Research Article

Computing Hypercrossed Complex Pairings in Digital Images

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We consider an additive group structure in digital images and introduce the commutator in digital images. Then we calculate the hypercrossed complex pairings which generates a normal subgroup in dimension 2 and in dimension 3 by using 8-adjacency and 26-adjacency.

1. Introduction

In this paper we denote the set of integers by \( \mathbb{Z} \). Then \( \mathbb{Z}^n \) represents the set of lattice points in Euclidean \( n \)-dimensional spaces. A finite subset of \( \mathbb{Z}^n \) with an adjacency relation is called a digital image.

Definition 1 (see [1, 2]). Consider the following.

1. Two points \( p \) and \( q \) in \( \mathbb{Z} \) are 2-adjacent if \( |p - q| = 1 \).
2. Two points \( p \) and \( q \) in \( \mathbb{Z}^2 \) are 8-adjacent if they are distinct and differ by at most 1 in each coordinate.
3. Two points \( p \) and \( q \) in \( \mathbb{Z}^2 \) are 4-adjacent if they are 8-adjacent and differ by exactly one coordinate.
4. Two points \( p \) and \( q \) in \( \mathbb{Z}^2 \) are 26-adjacent if they are distinct and differ by at most 1 in each coordinate.
5. Two points \( p \) and \( q \) in \( \mathbb{Z}^2 \) are 18-adjacent if they are 26-adjacent and differ in at most two coordinates.
6. Two points \( p \) and \( q \) in \( \mathbb{Z}^2 \) are 6-adjacent if they are 18-adjacent and differ by exactly one coordinate.

Definition 2. Let \( G \) be a subset of a digital image. A simplicial group \( G \) in digital images consists of a sequence of groups \( G \) and collections of group homomorphisms \( d_i : G_n \to G_{n-1} \) and \( s_i : G_n \to G_{n+1} \), \( 0 \leq i \leq n \), that satisfies the following axioms:

\[
\begin{align*}
    d_i d_j & = d_{j-1} d_i, \\
    d_i s_j & = s_{j-1} d_i, \\
    d_i s_j & = d_{j+1} s_i, \\
    s_i s_j & = s_{j+1} s_i, \\
    s_i s_j & = s_{j+1} s_i, \\n\end{align*}
\]

\( i < j \), \( i < j \), \( i > j + 1 \), \( i \leq j \).

Definition 3. Given a simplicial group \( G \) with \( \kappa \)-adjacency, the Moore complex \((NG, \partial)\) of \( G \) is the chain complex defined by

\[
\text{NG}_n = \bigcap_{i=0}^{n-1} \text{Ker} \, d_i, \quad \partial \circ d = d \circ \partial.
\]

The \( n \)th homology group of the Moore complex of \( G \) is

\[
H_n(NG, \partial) = \frac{\bigcap_{i=0}^{n} \text{Ker} \, d_i}{d_{n+1} \left( \bigcap_{i=0}^{n} \text{Ker} \, d_i \right)}. \quad (3)
\]

2. Hypercrossed Complex Pairings in Digital Images

First of all we adapt ideas from Carrasco and Cegarra [3–5] to get the construction in digital images. We define a set \( P(n) \)
consisting of pairs of elements \((\alpha, \beta)\) from \(S(n)\) with \(\alpha \cap \beta = \emptyset\) and \(\beta < \alpha\), with respect to lexicographic ordering in \(S(n)\) where \(\alpha = (i_1, \ldots, i_k)\) and \(\beta = (j_1, \ldots, j_l) \in S(n)\).

Consider the following diagram:

\[
\begin{array}{ccc}
NG_{n-\alpha} \times NG_{n-\beta} & \xrightarrow{F_{\alpha,\beta}} & NG_n \\
\downarrow s_{\alpha} \times s_{\beta} & & \downarrow \mu \\
G_n \times G_n & \xrightarrow{p} & G_n \\
\end{array}
\]

(4)

where

\[
s_{\alpha} = s_{i_1} \cdots s_{i_k} : NG_{n-\alpha} \rightarrow G_n, \\
s_{\beta} = s_{j_1} \cdots s_{j_l} : NG_{n-\beta} \rightarrow G_n,
\]

and define \(p : G_n \rightarrow NG_n\) and \(p(x) = p_{n-1} \cdots p_0(x)\) as \(p_j(z) = z - s_{d_j} z\) and \(j = 0, \ldots, n - 1\). Since a digital image has the additive group structure, define the commutator as

\[
[x, y] = xy - yx.
\]

