Research Article

Nonexistence of Homoclinic Solutions for a Class of Discrete Hamiltonian Systems

Xiaoping Wang

Department of Mathematics, Xiangnan College, Chenzhou, Hunan 423000, China

Correspondence should be addressed to Xiaoping Wang; wxp31415@sina.com

Received 3 September 2012; Accepted 27 November 2012

Academic Editor: Jinde Cao

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We give several sufficient conditions under which the first-order nonlinear discrete Hamiltonian system $\Delta x(n) = \alpha(n)x(n+1) + \beta(n)|y(n)|^{\mu-2}y(n)$, $\Delta y(n) = -\gamma(n)|x(n+1)|^{\nu-2}x(n+1) - \alpha(n)y(n)$ has no solution $(x(n), y(n))$ satisfying condition $0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^\nu + (1 + \beta(n))|y(n)|^\mu] < +\infty$, where $\mu, \nu > 1$ and $1/\mu + 1/\nu = 1$ and $\alpha(n), \beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$.

1. Introduction

In 1907, Lyapunov [1] established the first so-called Lyapunov inequality:

$$(b-a) \int_a^b q(t) dt > 4, \quad (1)$$

if Hill's equation

$$x''(t) + q(t)x(t) = 0 \quad (2)$$

has a real solution $x(t)$ such that

$$x(a) = x(b) = 0, \quad x(t) \not\equiv 0, \quad t \in [a, b], \quad (3)$$

and the constant 4 in (1) cannot be replaced by a larger number, where $q(t)$ is a piecewise continuous and nonnegative function defined on $\mathbb{R}$. Since this result has found applications in the study of various properties of solutions such as oscillation theory, disconjugacy, and eigenvalue problems of (2), a large number of Lyapunov-type inequalities were established in the literature which generalized or improved (1); see [1–20].

In 1983, Cheng [3] first obtained the discrete analogy of Lyapunov inequality (1) for the second-order difference equation:

$$\Delta^2 x(n) + q(n)x(n+1) = 0, \quad (4)$$

where, and in the sequel, Δ denotes the forward difference operator defined by $\Delta x(n) = x(n+1) - x(n)$.

When $a = -\infty$ and $b = +\infty$, that is, system (4) has a solution $x(n)$ satisfying $\lim_{|n| \rightarrow \infty} x(n) = 0$, which is called homoclinic solution, whether one can obtain Lyapunov-type inequalities for (4)? To the best of our knowledge, there are no results.

In 2003, Sh. Guseinov and Kaymakçalan [7] partly generalized the Cheng's result to the discrete linear Hamiltonian system:

$$\begin{aligned} \Delta x(n) &= \alpha(n)x(n+1) + \beta(n)y(n), \\ \Delta y(n) &= -\gamma(n)x(n+1) - \alpha(n)y(n), \end{aligned} \quad (5)$$

where $\alpha(n), \beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$ and a and b are not necessarily usual zeros, but rather, generalized zeros. Later, some better Lyapunov-type inequalities for system (5) were obtained in [19, 20].

Very recently, He and Zhang [10] further generalized the result in [19] to the following first-order nonlinear difference system:

$$\begin{aligned} \Delta x(n) &= \alpha(n)x(n+1) + \beta(n)|y(n)|^{\mu-2}y(n), \\ \Delta y(n) &= -\gamma(n)|x(n+1)|^{\nu-2}x(n+1) - \alpha(n)y(n), \end{aligned} \quad (6)$$

where $\mu, \nu > 1$ and $1/\mu + 1/\nu = 1$ and $\alpha(n), \beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$.

