Research Article

A One Step Optimal Homotopy Analysis Method for Propagation of Harmonic Waves in Nonlinear Generalized Magnetothermoelasticity with Two Relaxation Times under Influence of Rotation

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The aim of this paper is to apply OHAM to solve numerically the problem of harmonic wave propagation in a nonlinear thermoelasticity under influence of rotation, thermal relaxation times, and magnetic field. The problem is solved in one-dimensional elastic half-space model subjected initially to a prescribed harmonic displacement and the temperature of the medium. The HAM contains a certain auxiliary parameter which provides us with a simple way to adjust and control the convergence region and rate of convergence of the series solution. This optimal approach has a general meaning and can be used to get fast convergent series solutions of the different type of nonlinear fractional differential equation. The displacement and temperature are calculated for the models with the variations of the magnetic field, relaxation times, and rotation. The results obtained are displayed graphically to show the influences of the new parameters.

1. Introduction

In the past recent years, much attention has been devoted to simulate some real-life problems which can be described by nonlinear coupled differential equations using reliable and more efficient methods. Nonlinear partial differential equations are useful in describing various phenomena in disciplines. The nonlinear coupled systems of partial differential equations often appear in the study of circled fuel reactor, high-temperature hydrodynamics, and thermoelasticity problems, see [1–4]. From the analytical point of view, a lot of work has been done for such systems. With the rapid development of nanotechnology, there appears an ever increasing interest of scientists and researchers in this field of science. Nanomaterials, because of their exceptional mechanical, physical, and chemical properties, have been

the main topic of research in many scientific publications. Wave generation in nonlinear thermoelasticity problems has gained a considerable interest for its utilitarian aspects in understanding the nature of interaction between the elastic and thermal fields as well as the system of PDEs for its applications. A lot of applications were paid on existence, uniqueness, and stability of the solution of the problem, see [5–7].

Recently, much attention has been devoted to numerical methods, which do not require discretization of space-time variables or linearization of the nonlinear equations, among the homotopy analysis methods. Since most of the nonlinear FDEs cannot be solved exactly, approximate and numerical methods must be used. Some of the recent analytical methods for solving nonlinear problems include the homotopy analysis method HAM [8–14]. The HAM, first proposed in 1992 by Liao [8], has been successfully applied to solve many

problems in physics and science. This method is applied to solve linear and nonlinear systems.

The homotopy perturbation method HPM has the merits of simplicity and easy execution. The homotopy perturbation method was first proposed by He [15]. Unlike the traditional numerical methods, the HPM does not need discretization and linearization. Most perturbation methods assume that a small parameter exists, but most nonlinear problems have no small parameter at all. Many new methods have been proposed to eliminate the small parameter. Recently, the applications of homotopy theory among scientists appeared, and the homotopy theory became a powerful mathematical tool, when it is successfully coupled with perturbation theory.

Recently, Gepreel et al. [16] investigated the homotopy perturbation method and variational iteration method for harmonic waves propagation in nonlinear magnetothermoelasticity with rotation. Abd-Alla and Abo-Dahab [17] investigated the effect of rotation and initial stress on an infinite generalized magnetothermoelastic diffusion body with a spherical cavity. Abo-Dahab and Mohamed [18] studied the influence of magnetic field and hydrostatic initial stress on reflection phenomena of P (Primary) and SV (Shear Vertical) waves from a generalized thermoelastic solid half space. Abd-Alla and Mahmoud [19] investigated the magnetothermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model. Abd-Alla et al. [20] studied the thermal stresses effect in a non-homogeneous orthotropic elastic multilayered cylinder. Abd-Alla et al. [21] studied the generalized magnetothermoelastic Rayleigh waves in a granular medium under the influence of a gravity field and initial stress. Abd-Alla and Abo-Dahab [22] investigated the time-harmonic sources in a generalized magnetothermoviscoelastic continuum with and without energy dissipation.

In the present paper, investigation is devoted for solving numerically the problem of harmonic wave propagation in a nonlinear thermoelasticity under influence of magnetic field, thermal relaxation times, and rotation. The problem is solved in one-dimensional elastic half-space model subjected initially to a prescribed harmonic displacement and the temperature of the medium.

The HAM contains a certain auxiliary parameter which provides us with a simple way to adjust and control the convergence region and rate of convergence of the series solution. The h -curve of the third-order approximate solutions is displayed graphically to show the interval that the exact and approximate solutions take the same values. The displacement and temperature are calculated for the methods with the variations of the magnetic field and rotation. The results obtained are displayed graphically to show the influences of the new parameters.