Thus

\[
\mu : G_n \times G_n \rightarrow G_n, \\
F_{\alpha,\beta}(x_{\alpha}, y_{\beta}) = p\mu(s_{\alpha} \times s_{\beta})(x_{\alpha}, y_{\beta}) = p[s_{\alpha} x_{\alpha}, s_{\beta} y_{\beta}].
\]

The normal subgroup \(NG_n\) of \(G_n\) is generated by the elements of the form

\[
F_{\alpha,\beta}(x_{\alpha}, y_{\beta}),
\]

where \(x_{\alpha} \in NG_{n-\alpha}\) and \(y_{\beta} \in NG_{n-\beta}\).

**Theorem 4.** 2-dimensional normal subgroup \(N_2\) with 8-adjacency is generated by the elements of the form

\[
[s_0 x_1 - s_1 x_1, s_1 y_1].
\]

**Proof.** Let \(\alpha = (1)\) and \(\beta = (0)\) for \(n = 2\). For \(x_1\) and \(y_1 \in NG_1 = \text{Ker} d_0\),

\[
F_{(0,1)}(x_1, y_1) = p_1 p_0 [s_0 x_1, s_1 y_1] = p_1 [s_0 x_1, s_1 y_1] - s_0 d_0 [s_0 x_1, s_1 y_1] = [s_0 x_1 - s_1 x_1, s_1 y_1].
\]

(10)

Thus \(F_{(0,1)}(x_1, y_1) = [s_0 x_1 - s_1 x_1, s_1 y_1]\) and this is the element generating \(N_2\) normal subgroups.

**Proposition 5.** 3-dimensional normal subgroup \(N_3\) with 26-adjacency is generated by the elements of the following forms:

(i) \([s_1 s_0 x_1 - s_0 s_1 x_1, s_2 y_2]\),

(ii) \([s_2 s_0 x_1 - s_0 s_1 x_1, s_1 y_2 - s_2 y_2]\),

(iii) \([s_0 x_2 - s_1 x_2 + s_2 x_2, s_1 s_1 y_1]\),

(iv) \([s_1 x_2 - s_2 x_2, s_2 y_2]\),

(v) \([s_0 x_2, s_2 y_2]\),

(vi) \([s_0 x_2 - s_1 x_2, s_1 y_2] + [s_2 x_2, s_2 y_2]\).

**Proof.** For \(n = 3\) the possible pairings are the following:

(i) \(F_{(1,0)(2)}(x_1, y_2) = p [s_1 s_0 x_1, s_2 y_2]\)

\[
= p_2 p_1 p_0 [s_1 s_0 x_1, s_2 y_2] = [s_1 s_0 x_1 - s_2 s_1 x_1, s_2 y_2],
\]

(11)

(ii) \(F_{(2,0)(1)}(x_1, y_2) = p [s_2 s_0 x_1, s_1 y_2]\)

\[
= p_2 p_1 p_0 [s_2 s_0 x_1, s_1 y_2] = [s_2 s_0 x_1 - s_2 s_1 x_1, s_1 y_2],
\]

(12)

For all \(x_1 \in NG_1\) and \(y_2 \in NG_2\) the corresponding generators of \(N_3\) are the following with \(F_{\alpha,\beta} : NG_1 \times NG_2 \rightarrow NG_3\) and for \(n = 3\), \(p(x) = p_2 p_1 p_0(x)\):

(iii) \(F_{(0)(2,1)}(x_2, y_1) = p [s_0 x_2, s_2 s_1 y_1]\)

\[
= p_2 p_1 p_0 [s_0 x_2, s_2 s_1 y_1] = [s_0 x_2 - s_2 x_2 + s_2 x_2, s_2 s_1 y_1],
\]

(13)

For all \(x_2 \in NG_2\) and \(y_1 \in NG_1\), and considering the map \(F_{\alpha,\beta} : NG_2 \times NG_1 \rightarrow NG_3\), the corresponding generator of \(N_3\) is

(iv) \(F_{(1)(2)}(x_2, y_2) = p [s_1 x_2, s_2 y_2]\)

\[
= p_2 p_1 p_0 [s_1 x_2, s_2 y_2] = [s_1 x_2 - s_2 x_2, s_2 y_2],
\]

(14)