When $\mu = \nu = 2$, system (6) reduces to (5). In addition, the special forms of system (6) contain many well-known difference equations which have been studied extensively and have much applications in the literature [21–23], such as the second-order linear difference equation:

$$\Delta [p(n) \Delta x(n)] + q(n) x(n+1) = 0, \quad (7)$$

and the second-order half-linear difference equation:

$$\Delta [p(n) |\Delta x(n)|^{r-2} \Delta x(n)] + q(n) |x(n+1)|^{r-2} x(n+1) = 0, \quad (8)$$

where $r > 1$, $p(n)$ and $q(n)$ are real-valued functions defined on $\mathbb{Z}$ and $p(n) > 0$. Let

$$y(n) = p(n) |\Delta x(n)|^{r-2} \Delta x(n), \quad (9)$$

then (8) can be written as the form of (6):

$$\begin{aligned} \Delta x(n) &= [p(n)]^{1/(1-r)} |y(n)|^{(2-r)/(r-1)} y(n), \\ \Delta y(n) &= -q(n) |x(n+1)|^{r-2} x(n+1), \end{aligned} \quad (10)$$

where $\mu = r/(r-1)$, $\nu = r$ and $\alpha(n) = 0$, $\beta(n) = [p(n)]^{1/(1-r)}$ and $\gamma(n) = q(n)$.

In this paper, we will establish several Lyapunov-type inequalities for systems (5) and (6) if they have a solution $(x(n), y(n))$ satisfying conditions

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^2 + (1 + \beta(n)) |y(n)|^2] < +\infty, \quad (11)$$

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^\nu + (1 + \beta(n)) |y(n)|^\mu] < +\infty, \quad (12)$$

respectively. Taking advantage of these Lyapunov-type inequalities, we are able to establish some criteria for nonexistence of homoclinic solutions of systems (5) and (6). As we know, there are no results on non-existence of homoclinic solutions for Hamiltonian systems in previous literature.

2. Lyapunov-Type Inequalities for System (6)

In this section, we shall establish some Lyapunov-type inequalities for system (6). For the sake of convenience, we list some assumptions on $\alpha(n)$ and $\beta(n)$ as follows:

(A0) $\alpha(n) < 1$, for all $n \in \mathbb{Z}$, $\prod_{s=-\infty}^{+\infty} [1 - \alpha(s)]^{-1} < +\infty$;

(A1) $\alpha(n) < 1$, for all $n \in \mathbb{Z}$, $\sum_{s=-\infty}^{+\infty} |\alpha(s)| < +\infty$;

(B0) $\beta(n) \geq (\neq) 0$, for all $n \in \mathbb{Z}$;

(B1) $\sum_{\tau=-\infty}^0 \beta(\tau) \prod_{s=\tau}^0 [1 - \alpha(s)]^{-\mu}$
 $+ \sum_{\tau=1}^{+\infty} \beta(\tau) \prod_{s=0}^{\tau-1} [1 - \alpha(s)]^\mu < +\infty$.

Denote

$$\begin{aligned} \zeta(n) &:= \left[\sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-\mu} \right]^{v/\mu}, \\ \eta(n) &:= \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^\mu \right]^{v/\mu}. \end{aligned} \quad (13)$$

Theorem 1. Suppose that hypotheses (A0), (B0), and (B1) are satisfied. If system (6) has a solution $(x(n), y(n))$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^\nu + (1 + \beta(n)) |y(n)|^\mu] < +\infty, \quad (14)$$

then one has the following inequality:

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n) \eta(n)}{\zeta(n) + \eta(n)} \gamma^+(n) \geq 1, \quad (15)$$

where $\gamma^+(n) = \max\{\gamma(n), 0\}$.