2. A One-Step Optimal Homotopy Analysis Method for PDEs

To describe the basic ideas of the HAM, we consider the following general nonlinear differential equation:

$$N[u(x, t)] = 0, \quad (1)$$

where N is a nonlinear operator for this problem, x and t denote independent variables, and $u(x, t)$ is an unknown function.

By means of the HAM, one first constructs the following zero-order deformation equation:

$$(1 - q) \mathcal{L}(\phi(x, t; q) - u_0(x, t)) = qhH(t)N[\phi(x, t; q)], \quad (2)$$

where $q \in [0, 1]$ is the embedding parameter, $h \neq 0$ is an auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, $\mathcal{L}$ is an auxiliary linear operator, and $u_0(x, t)$ is an initial guess. Obviously, when $q = 0$ and $q = 1$, it holds

$$\phi(x, t; 0) = u_0(x, t), \quad \phi(x, t; 1) = u(x, t). \quad (3)$$

Liao [8, 9] expanded $\phi(x, t; q)$ in Taylor series with respect to the embedding parameter q , as follows:

$$\phi(x, t; q) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t) q^m, \quad (4)$$

where

$$u_m(x, t) = \frac{1}{m!} \left. \frac{\partial^m \phi(x, t; q)}{\partial q^m} \right|_{q=0}. \quad (5)$$

Assume that the auxiliary linear operator, the initial guess, the auxiliary parameter h , and the auxiliary function $H(t)$ are selected such that the series (4) is convergent at $q = 1$; then we have from (4)

$$u(x, t) = u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t). \quad (6)$$

Let us define the vector

$$\vec{u}_n(x, t) = \{u_0(x, t), u_1(x, t), u_2(x, t), \dots, u_n(x, t)\}. \quad (7)$$

Differentiating (2) m times with respect to q , then setting $q = 0$ and dividing then by $m!$, we have the following m th-order deformation equation:

$$\mathcal{L}(u_m(x, t) - \kappa_m u_{m-1}(x, t)) = hH(t) \mathcal{R}_m(\vec{u}_{m-1}), \quad (8)$$

where

$$\mathcal{R}_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \left. \frac{\partial^{m-1} \mathcal{N}[\phi(x, t; q)]}{\partial q^{m-1}} \right|_{q=0}, \quad (9)$$

$$\kappa_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases}$$

Applying the integral operator on both sides of (8), we have

$$u_m(x, t) = \kappa_m u_{m-1}(x, t) + h \int_0^t H(t) \mathcal{R}_m(\vec{u}_{m-1}) dt, \quad (10)$$

where the m th-order deformation equation (8) can be easily solved, especially by means of symbolic computation software

such as Mathematica, Maple, and MathLab. The convergence of the homotopy analysis method for solving these equations is discussed in [23].

Abbasbandy and Jalili [24] and Turkyilmazoglu [25–29] applied the homotopy analysis method to nonlinear ODEs and suggested the so-called optimization method to find out the optimal convergence control parameters by minimum of the square residual error integrated in the whole region having physical meaning. Their approach is based on the square residual error.

Let $\Delta(h)$ denote the square residual error of the governing equation (1) and express it as

$$\Delta(h) = \int_{\Omega} (N[\tilde{u}_n(t)])^2 d\Omega, \quad (11)$$

where

$$\tilde{u}_m(t) = u_0(t) + \sum_{k=1}^m u_k(t); \quad (12)$$

the optimal value of h is given by a nonlinear algebraic equation:

$$\frac{d\Delta(h)}{dh} = 0. \quad (13)$$

3. Application of HAM on the Nonlinear Magnetothermoelastic with Rotation Equations

In this section, we use the homotopy analysis method to calculate the approximate solutions of the following nonlinear magnetothermoelastic model with rotation equations

$$\begin{aligned} (1 + \sigma_1) u_{tt} + \Omega u_t - u_{xx} (1 - \sigma_2 + 2\gamma u_x + 3\delta u_x^2) \\ - \beta_1 (1 - i\omega\tau_2) \theta_x - \beta_2 (\theta u_x)_x = 0, \\ \left(\theta (1 - i\omega\tau_1) - a u_x (1 - i\omega\delta\tau_1) - \frac{1}{2} b u_x^2 \right)_t \\ - [(1 + \alpha u_x) \theta_x]_x = 0, \end{aligned} \quad (14)$$

where $\sigma_1, \sigma_2, \Omega, \gamma, \beta_1, \beta_2, a, b$, and α are arbitrary constants with the initial conditions

$$\begin{aligned} u(x, 0) = \theta(x, 0) = A(1 - \cos(x)), \\ u_t(x, 0) = \theta_t(x, 0) = 0, \end{aligned} \quad (15)$$

where A is an arbitrary constant and the boundary conditions

$$\begin{aligned} u(0, t) = \theta(0, t) = 0, \\ u_t(0, t) = \theta_t(0, t) = 0. \end{aligned} \quad (16)$$