(v) \(F_{(0)(2)}(x_2, y_2) = p [s_0 x_2, s_2 y_2]\)

\[
= p_2 p_1 p_0 [s_0 x_2, s_2 y_2] = [s_0 x_2, s_2 y_2],
\]

(15)
In $\text{Ker} \ d_0$, we get

$$ F_{(0,1)}(x_1, y_1) \in \text{Ker} \ d_1, \text{Ker} \ d_0 \] \tag{20} $$

\[ \square \]

**Theorem 7.** Let $NG_3$ be a 3-dimensional Moore complex of a simplicial group $G$ with 26-adjacency. Then

$$ \partial_3 \ (NG_3) \subseteq [\text{Ker} \ d_2, \text{Ker} \ d_0 \cap \text{Ker} \ d_1] $$

$$ + \ [\text{Ker} \ d_1, \text{Ker} \ d_0 \cap \text{Ker} \ d_2] $$

$$ + \ [\text{Ker} \ d_1 \cap \text{Ker} \ d_2, \text{Ker} \ d_0] $$

$$ + \ [\text{Ker} \ d_1, \text{Ker} \ d_0] $$

$$ + \ [\text{Ker} \ d_0 \cap \text{Ker} \ d_2, \text{Ker} \ d_0 \cap \text{Ker} \ d_1] $$

$$ + \ [\text{Ker} \ d_2, \text{Ker} \ d_0 \cap \text{Ker} \ d_1], $$

where $\partial_3$ is induced from $d_3$ by restriction.

**Proof.** For $n = 3$ investigate $d_3 F_{a,b}$ where $x_1 \in NG_1$ and $y_2 \in NG_2 = \text{Ker} \ d_0 \cap \text{Ker} \ d_1$.

From Proposition 5 we have $F_{(1,0),(2)}(x_1, y_2) = [s_1 s_0 x_1 - s_2 s_0 x_1, s_2 y_2]$. Then applying $d_3$ to $F_{(1,0),(2)}(x_1, y_2)$, we get the following:

$$ d_3 F_{(1,0),(2)}(x_1, y_2) = \begin{bmatrix} d_3 s_1 s_0 x_1 - d_3 s_2 s_0 x_1, & d_3 s_2 y_2 \end{bmatrix} $$

$$ = \begin{bmatrix} s_1 d_3 s_0 x_1 - s_0 x_1, & y_2 \end{bmatrix} $$

$$ = \begin{bmatrix} s_1 d_3 x_1 - s_0 x_1, & y_2 \end{bmatrix}. $$

Firstly, examine whether $s_1 s_0 d_1 x_1 - s_0 x_1$ is in $\text{Ker} \ d_0$ or not:

$$ d_0 (s_1 s_0 d_1 x_1 - s_0 x_1) = d_0 s_1 s_0 d_1 x_1 - d_0 s_0 x_1 $$

$$ = s_0 d_0 s_0 d_1 x_1 - d_0 s_0 x_1 $$

$$ = s_0 d_1 x_1 - x_1. $$

So $s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_0$.

Secondly we investigate whether $s_1 s_0 d_1 x_1 - s_0 x_1$ is in $\text{Ker} \ d_1$ or not:

$\begin{align*}
\text{If } s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_0, \\
\text{Then } s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_1.
\end{align*}$

Therefore $s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_1$.

Finally we check whether $s_1 s_0 d_1 x_1 - s_0 x_1$ is in $\text{Ker} \ d_2$ or not.

$\begin{align*}
\text{If } s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_0, \\
\text{Then } s_1 s_0 d_1 x_1 - s_0 x_1 \notin \text{Ker} \ d_2.
\end{align*}$

We get

$$ F_{(0,1),(2)}(x_1, y_2) \in [\text{Ker} \ d_2, \text{Ker} \ d_0 \cap \text{Ker} \ d_1], \tag{26} $$

since $y_2 \in \text{Ker} \ d_0 \cap \text{Ker} \ d_1$.