Proof. Hypothesis (B1) implies that functions $\zeta(n)$ and $\eta(n)$ are well defined on $\mathbb{Z}$. Without loss of generality, we can assume that

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n) \eta(n)}{\zeta(n) + \eta(n)} \gamma^+(n) < +\infty. \quad (16)$$

From (14) and (B0), one has

$$\lim_{|n| \rightarrow \infty} |x(n)| = \lim_{|n| \rightarrow \infty} |y(n)| = 0, \quad (17)$$

$$\sum_{\tau=-\infty}^{+\infty} \beta(\tau) |y(\tau)|^\mu < +\infty. \quad (18)$$

It follows from (13), (18), and the Hölder inequality that

$$\begin{aligned} &\sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^{\mu-1} \prod_{s=\tau}^n [1 - \alpha(s)]^{-1} \\ &\leq \left[\sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-\mu} \right]^{1/\mu} \left[\sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^\mu \right]^{1/\nu} \\ &= [\zeta(n)]^{1/\nu} \left[\sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^\mu \right]^{1/\nu} \\ &< +\infty, \quad \forall n \in \mathbb{Z}, \end{aligned} \quad (19)$$

$$\begin{aligned} &\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^{\mu-1} \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)] \\ &\leq \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^\mu \right]^{1/\mu} \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^\mu \right]^{1/\nu} \\ &= [\eta(n)]^{1/\nu} \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^\mu \right]^{1/\nu} \\ &< +\infty, \quad \forall n \in \mathbb{Z}. \end{aligned} \quad (20)$$

From (A0), (17), (19), (20), and the first equation of system (6), we have

$$x(n+1) = \sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^{\mu-2} y(\tau) \prod_{s=\tau}^n [1-\alpha(s)]^{-1}, \quad \forall n \in \mathbb{Z}, \quad (21)$$

$$x(n+1) = - \sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^{\mu-2} y(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)], \quad \forall n \in \mathbb{Z}. \quad (22)$$

Combining (19) with (21), one has

$$\begin{aligned} |x(n+1)|^v &= \left| \sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^{\mu-2} y(\tau) \prod_{s=\tau}^n [1-\alpha(s)]^{-1} \right|^v \\ &\leq \zeta(n) \sum_{\tau=-\infty}^n \beta(\tau) |y(\tau)|^\mu, \quad \forall n \in \mathbb{Z}. \end{aligned} \quad (23)$$

Similarly, it follows from (20) and (22) that

$$\begin{aligned} |x(n+1)|^v &= \left| \sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^{\mu-2} y(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)] \right|^v \\ &\leq \eta(n) \sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^\mu, \quad \forall n \in \mathbb{Z}. \end{aligned} \quad (24)$$

Combining (23) with (24), one has

$$|x(n+1)|^v \leq \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \sum_{\tau=-\infty}^{+\infty} \beta(\tau) |y(\tau)|^\mu, \quad \forall n \in \mathbb{Z}. \quad (25)$$

Now, it follows from (16), (18), and (25) that

$$\begin{aligned} &\sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v \\ &\leq \left[\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) \right] \\ &\quad \times \sum_{n=-\infty}^{+\infty} \beta(n) |y(n)|^\mu < +\infty. \end{aligned} \quad (26)$$

By (6), we obtain

$$\Delta(x(n)y(n)) = \beta(n) |y(n)|^\mu - \gamma(n) |x(n+1)|^v. \quad (27)$$

Summing the above from $-\infty$ to $+\infty$ and using (17) and (18), we obtain

$$\sum_{n=-\infty}^{+\infty} \gamma(n) |x(n+1)|^v = \sum_{n=-\infty}^{+\infty} \beta(n) |y(n)|^\mu, \quad (28)$$

which, together with (26), implies that

$$\begin{aligned} &\sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v \\ &\leq \left[\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) \right] \sum_{n=-\infty}^{+\infty} \beta(n) |y(n)|^\mu \\ &= \left[\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) \right] \sum_{n=-\infty}^{+\infty} \gamma(n) |x(n+1)|^v \\ &\leq \left[\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) \right] \sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v. \end{aligned} \quad (29)$$

We claim that

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v > 0. \quad (30)$$

If (30) is not true, then

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v = 0. \quad (31)$$

From (28) and (31), we have

$$\begin{aligned} 0 &\leq \sum_{n=-\infty}^{+\infty} \beta(n) |y(n)|^\mu = \sum_{n=-\infty}^{+\infty} \gamma(n) |x(n+1)|^v \\ &\leq \sum_{n=-\infty}^{+\infty} \gamma^+(n) |x(n+1)|^v = 0. \end{aligned} \quad (32)$$