To demonstrate the effectiveness of the method, we consider the system of nonlinear initial-value problem (14) with the

initial conditions (15) and the boundary conditions (16) by choosing the linear operators

$$\begin{aligned} \mathcal{L}_1[\phi_1(x, t; q)] &= \frac{\partial^2 \phi_1(x, t; q)}{\partial t^2}, \\ \mathcal{L}_2[\phi_2(x, t; q)] &= \frac{\partial \phi_2(x, t; q)}{\partial t}, \end{aligned} \quad (17)$$

with the property $\mathcal{L}_1[c_1 + c_2 t] = 0$, $\mathcal{L}_2[c_3]$, where c_i , ($i = 1, 2, 3$) are the integral constants and the nonlinear operators are defined as

$$\begin{aligned} N_1[\phi_1, \phi_2] &= (1 + \sigma_1) \frac{\partial^2 \phi_1}{\partial t^2} + \Omega \frac{\partial \phi_1}{\partial t} - \frac{\partial^2 \phi_1}{\partial x^2} \\ &\times \left(1 - \sigma_2 + 2\gamma \frac{\partial \phi_1}{\partial x} + 3\delta \left(\frac{\partial \phi_1}{\partial x} \right)^2 \right) - \beta_1 \frac{\partial \phi_2}{\partial x} \\ &+ i\omega\tau_2\beta_1 \frac{\partial \phi_2}{\partial x} - \beta_2 \frac{\partial}{\partial x} \left(\phi_2 \frac{\partial \phi_1}{\partial x} \right), \\ N_2[\phi_1, \phi_2] &= \frac{\partial}{\partial t} \left((1 - i\omega\tau_1) \phi_2 - a(1 - i\omega\tau_1\delta) \frac{\partial \phi_1}{\partial x} \right. \\ &\left. - \frac{1}{2} b \left(\frac{\partial \phi_1}{\partial x} \right)^2 - \frac{\partial}{\partial x} \left(1 + \alpha \frac{\partial \phi_1}{\partial x} \right) \frac{\partial \phi_2}{\partial x} \right). \end{aligned} \quad (18)$$

Choosing $H_i(t) = 1$ for $i = 1, 2$, the zeroth-order deformation equations are

$$\begin{aligned} (1 - q) \mathcal{L}_1[\phi_1(x, t; q) - u_0(x, t)] \\ = qh_1 N_1[\phi_1(x, t; q), \phi_2(x, t; q)], \\ (1 - q) \mathcal{L}_2[\phi_2(x, t; q) - v_0(x, t)] \\ = qh_2 N_2[\phi_1(x, t; q), \phi_2(x, t; q)], \end{aligned} \quad (19)$$

where

$$\begin{aligned} \phi_1(x, t; 0) = u_0(x, t), \quad \phi_1(x, t; 1) = u(x, t), \\ \phi_2(x, t; 0) = v_0(x, t), \quad \phi_2(x, t; 1) = \theta(x, t). \end{aligned} \quad (20)$$

Then, the m th-order deformation equations become

$$\begin{aligned} \mathcal{L}_1[u_m(x, t) - \kappa_m u_{m-1}(x, t)] &= h_1 \mathcal{R}_{1m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}), \\ \mathcal{L}_2[\theta_m(x, t) - \kappa_m \theta_{m-1}(x, t)] &= h_2 \mathcal{R}_{2m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}), \end{aligned} \quad (21)$$