If

$$ F_{(2,0),(1)}(x_1, y_2) = [s_2 s_0 x_1 - s_2 s_1 x_1, s_1 y_2 - s_2 y_2], \tag{27} $$

then

$$ d_3 F_{(2,0),(1)}(x_1, y_2) = d_3 \begin{bmatrix} s_2 s_0 x_1 - s_2 s_1 x_1, & s_1 y_2 - s_2 y_2 \end{bmatrix} $$

$$ = \begin{bmatrix} d_3 s_2 s_0 x_1 - d_3 s_2 s_1 x_1, & d_3 s_1 y_2 - d_3 s_2 y_2 \end{bmatrix} $$

$$ = \begin{bmatrix} s_0 x_1 - s_1 x_1, & s_1 d_2 y_2 - y_2 \end{bmatrix}. $$

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**Abstract and Applied Analysis**

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At first we check whether \( s_0 x_1 - s_1 x_1 \) is in \( \text{Ker} \, d_0, \text{Ker} \, d_1, \) and \( \text{Ker} \, d_2 \) or not.

\[
d_0 (s_0 x_1 - s_1 x_1) = \frac{d_0 s_0 x_1 - d_0 s_1 x_1}{id} = x_1 - d_0 s_1 x_1. \tag{29}
\]

Thus \( s_0 x_1 - s_1 x_1 \notin \text{Ker} \, d_0. \)

Next, since

\[
d_1 (s_0 x_1 - s_1 x_1) = \frac{d_1 s_0 x_1 - d_1 s_1 x_1}{id} = x_1 - x_1 = 0, \tag{30}
\]

Thus \( s_0 x_1 - s_1 x_1 \in \text{Ker} \, d_1, \)

and, finally,

\[
d_2 (s_0 x_1 - s_1 x_1) = \frac{d_2 s_0 x_1 - d_2 s_1 x_1}{id} = s_0 d_1 x_1 - x_1 \notin \text{Ker} \, d_2. \tag{31}
\]

Now examine whether \( s_1 d_2 y_2 - y_2 \) is in \( \text{Ker} \, d_0, \text{Ker} \, d_1, \) and \( \text{Ker} \, d_2 \) or not:

\[
d_0 (s_1 d_2 y_2 - y_2) = \frac{d_0 s_1 d_2 y_2 - d_0 y_2}{id} = s_0 d_1 d_2 y_2 - d_0 y_2 = 0. \tag{32}
\]

Therefore \( s_1 d_2 y_2 - y_2 \in \text{Ker} \, d_0. \) We have the following:

\[
d_1 (s_1 d_2 y_2 - y_2) = \frac{d_1 s_1 d_2 y_2 - d_1 y_2}{id} = d_2 y_2 \notin \text{Ker} \, d_1; \tag{33}
\]

\[
d_2 (s_1 d_2 y_2 - y_2) = \frac{d_2 s_1 d_2 y_2 - d_2 y_2}{id} = d_2 y_2 - d_2 y_2 = 0
\]

\[\implies s_1 d_2 y_2 - y_2 \in \text{Ker} \, d_2. \]

So \( F_{(2,0)(1)}(x_1, y_2) = [\text{Ker} \, d_1, \text{Ker} \, d_0 \cap \text{Ker} \, d_2]. \)

For all \( x_2 \in \text{NG}_2 \) and \( y_1 \in \text{NG}_1 \) if

\[
F_{(0)(2,1)}(x_2, y_1) = [s_0 x_2 - s_1 x_2 + s_2 x_2, s_1 y_1], \tag{34}
\]

then

\[
d_3 F_{(0)(2,1)}(x_2, y_1)
= d_3 \left( [s_0 x_2 - s_1 x_2 + s_2 x_2, s_1 y_1] \right)
= \left[ d_3 s_0 x_2 - d_3 s_1 x_2 + d_3 s_2 x_2, \frac{d_3 s_2 y_1}{id} \right] \tag{35}
= [s_0 d_2 x_2 - s_1 d_2 x_2 + s_2 y_1].
\]

Firstly investigate whether \( s_0 d_2 x_2 - s_1 d_2 x_2 + x_2 \) is in \( \text{Ker} \, d_0, \text{Ker} \, d_1, \) and \( \text{Ker} \, d_2 \) or not:

\[
d_0 (s_0 d_2 x_2 - s_1 d_2 x_2 + x_2) = \frac{d_0 s_0 d_2 x_2 - d_0 s_1 d_2 x_2 + d_0 x_2}{id} = 0. \tag{36}
\]

Thereby \( s_0 d_2 x_2 - s_1 d_2 x_2 + x_2 \notin \text{Ker} \, d_0. \) We have

\[
d_1 (s_0 d_2 x_2 - s_1 d_2 x_2 + x_2)
= \frac{d_1 s_0 d_2 x_2 - d_1 s_1 d_2 x_2 + d_1 x_2}{id} = 0
= d_2 x_2 - d_2 x_2 = 0. \tag{37}
\]