It follows that

$$\beta(n) |y(n)|^{\mu-2} y(n) \equiv 0, \quad \forall n \in \mathbb{Z}. \quad (33)$$

Combining (21) with (33), we obtain that

$$x(n) \equiv 0, \quad \forall n \in \mathbb{Z}, \quad (34)$$

which, together with the second equation of system (6), implies that

$$\Delta y(n) = -\alpha(n) y(n), \quad \forall n \in \mathbb{Z}. \quad (35)$$

Combining the above with (17), one has

$$y(n) \equiv 0, \quad \forall n \in \mathbb{Z}. \quad (36)$$

Both (34) and (36) contradict with (14). Therefore, (30) holds. Hence, it follows from (29) and (30) that (15) holds. $\square$

Corollary 2. Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If system (6) has a solution $(x(n), y(n))$ satisfying (14), then one has the following inequality:

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{v/2\mu} \geq 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{v/2}, \quad (37)$$

where and in the sequel,

$$\begin{aligned}\Theta[\alpha(n)] &= \min\{1 - \alpha^+(n), [1 + \alpha^-(n)]^{-1}\}, \\ \alpha^+(n) &= \max\{\alpha(n), 0\}, \quad \alpha^-(n) = \max\{-\alpha(n), 0\}.\end{aligned}\quad (38)$$

Proof. Obviously, (A1) implies that

$$0 < \prod_{s=-\infty}^{+\infty} [1 - \alpha(s)] < +\infty, \quad (39)$$

and so (A0) holds, and which, together with (B1), implies that $\sum_{\tau=-\infty}^{+\infty} \beta(\tau) < +\infty$. Since

$$\zeta(n) + \eta(n) \geq 2[\zeta(n)\eta(n)]^{1/2}, \quad (40)$$

it follows that

$$\begin{aligned}1 &\leq \sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n) + \eta(n)} \gamma^+(n) \\ &\leq \frac{1}{2} \sum_{n=-\infty}^{+\infty} [\zeta(n)\eta(n)]^{1/2} \gamma^+(n) \\ &= \frac{1}{2} \sum_{n=-\infty}^{+\infty} \gamma^+(n) \left\{ \sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-\mu} \right. \\ &\quad \times \left. \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^{\mu} \right\}^{v/2\mu} \\ &\leq \frac{1}{2} \sum_{n=-\infty}^{+\infty} \gamma^+(n) \left\{ \sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha^+(s)]^{-\mu} \right. \\ &\quad \times \left. \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 + \alpha^-(s)]^{\mu} \right\}^{v/2\mu} \quad (41) \\ &\leq \frac{1}{2} \sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{v/2\mu} \\ &\quad \times \prod_{s=-\infty}^n [1 - \alpha^+(s)]^{-v/2} \prod_{s=n+1}^{+\infty} [1 + \alpha^-(s)]^{v/2} \\ &\leq \frac{1}{2} \sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^b \beta(\tau) \right]^{v/2\mu} \\ &\quad \times \prod_{s=-\infty}^{+\infty} \{\Theta[\alpha(s)]\}^{-v/2},\end{aligned}$$

which implies that (37) holds. $\square$

Since

$$\left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{1/2} \leq \frac{1}{2} \sum_{n=-\infty}^{+\infty} \beta(n), \quad (42)$$

then it follows from (37) that the following corollary is true.

Corollary 3. Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If system (6) has a solution $(x(n), y(n))$ satisfying (14), then

$$\left(\sum_{n=-\infty}^{+\infty} \beta(n) \right)^{1/\mu} \left(\sum_{n=-\infty}^{+\infty} \gamma^+(n) \right)^{1/\nu} \geq 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{1/2}. \quad (43)$$

Applying Theorem 1 and Corollary 2 to system (8) (i.e., (10)), we have immediately the following two corollaries.