where

$$\begin{aligned}
& \mathcal{R}_{1m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}) \\
&= \frac{\partial^2 u_{m-1}}{\partial t^2} \\
&+ \frac{1}{1 + \sigma_1} \left(\Omega(u_{m-1})_t - (u_{m-1})_{xx} + \sigma_2(u_{m-1})_{xx} \right. \\
&\quad - 2\gamma \sum_{j=0}^{m-1} (u_j)_{xx} (u_{m-1-j})_x \\
&\quad - 3\delta \sum_{i=0}^{m-1} \sum_{j=0}^i (u_j)_x (u_{i-j})_x (u_{m-1-i})_{xx} \\
&\quad - \beta_2 \sum_{j=0}^{m-1} (\theta_j)_x (u_{m-1-j})_x - \beta_1 (\theta_{m-1})_x \\
&\quad \left. - \beta_2 \sum_{j=0}^{m-1} (\theta_j) (u_{m-1-j})_{xx} + i\omega\tau_2 \beta_1 \theta_{m-1} \right), \\
& \mathcal{R}_{2m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}) \\
&= \frac{\partial \theta_{m-1}}{\partial t} + \frac{1}{1 - i\omega\tau_1} \left(-a(u_{m-1})_{xt} \right. \\
&\quad - \frac{1}{2} b \sum_{j=0}^{m-1} (u_j)_{xt} (u_{m-1-j})_{xt} \\
&\quad - (\theta_{m-1})_{xx} \\
&\quad - \alpha \sum_{j=0}^{m-1} (\theta_{m-1-j})_x (u_j)_{xx} \\
&\quad - \alpha \sum_{j=0}^{m-1} (\theta_{m-1-j})_{xx} (u_j)_x \\
&\quad \left. - (\theta_{m-1})_{xx} + ai\omega\tau_1 \delta (u_{m-1})_{xt} \right). \tag{22}
\end{aligned}$$

For simplicity, we suppose $h_1 = h_2$; the system (21) has the following general solutions:

$$\begin{aligned}
u_m(x, t) &= \kappa_m u_{m-1}(x, t) + h \iint_0^t \mathcal{R}_{1m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}) dt dt, \\
\theta_m(x, t) &= \kappa_m \theta_{m-1}(x, t) + h \int_0^t \mathcal{R}_{2m}(\vec{u}_{m-1}, \vec{\theta}_{m-1}) dt. \tag{23}
\end{aligned}$$

In this case, where u_0 and θ_0 are constants, the general solution of (23) is taking the following form:

$$\begin{aligned}
u(x, t) &= u_0(x, t) + \sum_{m=1}^{\infty} u_m(x, t), \\
\theta(x, t) &= \theta_0(x, t) + \sum_{m=1}^{\infty} \theta_m(x, t). \tag{24}
\end{aligned}$$

The problems above can be readily solved by symbolic computation packages such as Mathematica. Thereupon, successive solving of these problems yields

$$\begin{aligned}
u_0(x, 0) &= A(1 - \cos(x)), \\
\theta_0(x, 0) &= A(1 - \cos(x)), \\
u_1(x, t) &= \frac{Aht^2}{4(1 + \sigma_1)} \left(2A\beta_2(-\cos(x) + \cos(2x)) \right. \\
&\quad + \cos(x)(-2 - 3A^2\delta \\
&\quad + 3A^2\delta \cos(2x) \\
&\quad - 4A\gamma \sin(x) + 2\sigma_2) \\
&\quad \left. + 2i \sin(x) \beta_1(i + \omega\tau_2) \right), \\
\theta_1(x, t) &= \frac{Aht \cos(x)(1 + 2A\alpha \sin(x))}{-1 + i\omega\tau_1}, \\
u_2(x, t) &= \frac{Aht^2}{4(1 + \sigma_1)} \\
&\quad \times (2A\beta_2(-\cos(x) + \cos(2x)) + \cos(x) \\
&\quad \times (-2 - 3A^2\delta + 3A^2\delta \cos(2x) \\
&\quad - 4A\gamma \sin(x) + 2\sigma_2) \\
&\quad + 2i \sin(x) \beta_1(i + \omega\tau_2)) \\
&\quad + \frac{1}{384(1 + \sigma_1)^2} \\
&\quad \times (Ah^2 t^4 (-135A^4 \delta^2 \cos(5x) \\
&\quad + 240A^3 \delta \gamma \sin(4x) - 16A^2 \\
&\quad \times (2 \cos(x) - 5 \cos(2x) + 3 \cos(3x)) \\
&\quad \times \beta_2^2 - 96A\gamma \sin(2x) \\
&\quad \times (1 + 2A^2\delta - \sigma_2) + 3A^2 \cos(3x) \\
&\quad \times (48\delta + 63A^2 \delta^2 + 32\gamma^2 - 48\delta\sigma_2)
\end{aligned}$$