For this reason \( s_0 d_2 x_2 - s_1 d_2 x_2 + x_2 \in \text{Ker} \, d_1. \) We also have

\[
d_2 (s_0 d_2 x_2 - s_1 d_2 x_2 + x_2)
= \frac{d_2 s_0 d_2 x_2 - d_2 s_1 d_2 x_2 + d_2 x_2}{id} = 0
= s_0 d_1 d_2 x_2 - d_2 x_2 + d_2 x_2
= s_0 d_1 d_2 x_2 = 0. \tag{38}
\]

Hence \( s_0 d_2 x_2 - s_1 d_2 x_2 + x_2 \in \text{Ker} \, d_2. \)

Later on we research whether \( s_1 s_1 y_1 \) is in \( \text{Ker} \, d_0, \text{Ker} \, d_1, \) and \( \text{Ker} \, d_2 \) or not.

Since \( d_0 (s_1 y_1) = s_0 d_0 y_1 = 0, s_1 y_1 \in \text{Ker} \, d_0. \)

Since \( d_1 (s_1 y_1) = \frac{d_1 s_1 y_1}{id} = y_1, s_1 y_1 \notin \text{Ker} \, d_1. \)

Since \( d_2 (s_1 y_1) = \frac{d_2 s_1 y_1}{id} = y_1, s_1 y_1 \notin \text{Ker} \, d_2. \)

Thus \( d_3 F_{(3,0)(1)}(x_2, y_1) \in [\text{Ker} \, d_1 \cap \text{Ker} \, d_2, \text{Ker} \, d_0]. \)

For all \( x_2, y_2 \in \text{NG}_2 \) since \( F_{(0)(2,1)}(x_2, y_2) = [s_0 x_2, s_2 y_2], \)

\[
d_3 F_{(0)(2,1)}(x_2, y_2) = d_3 \left( [s_0 x_2, s_2 y_2] \right)
= \left[ \frac{d_3 s_0 x_2, d_3 s_2 y_2}{id} \right] \tag{39}
= [s_0 d_2 x_2, y_2].
\]

By using properties of the commutator we have

\[
[s_0 d_2 x_2 - s_1 d_2 x_2 + x_2, y_2]
= [s_0 d_2 x_2 + (x_2 - s_1 d_2 x_2), y_2]
= [s_0 d_2 x_2, y_2] + [x_2 - s_1 d_2 x_2, y_2], \tag{40}
\]

\[
= [s_0 d_2 x_2, y_2] + [y_2, x_2 - s_1 d_2 x_2]
= [s_0 d_2 x_2, y_2]
= d_3 F_{(0)(2,1)}(x_2, y_2).
\]