Corollary 4. Suppose that $r > 1$ and $p(n) > 0$ for $n \in \mathbb{Z}$, and that

$$\sum_{\tau=-\infty}^{+\infty} \frac{1}{[p(\tau)]^{1/(r-1)}} < +\infty. \quad (44)$$

If (8) has a solution $x(n)$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^r + p(n) \left(1 + [p(n)]^{1/(r-1)} \right) |\Delta x(n)|^r \right] < +\infty, \quad (45)$$

then

$$\begin{aligned}&\sum_{n=-\infty}^{+\infty} \left(\left\{ \sum_{\tau=-\infty}^n [p(\tau)]^{-1/(r-1)} \right\}^{r-1} \left\{ \sum_{\tau=n+1}^{+\infty} [p(\tau)]^{-1/(r-1)} \right\}^{r-1} \right. \\ &\quad \times \left(\left\{ \sum_{\tau=-\infty}^n [p(\tau)]^{-1/(r-1)} \right\}^{r-1} \right. \\ &\quad \left. \left. + \left\{ \sum_{\tau=n+1}^{+\infty} [p(\tau)]^{-1/(r-1)} \right\}^{r-1} \right)^{-1} \right) q^+(n) \geq 1.\end{aligned}\quad (46)$$

Corollary 5. Suppose that $r > 1$ and $p(n) > 0$ for $n \in \mathbb{Z}$, and that (44) holds. If (8) has a solution $x(n)$ satisfying (45), then

$$\sum_{n=-\infty}^{+\infty} q^+(n) \left\{ \sum_{\tau=-\infty}^n [p(\tau)]^{-1/(r-1)} \sum_{\tau=n+1}^{+\infty} [p(\tau)]^{-1/(r-1)} \right\}^{(r-1)/2} \geq 2. \quad (47)$$

3. Lyapunov-Type Inequalities for System (5)

When $\mu = \nu = 2$, assumption (B1) reduces the following form:

$$(B2) \quad \sum_{\tau=-\infty}^0 \beta(\tau) \prod_{s=\tau}^0 [1 - \alpha(s)]^{-2} + \sum_{\tau=1}^{+\infty} \beta(\tau) \prod_{s=0}^{\tau-1} [1 - \alpha(s)]^2 < +\infty.$$

Applying the results obtained in last section to the first-order linear Hamiltonian system (5), we have immediately the following corollaries.

Corollary 6. Suppose that hypotheses (A0), (B0), and (B2) are satisfied. If system (5) has a solution $(x(n), y(n))$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^2 + (1 + \beta(n)) |y(n)|^2] < +\infty, \quad (48)$$

then

$$\begin{aligned} & \sum_{n=-\infty}^{+\infty} \left(\left\{ \sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-2} \right\} \right. \\ & \quad \times \left\{ \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^2 \right\} \\ & \quad \times \left(\sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-2} \right. \\ & \quad \left. \left. + \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^2 \right)^{-1} \right) \gamma^+(n) \geq 1. \end{aligned} \quad (49)$$

Corollary 7. Suppose that hypotheses (A1), (B0), and (B2) are satisfied. If system (5) has a solution $(x(n), y(n))$ satisfying (48), then

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{1/2} \geq 2 \prod_{n=-\infty}^{+\infty} \Theta[\alpha(n)]. \quad (50)$$

Corollary 8. Suppose that $p(n) > 0$ for $n \in \mathbb{Z}$, and that

$$\sum_{\tau=-\infty}^{+\infty} \frac{1}{p(\tau)} < +\infty. \quad (51)$$

If (7) has a solution $x(n)$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^2 + p(n) (1 + p(n)) |\Delta x(n)|^2] < +\infty, \quad (52)$$

then

$$\sum_{n=-\infty}^{+\infty} q^+(n) \left[\sum_{\tau=-\infty}^n \frac{1}{p(\tau)} \sum_{\tau=n+1}^{+\infty} \frac{1}{p(\tau)} \right] \geq \sum_{n=-\infty}^{+\infty} \frac{1}{p(n)}. \quad (53)$$

4. Nonexistence of Homoclinic Solutions

Applying the results obtained in Sections 2 and 3, we can drive the following criteria for non-existence of homoclinic solutions of systems (5) and (6) immediately.