$$\begin{aligned}
& + 8A\beta_2 \left(-2 \left(2 + 3A^2\delta \right) \cos(x) \right. \\
& \quad + 10 \cos(2x) \\
& \quad + A \left(3A\delta \left(3 \cos(2x) \right. \right. \\
& \quad \quad + 6 \cos(3x) \\
& \quad \quad \quad - 7 \cos(4x) \left. \right) \\
& \quad \quad - 2\gamma \left(\sin(x) \right. \\
& \quad \quad \quad + 6 \sin(2x) \\
& \quad \quad \quad \left. \left. - 9 \sin(3x) \right) \right) \\
& \quad + 2 \left(2 \cos(x) \right. \\
& \quad \quad \left. \left. - 5 \cos(2x) \right) \sigma_2 \right) \\
& - 2 \cos(x) \left(8 + A^2 \right. \\
& \quad \times \left(3\delta \left(8 + 9A^2\delta \right) \right. \\
& \quad \quad + 16\gamma^2 \left. \right) \\
& \quad \left. + 8\sigma_2 \left(-2 - 3A^2\delta + \sigma_2 \right) \right) \\
& + 4i\beta_1 \left(4 \sin(x) \right. \\
& \quad + A \left(-8\gamma \cos(2x) \right. \\
& \quad \quad + 3A\delta \left(\sin(x) \right. \\
& \quad \quad \quad - 3 \sin(3x) \left. \right) \\
& \quad \quad \left. \left. - 4 \sin(x) \left(A \left(-1 + 2 \cos(x) \right) \right. \right. \right. \\
& \quad \quad \quad \left. \left. \times \beta_2 + \sigma_2 \right) \right) \\
& \quad \quad \left. \left. \times \left(i + \omega\tau_0 \right) \right) \right) \\
& - \frac{1}{24 \left(1 + \sigma_1 \right)^2 \left(i + \omega\tau_1 \right)} \\
& \times \left(Ah^2t^3 \left(-2 - 3A^2\delta + 3A^2\delta \cos(2x) \right. \right. \\
& \quad \left. \left. - 4A\gamma \sin(x) + 2\sigma_2 \right) \right. \\
& \quad \times \left(i + \omega\tau_1 \right) + 2A\beta_2 \\
& \quad \times \left(iA\alpha \sin(x) \left(1 + \sigma_1 \right) - 3Aia \sin(3x) \right. \\
& \quad \quad \times \left(1 + \sigma_1 \right) + 2\Omega \cos(x) \left(i + \omega\tau_1 \right) \\
& \quad \quad \left. \left. - 2i \cos(2x) \left(1 + \Omega + \sigma_1 - i\Omega\omega\tau_1 \right) \right) \right. \\
& \quad + 4\beta_1 \left(i + \omega\tau_2 \right) \\
& \quad \times \left(-2A\alpha \cos(2x) \left(1 + \sigma_1 \right) \right. \\
& \quad \quad \left. \left. + \sin(x) \left(1 + \Omega + \sigma_1 - i\Omega\omega\tau_1 \right) \right) \right),
\end{aligned}$$

$$\begin{aligned}
\theta_2(x, t) = & \frac{Ah(1+h)t \cos(x) (1 + 2A\alpha \sin(x))}{-1 + i\omega\tau_1} \\
& + \frac{A^2h^2t^3\alpha}{6(1+\sigma_1)(i+\omega\tau_1)} \\
& \times \left(-\frac{1}{2}i \left(2A\gamma \cos(x) - 6A\gamma \cos(3x) \right. \right. \\
& \quad - 9A^2\delta \sin(4x) \\
& \quad + 2A \left(\sin(x) + \sin(2x) \right. \\
& \quad \quad \left. \left. - 3 \sin(3x) \right) \beta_2 \right. \\
& \quad \left. + 2 \sin(2x) \left(1 + 3A^2\delta - \sigma_2 \right) \right) \\
& + \cos(2x) \beta_1 \left(i + \omega\tau_2 \right) \\
& + \frac{1}{8(i+\omega\tau_1)} \\
& \times \left(iAh^2t^2 \right. \\
& \quad \times \left(\frac{a}{1+\sigma_1} \right. \\
& \quad \quad \times \left(-4 \sin(x) \right. \\
& \quad \quad \quad + A \left(8\gamma \cos(2x) \right. \\
& \quad \quad \quad \left. \left. - 3A\delta \left(\sin(x) \right. \right. \right. \\
& \quad \quad \quad \left. \left. \quad \left. - 3 \sin(3x) \right) \right) \right. \\
& \quad \quad \quad \left. + 4 \sin(x) \left(A \left(-1 + 4 \cos(x) \right) \right. \right. \\
& \quad \quad \quad \left. \left. \times \beta_2 + \sigma_2 \right) \right. \\
& \quad \quad \quad \left. + 4 \cos(x) \beta_1 \left(1 - i\omega\tau_2 \right) \right) \\
& + \frac{i\omega\delta_1}{1+\sigma_1} \left(4 \sin(3x) \right. \\
& \quad + A \left(-8\gamma \cos(x) \right. \\
& \quad \quad + 3A\delta \left(\sin(x) \right. \\
& \quad \quad \quad \left. \left. - 3 \sin(3x) \right) \right) \\
& \quad - 4 \sin(x) \left(A \left(-1 \right. \right. \\
& \quad \quad \quad \left. \left. + 4 \cos(x) \right) \right. \\
& \quad \quad \quad \left. \times \beta_2 + \sigma_2 \right) \\
& \quad \quad + 4i \cos(x) \beta_1 \left(i + \omega\tau_2 \right) \left. \right) \\
& \times \tau_1 - \frac{1}{i + \omega\tau_1} \\
& \times 4i \left(\left(1 + A^2 \right. \right. \\
& \quad \left. \left. + \alpha^2 \cos(x) + A\alpha \right. \right.
\end{aligned}$$

