda_{\max} \left( S \right) + c_2^2 \rho_i \lambda_{\max} \left( \bar{R} \right) < 0, \]

(22)

where \( \bar{P}_i = G^{-1/2} P_i G^{-1/2}, \) \( \bar{Q} = G^{-1/2} Q G^{-1/2}, \) \( \bar{R} = G^{-1/2} RG^{-1/2}, \) and \( \lambda_{\max}(\cdot), \) \( \lambda_{\min}(\cdot) \) indicate the maximal and minimal eigenvalues of the augment, respectively.

**Proof.** For the closed-loop system (19), choose a stochastic Lyapunov function candidate as

\[ V_i (k) = x_k^T P_i x_k + \sum_{f=k-d}^{k-1} x_f^T Q x_f. \]

(23)

Simple calculation shows that

\[ E \left[ V_i (k+1) \right] - V_i (k) \]

\[ = x_k^T \left( \overline{A}_i^T P_i \overline{A}_i - P + Q \right) x_k + 2 x_k^T \overline{A}_i^T P_i A_i x_k - \Delta f_i (x_k) \]

\[ + x_k^T \left( \overline{A}_i^T P_i A_i - Q \right) x_k - 2 x_k^T \overline{A}_i^T P_i B_i w_k - 2 x_k^T \overline{A}_i^T P_i C_i \Delta f_i (x_k) \]

\[ + w_k^T B_i^T P_i B_i w_k + 2 w_k^T \overline{A}_i^T P_i B_i w_k \]

\[ = \xi_k \Omega \xi_k, \]

(24)

where

\[ \Omega_k = \begin{bmatrix} A_i^T P_i A_i - Q & A_i^T P_i A_i - Q & * & * \\ A_i^T P_i A_i & A_i^T P_i A_i - Q & * & * \\ B_i^T P_i A_i & B_i^T P_i A_i & B_i^T P_i B_i & * \\ C_i^T P_i A_i & C_i^T P_i A_i & C_i^T P_i B_i & C_i^T P_i C_i \end{bmatrix}. \]

(25)

Conditions (21) and (24) imply that

\[ E \left[ V_i (k+1) \right] \leq (1 + \alpha) x_k^T P_i x_k + (1 + \alpha) w_k^T S w_k + (1 + \alpha) \Delta f_i (x_k) R A \Delta f_i (x_k) + (1 + \alpha) \sum_{f=k-d}^{k-1} x_f^T Q x_f \]

\[ = (1 + \alpha) V_i (k) + (1 + \alpha) w_k^T S w_k + (1 + \alpha) \Delta f_i (x_k) R A \Delta f_i (x_k). \]

(26)

Noting that \( \alpha \geq 0 \), we can obtain from (26) that

\[ V_i (k) \leq (1 + \alpha) V_i (0) + \sum_{f=1}^{k} (1 + \alpha)^{k-f+1} w_f^T S w_{f-1} + \sum_{f=1}^{k} (1 + \alpha)^{k-f+1} c_2^2 \rho_i \lambda_{\max} \left( \bar{R} \right). \]
\[(1 + \alpha)^k \left[ x_0^T P_0 x_0 + \sum_{f=\text{cd}}^1 x_0^T Q x_f \right. \\
+ \sum_{f=1}^k (1 + \alpha)^{1-f} w_{f-1}^T S w_{f-1} \left. \\
+ \sum_{f=1}^k (1 + \alpha)^{1-f} c_2^2 \bar{P}_f^T \lambda_{\max}(\bar{R}) \right] \\
\leq (1 + \alpha)^N \left[ c_1^2 \max_{i \in M} \{ \lambda_{\max}(\bar{P}) \} + c_2^2 d \lambda_{\max}(\bar{Q}) \right. \\
+ \left. \delta^2 \lambda_{\max}(S) + c_2 \bar{P}_1^T \lambda_{\max}(\bar{R}) \right]. \quad (27)