Corollary 9. Suppose that hypotheses (A0), (B0), and (B1) are satisfied. If

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n) \eta(n)}{\zeta(n) + \eta(n)} \gamma^+(n) < 1, \quad (54)$$

then system (6) has no solution $(x(n), y(n))$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^\nu + (1 + \beta(n)) |y(n)|^\mu] < +\infty. \quad (55)$$

Corollary 10. Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{v/2\mu} < 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{v/2}, \quad (56)$$

then system (6) has no solution $(x(n), y(n))$ satisfying (55).

Corollary 11. Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If

$$\left(\sum_{n=-\infty}^{+\infty} \beta(n) \right)^{1/\mu} \left(\sum_{n=-\infty}^{+\infty} \gamma^+(n) \right)^{1/\nu} < 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{1/2}, \quad (57)$$

then system (6) has no solution $(x(n), y(n))$ satisfying (55).

Corollary 12. Suppose that hypotheses (A0), (B0), and (B2) are satisfied. If

$$\begin{aligned} & \sum_{n=-\infty}^{+\infty} \left(\left\{ \sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-2} \right\} \right. \\ & \quad \times \left\{ \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^2 \right\} \\ & \quad \times \left(\sum_{\tau=-\infty}^n \beta(\tau) \prod_{s=\tau}^n [1 - \alpha(s)]^{-2} \right. \\ & \quad \left. \left. + \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1 - \alpha(s)]^2 \right)^{-1} \right) \gamma^+(n) < 1, \end{aligned} \quad (58)$$

then system (5) has no solution $(x(n), y(n))$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} [|x(n)|^2 + (1 + \beta(n)) |y(n)|^2] < +\infty. \quad (59)$$

Corollary 13. Suppose that hypotheses (A1), (B0), and (B2) are satisfied. If

$$\sum_{n=-\infty}^{+\infty} \gamma^+(n) \left[\sum_{\tau=-\infty}^n \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{1/2} < 2 \prod_{n=-\infty}^{+\infty} \Theta[\alpha(n)], \quad (60)$$

then system (5) has no solution $(x(n), y(n))$ satisfying (59).

Corollary 14. Suppose that $p(n) > 0$ for $n \in \mathbb{Z}$, and that (51) holds. If

$$\sum_{n=-\infty}^{+\infty} q^+(n) \left(\sum_{\tau=-\infty}^n \frac{1}{p(\tau)} \sum_{\tau=n+1}^{+\infty} \frac{1}{p(\tau)} \right) < \sum_{n=-\infty}^{+\infty} \frac{1}{p(n)}, \quad (61)$$

then (7) has no solution $x(n)$ satisfying (52).

Example 15. Consider the second-order difference equation:

$$\Delta \left[(1 + n^2) \Delta x(n) \right] + q(n) x(n+1) = 0, \quad (62)$$

where $q(n)$ is real-valued function defined on $\mathbb{Z}$. In view of Corollary 14, if

$$\sum_{n=-\infty}^{+\infty} \left[\left(\sum_{\tau=-\infty}^n \frac{1}{1+\tau^2} \sum_{\tau=n+1}^{+\infty} \frac{1}{1+\tau^2} \right) \right] q^+(n) < \sum_{n=-\infty}^{+\infty} \frac{1}{1+n^2}, \quad (63)$$

then (62) has no solution $x(n)$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^2 + (1 + n^2)^2 |\Delta x(n)|^2 \right] < +\infty. \quad (64)$$

Acknowledgment

This work is supported by the Scientific Research Fund of Hunan Provincial Education Department (07A066).

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