TABLE 1: The optimal values of h at third-order approximate solutions of (14) when $x = 1.0$, $t = 1.0$, for $\delta = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

τ_1	τ_2	Optimal value of h	Minimum value
$u(1, 1)$			
1	0	-0.95623	1.37312×10^{-3}
0	1	-0.97543	7.9675×10^{-7}
$\theta(1, 1)$			
1	0	-0.95623	1.3242×10^{-2}
0	1	-0.97543	2.1432×10^{-5}

TABLE 2: The optimal values of h at 5th-order approximate solutions of (14) when $x = 1.0$, $t = 1.0$, for $\delta = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

τ_1	τ_2	Optimal value of h	Minimum value
$u(1, 1)$			
1	0	-0.990321	1.37312×10^{-7}
0	1	-0.995339	7.9675×10^{-9}
$\theta(1, 1)$			
1	0	-0.990321	1.3242×10^{-4}
0	1	-0.995339	2.1432×10^{-6}

$$\times (-3A \times \alpha \cos(3x) + 5 \sin(2x))))). \quad (25)$$

Now we make calculations for the results obtained by the HAM using the Mathematica software package with the following arbitrary constants:

$$\begin{aligned} a = 0.5, \quad A = 0.001, \quad b = 0.5, \quad \alpha = 1, \\ \beta_1 = 0.5, \quad \beta_2 = 0.05, \quad \gamma = 1, \quad \delta = 0.8, \quad (26) \\ \Omega = 1, \quad \sigma_1 = 0.2, \quad \sigma_2 = 0.1. \end{aligned}$$

To investigate the influence of h on the convergence of the solution series given by the HAM, we first plot the so-called h -curves of $u(1, 1)$ and $\theta(1, 1)$. According to the h -curves, it is easy to discover the valid region of h . We used 3 terms in evaluating the approximate solution $u(x, t) \cong \sum_{i=0}^2 u_i(x, t)$ and $\theta(x, t) \cong \sum_{i=0}^2 \theta_i(x, t)$. Note that the solution series contains the auxiliary parameter h which provides us with a simple way to adjust and control the convergence of the solution series. In general, by means of the so-called h -curve that is, a curve of h versus h . As pointed by Liao [8] and Turkyilmazoglu [25], the valid region of h is a horizontal line segment. Therefore, it is straightforward to choose an appropriate range for h which ensures the convergence of the solution series (Tables 1 and 2). We sketch the h -curve of $u(1, 1)$ and $\theta(1, 1)$ in Figure 1, which shows that the solution series is convergent when $-1.45 < h < -0.5$.

4. Discussion

In order to gain physical insight, the temperature T and radial displacement u have been discussed by assigning numerical values to the parameter encountered in the problem in which the numerical results are displayed with the graphical illustrations in 2D and 3D formats. The variations are shown in Figures 1–15, with the view of illustrating the theoretical results obtained in the preceding sections; a numerical result is calculated for the homotopy analysis method.

Figures 1 and 2 display the h -curve of the third-order approximate solutions (14) when $x = 1.0$, $t = 1.0$; it

is concluded that the displacement $u(1, 1)$ and temperature $\theta(1, 1)$ increase with increasing the values of h to their maxima and then decrease with the high values of h ; also, it is shown that $u(1, 1)$ is convergent when $-0.9 < h < -0.6$ and $\theta(1, 1)$ is convergent when $-0.6 < h < -0.4$.