Thus

\[
d_3 F_{(0)(2,1)}(x_2, y_2)
= \frac{d_3 s_0 x_2, d_3 s_2 y_2}{id}
\in [\text{Ker} \, d_1 \cap \text{Ker} \, d_2, \text{Ker} \, d_1 \cap \text{Ker} \, d_0] \tag{41}
+ [\text{Ker} \, d_0 \cap \text{Ker} \, d_1, \text{Ker} \, d_0 \cap \text{Ker} \, d_2].
\]
If \( F_{(1)(2)}(x_2, y_2) = [s_1x_2 - s_2x_2, s_2y_2] \), then
\[
d_3 F_{(1)(2)} (x_2, y_2) = d_3 \left( \left[ s_1x_2 - s_2x_2, s_2y_2 \right] \right)
= \left[ d_3 s_1x_2 - \frac{d_3 s_2x_2 d_3 s_2y_2}{i f}, \frac{d_3 s_2x_2 d_3 s_2y_2}{i f} \right]
= [s_1 d_2x_2 - x_2, y_2].
\]
Firstly we check whether \( s_1 d_2x_2 - x_2 \) is in \( \text{Ker} \, d_0, \text{Ker} \, d_1, \) and \( \text{Ker} \, d_2 \) or not:
\[
d_0 \left( s_1 d_2x_2 - x_2 \right) = d_0 s_1 d_2x_2 - \frac{d_0 x_2}{i f} = 0
\]
\[
= s_0 d_0 d_2x_2
= s_0 d_1 d_0 x_2
= 0.
\]
Therefore \( s_1 d_2x_2 - x_2 \in \text{Ker} \, d_0. \) Since
\[
d_1 \left( s_1 d_2x_2 - x_2 \right) = d_1 s_1 d_2x_2 - \frac{d_1 x_2}{i f} = 0
\]
\[
= d_2x_2,
\]
\( s_1 d_2x_2 - x_2 \notin \text{Ker} \, d_1. \) We have
\[
d_2 \left( s_1 d_2x_2 - x_2 \right) = d_2 s_1 d_2x_2 - \frac{d_2 x_2}{i f} = 0
\]
\[
= d_2x_2 - d_2x_2
= 0.
\]
Hence \( s_1 d_2x_2 - x_2 \in \text{Ker} \, d_2. \)
Because of the case \( y_2 \in \text{Ker} \, d_0 \cap \text{Ker} \, d_1, \)
\[
d_3 F_{(1)(2)} (x_2, y_2) \in [\text{Ker} \, d_0 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1].
\]
If \( F_{(0)(1)}(x_2, y_2) = [s_0x_2 - s_1x_2, s_1y_2] + [s_2x_2, s_2y_2], \) then
\[
d_3 F_{(0)(1)} (x_2, y_2)
= [d_3 s_0x_2 - d_3 s_1x_2, s_1y_2] + \left[ d_3 s_2x_2, \frac{d_3 s_2x_2 d_3 s_2y_2}{i f} \right]
= [s_0 d_2x_2 - s_1 d_2x_2, s_1 d_2y_2] + [x_2, y_2].
\]
Consider the following commutator:
\[
[s_0 d_2x_2 - s_1 d_2x_2 + x_2, s_1 d_2y_2 - y_2],
\]
and code the terms of this commutator such as
\[
a = s_0 d_2x_2, \quad b = s_1 d_2y_2,
\]
\[
c = s_1 d_2x_2, \quad d = x_2, \quad e = y_2.
\]
in order to simplify the algebraic operations. Thus, by using the properties and definition of the commutator we obtain the following:
\[
[a - c + d, b - e] = [a - c, b] + [d, e],
\]
\[
(a - c + d)(b - e) - (b - e)(a - c + d)
= ab - cb + db - ae + ce - de
- \{ba - bc + bd - ea + ec - ed\}.
\]
Consider the following cases:
\[
ab - cb - ba + bc = (a - c)b - b(a - c)
= [a - c, b],
\]
\[
(ce - de - ec + ed) = (c - d)e - e(c - d)
= [c - d, e].
\]
And from the remaining terms we get
\[
db - bd - [d, e] = db - bd - de + ed
= d(b - e) - (b - e) d
= [d, b - e].
\]
Consequently for \( n = 3 \) we have
\[
\partial_3 (NG_3) \subseteq \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_1, \text{Ker} \, d_0 \cap \text{Ker} \, d_2 \right]
+ \left[ \text{Ker} \, d_1 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \right]
+ \left[ \text{Ker} \, d_0 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right].
\]
\[
\partial_3 (NG_3) \subseteq \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_1, \text{Ker} \, d_0 \cap \text{Ker} \, d_2 \right]
+ \left[ \text{Ker} \, d_1 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \right]
+ \left[ \text{Ker} \, d_0 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right].
\]
\[
\partial_3 (NG_3) \subseteq \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_1, \text{Ker} \, d_0 \cap \text{Ker} \, d_2 \right]
+ \left[ \text{Ker} \, d_1 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \right]
+ \left[ \text{Ker} \, d_0 \cap \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right]
+ \left[ \text{Ker} \, d_2, \text{Ker} \, d_0 \cap \text{Ker} \, d_1 \right].
\]
Proof. Otherwise inclusion for the previous theorem is obtained from \([4, 5]\). Therefore

\[
\partial_3 (NG_3) = [\text{Ker } d_2, \text{Ker } d_0 \cap \text{Ker } d_1]
\]

\[
+ [\text{Ker } d_1, \text{Ker } d_0 \cap \text{Ker } d_2]
\]

\[
+ [\text{Ker } d_1 \cap \text{Ker } d_2, \text{Ker } d_0]
\]

\[
+ [\text{Ker } d_1, \text{Ker } d_0]
\]

\[
+ [\text{Ker } d_0 \cap \text{Ker } d_2, \text{Ker } d_0 \cap \text{Ker } d_1]
\]

\[
+ [\text{Ker } d_2, \text{Ker } d_0 \cap \text{Ker } d_1].
\]

(55)

3. Conclusion

In this paper for dimension 2 and dimension 3, we obtained the Moore complex of simplicial groups generated by hypercrossed complex pairings in digital images.

References