Note that

\[ V_j(k) = x_j^T P_j x_j + \sum_{f=\text{cd}}^1 x_j^T Q x_f \]
\[ \geq x_j^T P_j x_j \]
\[ \geq \min_{i \in M} \{ \lambda_{\min}(\bar{P}) \} x_j^T G x_j. \quad (28) \]

According to (27)-(28), one has

\[ x_j^T G x_j \leq \left( (1 + \alpha)^N \left( c_1^2 \max_{i \in M} \{ \lambda_{\max}(\bar{P}) \} + c_2^2 d \lambda_{\max}(\bar{Q}) \right. \\
+ \left. \delta^2 \lambda_{\max}(S) + c_2 \bar{P}_1^T \lambda_{\max}(\bar{R}) \right) \right) \]
\[ \times \left( \min_{i \in M} \{ \lambda_{\min}(\bar{P}) \} \right)^{-1}. \quad (29) \]

Condition (19) implies that, for \( k \in \{1, 2, \ldots, N\}, E[x_k^T G x_k] < c_1^2 \). This completes the proof. \( \square \)

Now, we direct our attention to present a solution to the problem of finite-time stabilizing controller design. Such controller is provided by the following theorem.

**Theorem 5.** The closed-loop system (19) is stochastic finite-time stabilizable via state feedback with respect to the given \((c_1, c_2, G, N, \delta)\) and scalar \(\alpha \geq 0\), if there exist matrices \(X_i = X_i^T > 0, Y_i, H = H^T > 0, S = S^T > 0, \) and \( R = R^T > 0 \) and scalars \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0 \) such that

\[
\begin{bmatrix}
- (1 + \alpha) X_i & N_i^T \\
N_i & -M_{i1} + M_{i1}^T & 0 & 0 & 0 & 0 & X_i \\
0 & 0 & M_{i1}^T & - (1 + \alpha) S & 0 & 0 & 0 \\
0 & 0 & M_{i1} & 0 & -(1 + \alpha) R & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & -H \\
\end{bmatrix} \leq 0,
\]
\[
\begin{bmatrix}
- \frac{c_1^2}{(1 + \alpha)^N} + \delta^2 \lambda_1 + c_2^2 \bar{P}_1^T \lambda_4 & c_1^2 \lambda_1 & \sqrt{\delta} c_1 \\
\end{bmatrix} \leq 0. \quad (32)
\]

**Proof.** By using Schur complement, from condition (21) in Lemma 4, it follows that

\[
\begin{bmatrix}
- (1 + \alpha) P_j + Q & * & * & * & * \\
0 & -Q & * & * & * \\
0 & 0 & (1 + \alpha) S & * & * \\
0 & 0 & 0 & -(1 + \alpha) R & * \\
M_{i1} & M_{i2} & M_{i3} & M_{i4} & -M_{i5} \\
\end{bmatrix} \leq 0, \quad (33)
\]

where

\[
M_{i1} = \left[ \sqrt{\lambda_{i1} A_{i1}^T}, \ldots, \sqrt{\lambda_{i1} A_{i1}} \right]^T, \]
\[
M_{i2} = \left[ \sqrt{\lambda_{i2} A_{i2}^T}, \ldots, \sqrt{\lambda_{i2} A_{i2}} \right]^T, \]
\[
M_{i3} = \left[ \sqrt{\lambda_{i3} B_{i3}}, \ldots, \sqrt{\lambda_{i3} B_{i3}} \right]^T, \]
\[
M_{i4} = \left[ \sqrt{\lambda_{i4} C_{i4}}, \ldots, \sqrt{\lambda_{i4} C_{i4}} \right]^T, \]
\[
M_{i5} = \text{diag} \{ P_1^{-1}, \ldots, P_r^{-1} \}. \quad (34)
\]