Figures 2–7 show the variations of the radial displacement and temperature with respect to axial x , respectively, for different values of the time t , rotation Ω , and sensitive parts of the magnetic fields σ_1 and σ_2 . In both figures, it is clear that the radial displacement and temperature have a zero value only in a bounded region of space. It is observed from Figure 3 that the displacement u and the temperature θ start from their maximum values, decrease, and increase periodically with an increase of the coordinate x ; also, it is obvious that their values take the minimum values and increases with the increasing values of the time t . From Figure 4, one can see that u and θ decrease with an increase of the rotation Ω . It is shown that the components of the displacement u and the temperature θ start from the minimum values near zero, increase, and then decrease periodically with the coordinate x ; it is clear also that there is a slight increase with an increase of the sensitive parts of the magnetic field (see, Figures 5 and 6). It is shown that the increasing of the coordinate x sensitive an increasing and decreasing on them periodically due to appearance of the pairs $(\cos, \sin)$ in the initial condition and the approximate solutions; it is also clear that the components begin from their minimum values and increase absolutely with the variation of the time t . The variations of the rotation and magnetic field tend to slightly affect the displacement and the temperature.

From Figures 7 and 8 (GL model), it is clear that the displacement component and temperature if the rotation and magnetic field are vanish, take larger values than the corresponding values with the rotation and magnetic field effects.

Figures 9, 10, and 11 show the variations of the displacement and temperature with respect to the time t with LS and GL models; it is shown that the radial displacement and the temperature increase with an increase of t that takes a slight change with the rotation Ω if $\sigma_1 = \sigma_2 = 0$, σ_1 if $\Omega = \sigma_2 = 0$, and σ_2 if $\Omega = \sigma_1 = 0$.

Figures 12 and 13 show clearly the variations of the displacement and temperature in the presence and absence of the rotation and sensitive magnetic field; it is observed that u and θ in presence of the parameters are smaller than the

FIGURE 1: The h -curve of the third-order approximate solutions of (14) when $x = 1.0$, $t = 1.0$; for LS model when $\tau_2 = 0$, $\tau_1 = 1$, $\delta = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

FIGURE 2: The h -curve of the third-order approximate solutions of (14) when $x = 1.0$, $t = 1.0$; for GL model when $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

FIGURE 3: Variations of the displacement u and the temperature θ for various values of the x -axis and time t when $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

FIGURE 4: Variations of the displacement u and the temperature θ for various values of the x -axis and rotation Ω when $t = 0.1$, $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, and $\omega = 0.02$.

FIGURE 5: Variations of the displacement u and the temperature θ for various values of the x -axis and magnetic field σ_1 when $t = 0.1$, $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, $\sigma_2 = 0.1$, $\Omega = 0.1$, and $\omega = 0.02$.

corresponding values in the absence of σ_1 , σ_2 , and Ω , but there is a slight change for LS and GL models.

Finally, Figures 14 and 15 show the variations of the displacement and the temperature with respect to the time t for different values of $\beta = 0.5, 0.005$ and with and without rotation and magnetic field effects, respectively. It is obvious that the radial displacement and the temperature increase with an increase of t ; the displacement increases with an increase of β parameter, but the temperature is not affected by β . It is also seen that the radial displacement and the temperature take large values with the rotation and magnetic field effects. Also,

it is concluded that takes large values for LS comparing with those in GL model, vice versa for the temperature.

It is obvious that if the rotation and the sensitive part of the magnetic field are neglected, the approximate solutions obtained by HAM agree with the results obtained by Sweilam and Khader [1], taking into consideration VIM. Finally, it is obvious that the displacement takes large values if there are no rotation, thermal relaxation times, and sensitive part of the magnetic field parameters compared with the corresponding value with the rotation and magnetic fields parameters.

FIGURE 6: Variations of the displacement u and temperature θ for various values of the x -axis and magnetic field σ_2 when $t = 0.1$, $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, and $\omega = 0.02$.

FIGURE 7: Variations of the displacement u and the temperature θ for various values of the x -axis and time t when $\Omega = \sigma_1 = \sigma_2 = 0$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 1$, and $\omega = 0.02$.

The results indicate that the effect of the rotation and the magnetic field on the radial displacement and the temperature is very pronounced.

5. Conclusion

Due to the complicated nature of the governing equations of the magnetothermoelastic, the finished works in this field are unfortunately limited. The method used in this study

provides a quite successful in dealing with such problems. This method gives numerical solutions in the elastic medium without any restrictions on the actual physical quantities that appear in the governing equations of the considered problem. Important phenomena are observed in these computations.

- (i) The homotopy analysis method has been successfully applied to obtain the numerical solutions of the non-linear equation with initial conditions. The reliability of this method and reduction in computations give

FIGURE 8: Variations of the displacement u and the temperature θ for various values of the x -axis and time t when $\Omega = \sigma_1 = \sigma_2 = 0$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, and $\omega = 0.02$.

FIGURE 9: Variations of the displacement u and the temperature θ for various values of the x -axis and rotation Ω when $\sigma_1 = \sigma_2 = 0$, $t = 0.1$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, and $\omega = 0.02$.

this method a wider applicability. HAM contains a certain auxiliary parameter h which provides us with a simple way to adjust and control the convergence region and rate of convergence of the series solution. It was also demonstrated that the Adomian decomposition method, homotopy perturbation method, and variational iteration method are specialcases of. HAM is clearly a very efficient and powerful technique for finding the numerical solutions of the proposed equation. It therefore provides more realistic series solutions that generally converge very rapidly in real physical problems. HAM provides us with a convenient way of controlling the convergence of approximation series, which is a fundamental qualitative difference in analysis between HAM and

other methods. The illustrative examples suggest that HAM is a powerful method for nonlinear problems in science and engineering. Mathematica has been used for computations in this paper.

- (ii) It was found that for large values of time the large and the generalization give numerical results. The case is quite different when we consider small values of rotation and magnetic field. The coupled theory predicts infinite speeds of wave propagation. The solutions obtained in the context of generalized thermoelasticity theory, however, exhibit the behavior of finite speeds of wave propagation.
- (iii) By comparing Figures 1–15 for thermoelastic medium with presence and absence of the rotation and

FIGURE 10: Variations of the displacement u and the temperature θ for various values of the x -axis and magnetic field σ_1 when $\Omega = \sigma_2 = 0$, $t = 0.1$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, and $\omega = 0.02$.

FIGURE 11: Variations of the displacement u and the temperature θ for various values of the x -axis and magnetic field σ_2 when $\Omega = \sigma_1 = 0$, $t = 0.1$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, and $\omega = 0.02$.

magnetic field, it was found that they have the same behavior in both media. The effect of rotation and sensitive parts of the magnetic field is strongly effective in the displacement and temperature of the propagation of the harmonic waves propagation in nonlinear thermoelasticity.

(iv) The results presented in this paper will be very helpful for researchers concerned with material science, designers of new materials, and low-temperature physicists, as well as for those working on the development of a theory of hyperbolic propagation of hyperbolic thermoelastic. The study of the phenomenon of

FIGURE 12: The displacement as a function of time and the temperature without and with rotation and magnetic field (LS) at $x = 100$, $a = 0.5$, $\beta_1 = 0.5$, $A = 0.001$, $b = 0.5$, $\alpha = 1$, $\beta_2 = 0.05$, $\gamma = 1$, $\delta = 0.8$, $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, $\omega = 0.02$ ($\Omega = \sigma_1 = \sigma_2 = 0$, —), and ($\Omega = 0.1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, ---).

FIGURE 13: The displacement as a function of time and the temperature without and with rotation and magnetic field (GL) at $x = 100$, $a = 0.5$, $\beta_1 = 0.5$, $A = 0.001$, $b = 0.5$, $\alpha = 1$, $\beta_2 = 0.05$, $\gamma = 1$, $\delta = 0.8$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, $\omega = 0.02$ ($\Omega = \sigma_1 = \sigma_2 = 0$, —), and ($\Omega = 0.1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, ---).

FIGURE 14: The displacement as a function of time and the temperature for two values of β_1 (LS) at $x = 100$, $a = 0.5$, $A = 0.001$, $b = 0.5$, $\alpha = 1$, $\beta_2 = 0.05$, $\gamma = 1$, $\delta = 0.8$, $\Omega = 0.1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\tau_2 = 0$, $\tau_1 = 0.1$, $\delta = 1$, $\omega = 0.02$, — $\beta_1 = 0.5$, and --- $\beta_1 = 0.005$.

FIGURE 15: The displacement as a function of time and the temperature for two values of β_1 (GL) at $x = 100$, $a = 0.5$, $A = 0.001$, $b = 0.5$, $\alpha = 1$, $\beta_2 = 0.05$, $\gamma = 1$, $\delta = 0.8$, $\Omega = 0.1$, $\sigma_1 = 0.2$, $\sigma_2 = 0.1$, $\tau_2 = 0.2$, $\tau_1 = 0.1$, $\delta = 0$, $\omega = 0.02$, — $\beta_1 = 0.5$, and - - - $\beta_1 = 0.005$.

rotation and magnetic field is also used to improve the conditions of oil extractions.

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