Performing matrix elementary transformation to the above inequality, we have

\[
\begin{bmatrix}
- (1 + \alpha) P_j + Q & M_{i1}^T & 0 & 0 & 0 & 0 \\
M_{i1} & -M_{i3} & M_{i3} & M_{i4} & M_{i2} & 0 \\
0 & M_{i3} & -(1 + \alpha) S & 0 & 0 & 0 \\
0 & M_{i4} & 0 & -(1 + \alpha) R & 0 & 0 \\
0 & M_{i2} & 0 & 0 & -Q & 0 \\
\end{bmatrix} \leq 0. \quad (35)
\]

Performing a congruence to the above condition by \( \text{diag} \{ P_1^{-1}, I, I, I, I \} \), using Schur complement, and letting \( X_j = P_{j-1} \) and \( Y_j = K_j X_j \), we get

\[
\begin{bmatrix}
- (1 + \alpha) X_j + X_j Q X_j & N_j^T \\
N_j & -M_{j1} + M_{j1}^T & 0 & 0 & 0 & 0 \\
0 & 0 & M_{j1}^T & -(1 + \alpha) S & 0 & 0 \\
0 & 0 & M_{j1} & 0 & -(1 + \alpha) R & 0 \\
X_j & 0 & 0 & 0 & 0 & -H \\
\end{bmatrix} \leq 0, \quad (36)
\]
where

\[
N_{1i} = \left[ \sqrt{\pi_{1i}} (A_i X_i + B_i Y_i)^T, \ldots, \sqrt{\pi_{si}} (A_i X_i + B_i Y_i)^T \right]^T,
\]

\[
N_{5i} = \begin{bmatrix}
\pi_{1i} A_d H A_d^T & \sqrt{\pi_{1i}} \pi_{2i} A_d H A_d^T & \cdots & \sqrt{\pi_{si}} \pi_{2i} A_d H A_d^T \\
\sqrt{\pi_{3i}} \pi_{2i} A_d H A_d^T & \pi_{1i} A_d H A_d^T & \cdots & \sqrt{\pi_{si}} \pi_{2i} A_d H A_d^T \\
\cdots & \cdots & \cdots & \cdots \\
\sqrt{\pi_{si}} \pi_{1i} A_d H A_d^T & \sqrt{\pi_{si}} \pi_{2i} A_d H A_d^T & \cdots & \pi_{1i} A_d H A_d^T
\end{bmatrix},
\]

\[N = 7\]. The model path from time step 0 to time

\[\text{Remark 7.}\] The coupling relationship between time delay and given finite-time horizon of the underlying system is obtained through a finite-time stable constraint (32) in Theorem 5. From condition (32), it can be seen that, in given finite-time horizon, if the time delay \(d\) is larger, constraint (32) is more difficult to be satisfied, which means that the existence of time delay increases the instability of system.

\section*{4. Numerical Example}

Consider discrete-time Markov jump nonlinear system (1) with three operation modes and the following data:

\[
A_1 = \begin{bmatrix}
0.88 & -0.05 \\
0.40 & -0.72
\end{bmatrix}, \quad A_{d1} = \begin{bmatrix}
0.2 & 0.1 \\
0.2 & 0.15
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
2 \\
1
\end{bmatrix}, \quad B_{w1} = \begin{bmatrix}
0.4 \\
0.5
\end{bmatrix}, \quad C_1 = \begin{bmatrix}
0 \\
0.1
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
2 & 0.24 \\
0.80 & 0.32
\end{bmatrix}, \quad A_{d2} = \begin{bmatrix}
-0.6 & 0.4 \\
0.2 & 0.6
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
1 \\
-1
\end{bmatrix}, \quad B_{w2} = \begin{bmatrix}
0.2 \\
0.6
\end{bmatrix}, \quad C_2 = \begin{bmatrix}
0 \\
0.3
\end{bmatrix},
\]

(41)

\[
A_3 = \begin{bmatrix}
-0.8 & 0.16 \\
0.80 & 0.64
\end{bmatrix}, \quad A_{d3} = \begin{bmatrix}
-0.3 & 0.1 \\
0.2 & 0.5
\end{bmatrix},
\]

\[
B_3 = \begin{bmatrix}
1 \\
1
\end{bmatrix}, \quad B_{w3} = \begin{bmatrix}
0.1 \\
0.3
\end{bmatrix}, \quad C_3 = \begin{bmatrix}
0 \\
0.5
\end{bmatrix},
\]

\[
f_1(x_k) = f_2(x_k) = f_3(x_k) = \sin(x_{1k}) \cos(x_{2k}).
\]

Now, a single hidden layer neural network with 2 hidden neurons was chosen to approximate the nonlinear functions \(f_i(x_k)\). All parameters of activation functions associated with the hidden layer were chosen to be \(q_1 = 0.5\) and \(\lambda_1 = 1\). For these activation functions, we have \(s_i(0, \phi_i) = 0\) and \(s_i(1, \phi_i) = 1\). The connection weights are trained offline by using BP algorithm. The initial weights and state vector are placed by uniformly distributed random numbers in \([-1, 1]\). After 1000 training steps, the optimal approximation weights are as follows:

\[
W_1^* = \begin{bmatrix}
-0.86017 & -0.81881 \\
-0.95025 & 0.96405
\end{bmatrix},
\]

(42)

\[
W_2^* = \begin{bmatrix}
-0.57752 & -0.58342
\end{bmatrix}.
\]

The upper bound of approximation error is estimated as \(\rho_1 = 0.022\). Obviously, in this case, we have \(\Theta^2 = 2^2 \times 2^4\). According to (15), \(A_{\sigma_i}\) can be obtained as follows:

\[
A_{11} = A_{12} = A_{13} = A_{14} = A_{15} = A_{16[0,0,0]^T} = A_{08[i,j,k]^T} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}, \quad (i, j, k \in \{0, 1\}),
\]

(43)

\[
A_{16} = A_{16[1,0]^T} = \begin{bmatrix}
0 & 0.49677 & 0.47288
\end{bmatrix},
\]

\[
A_{17} = A_{17[0,1]^T} = \begin{bmatrix}
0 & 0 \\
0.55439 & -0.56245
\end{bmatrix},
\]

\[
A_{18} = A_{18[1,1]^T} = \begin{bmatrix}
0 & 0 \\
1.0512 & -0.089567
\end{bmatrix}.
\]

The initial state and initial mode are taken as \(x_0 = [-0.3 0.4]^T\) and \(r_0 = 1\), respectively. The iterative step is taken as \(N = 7\). The mode path from time step 0 to time
step 7 is generated randomly and it is shown in Figure 1. Let $c_1 = 0.5, c_2 = 2, N = 7, G = I, d = 0.5, \alpha = 0.5$, and $\delta^2 = 1$. By solving the matrix inequalities in Theorem 5, we have the following controller gains:

$$K_1 = \begin{bmatrix} -0.9304 & -0.0683 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -1.7231 & 0.3654 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 1.1486 & -0.1588 \end{bmatrix}.$$ (44)

The state trajectories of the free and controlled MJLS (16) are drawn in Figures 2 and 3, respectively. It could be seen that the free MJLS (16) is not stochastic FTB because the trajectory exceeds the given bound $c_2^2$. However, the trajectory is limited between the two ellipsoids regions by employing the proposed control move which satisfactorily justify that the closed-loop MJLS (16) is stochastic FTB.

It should be pointed out that, in the simulation example, as long as the choice of initial condition is satisfied with $\|x_0^TR_x\| \leq c_1$, then the system is robustly finite-time stabilizable; that is, system trajectories stay within a given bound.

5. Conclusions

The finite-time stabilization problem for discrete-time Markovian jump nonlinear system with time delay and norm-bounded exogenous disturbance is investigated in this paper. The nonlinearities are parameterized by multilayer neural network and the relationship between time delay and given finite-time horizon is explored with delay-independent conditions. The proposed framework is versatile and can accommodate a number of challenging design problems including finite-time control and filtering of discrete-time or continuous-time nonlinear MJLS with parameter uncertainties, time delays, and so on. The future work can consider some delay-dependent approaches or delay fractioning approaches to reduce the conservativeness introduced by time delay.

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References


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