Research Article

Stability of n -Jordan Homomorphisms from a Normed Algebra to a Banach Algebra

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We establish the hyperstability of n -Jordan homomorphisms from a normed algebra to a Banach algebra, and also we show that an n -Jordan homomorphism between two commutative Banach algebras is an n -ring homomorphism.

1. Introduction

Let A, B be two rings (algebras) and n a positive integer greater than 1. An additive mapping $g : A \rightarrow B$ is called an n -Jordan homomorphism if $g(a^n) = (g(a))^n$ for all $a \in A$ and an additive mapping $h : A \rightarrow B$ is called an n -ring homomorphism if $h(\prod_{i=1}^n a_i) = \prod_{i=1}^n h(a_i)$ for all $a_1, a_2, \dots, a_n \in A$.

In 2009, Gordji et al. [1] showed the following theorems.

Theorem 1. Let $n \in \{2, 3, 4, 5\}$ be fixed. Suppose that A, B are two commutative algebras. Let $h : A \rightarrow B$ be an n -Jordan homomorphism. Then h is an n -ring homomorphism.

Theorem 2. Let $n \in \{2, 3, 4, 5\}$ be fixed. Suppose that A, B are commutative Banach algebras. Let δ and ε be nonnegative real numbers, and let p, q be real numbers such that $(p-1)(q-1) > 0$, $q \geq 0$ or $(p-1)(q-1) > 0$, $q < 0$, and $f(0) = 0$. Assume that $f : A \rightarrow B$ satisfies the system of functional inequalities:

$$\begin{aligned} \|f(a+b) - f(a) - f(b)\| &\leq \varepsilon (\|a\|^p + \|b\|^p), \\ \|f(a^n) - f(a)^n\| &\leq \delta \|a\|^{nq}, \end{aligned} \quad (1)$$

for all $a, b \in A$. Then, there exists a unique n -ring homomorphism $h : A \rightarrow B$ such that

$$\|f(a) - h(a)\| \leq \frac{2\varepsilon}{|2 - 2^p|} \|a\|^p, \quad (2)$$

for all $a \in A$.

The stability problem of group homomorphisms was formulated by Ulam [2] in 1940. Bourgin [3] and Badora [4] solved the stability problem of ring homomorphisms (see [5]). The term hyperstability was used for the first time in [6]. Some recent results on hyperstability of Cauchy or linear equation can be founded in [5, 7, 8].

In this paper, we improve Theorems 1 and 2 into Theorems 4 and 8, respectively. In particular, we prove the hyperstability of n -Jordan homomorphisms between two commutative Banach algebras.

2. Generalization of Theorem 1

Lemma 3. Let n, k be fixed natural numbers with $n > k \geq 2$. Let A, B be two commutative algebras, and let $f : A \rightarrow B$ be an additive mapping. Assume that f satisfies the following equality:

$$\begin{aligned} &\sum_{i_1=k-1}^{n-1} \sum_{i_2=k-2}^{i_1-1} \cdots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{k-2}}{i_{k-1}} \\ &\quad \times f(x_1^{n-i_1} x_2^{i_1-i_2} x_3^{i_2-i_3} \cdots x_k^{i_{k-1}}) \\ &= \sum_{i_1=k-1}^{n-1} \sum_{i_2=k-2}^{i_1-1} \cdots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{k-2}}{i_{k-1}} f(x_1)^{n-i_1} \\ &\quad \times f(x_2)^{i_1-i_2} \cdots f(x_k)^{i_{k-1}}, \end{aligned} \quad (3)$$

for all $x_1, x_2, x_3, \dots, x_k \in A$. Then one gets

$$\begin{aligned} & \sum_{i_1=k}^{n-1} \sum_{i_2=k-1}^{i_1-1} \dots \sum_{i_k=1}^{i_{k-1}-1} \binom{n}{i_1} \binom{i_1}{i_2} \dots \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1^{n-i_1} x_2^{i_1-i_2} \dots x_{k+1}^{i_k}) \\ &= \sum_{i_1=k}^{n-1} \sum_{i_2=k-1}^{i_1-1} \dots \sum_{i_k=1}^{i_{k-1}-1} \binom{n}{i_1} \binom{i_1}{i_2} \dots \binom{i_{k-1}}{i_k} f(x_1)^{n-i_1} \\ & \quad \times f(x_2)^{i_1-i_2} \dots f(x_{k+1})^{i_k}, \end{aligned} \quad (4)$$

for all $x_1, x_2, x_3, \dots, x_{k+1} \in A$.

Proof. Replacing x_k by x_{k+1} in (3), we obtain

$$\begin{aligned} & \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} x_{k+1}^{i_{k-1}}) \\ &= \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1)^{n-i_1} \dots f(x_{k-1})^{i_{k-2}-i_{k-1}} \\ & \quad \times f(x_{k+1})^{i_{k-1}}, \end{aligned} \quad (5)$$

for all $x_1, x_2, x_3, \dots, x_{k-1}, x_{k+1} \in A$. In particular, the equality (3) implies that

$$\begin{aligned} & \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{0} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} x_k^{i_{k-1}}) \\ &= \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{0} \\ & \quad \times f(x_1)^{n-i_1} \dots f(x_{k-1})^{i_{k-2}-i_{k-1}} f(x_k)^{i_{k-1}}, \end{aligned} \quad (6)$$

for all $x_1, x_2, x_3, \dots, x_k \in A$. Recall that the equality,

$$(x_k + x_{k+1})^{i_{k-1}} = \sum_{i_k=0}^{i_{k-1}} \binom{i_{k-1}}{i_k} x_k^{i_{k-1}-i_k} x_{k+1}^{i_k}, \quad (7)$$

holds for all $x_k, x_{k+1} \in A$. Replacing x_k by $x_k + x_{k+1}$ in (3), we obtain

$$\begin{aligned} & \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \sum_{i_k=0}^{i_{k-1}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} x_k^{i_{k-1}-i_k} x_{k+1}^{i_k}) \\ &= \sum_{i_1=k-1}^{n-1} \sum_{i_2=k-2}^{i_1-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} \\ & \quad \times (x_k + x_{k+1})^{i_{k-1}}) \\ &= \sum_{i_1=k-1}^{n-1} \sum_{i_2=k-2}^{i_1-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \dots \binom{i_{k-2}}{i_{k-1}} \\ & \quad \times f(x_1)^{n-i_1} \dots f(x_k + x_{k+1})^{i_{k-1}} \\ &= \sum_{i_1=k-1}^{n-1} \sum_{i_2=k-2}^{i_1-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \dots \binom{i_{k-2}}{i_{k-1}} \\ & \quad \times f(x_1)^{n-i_1} \dots (f(x_k) + f(x_{k+1}))^{i_{k-1}} \\ &= \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \sum_{i_k=0}^{i_{k-1}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1)^{n-i_1} \dots f(x_k)^{i_{k-1}-i_k} f(x_{k+1})^{i_k}, \end{aligned} \quad (8)$$

for all $x_1, x_2, x_3, \dots, x_k, x_{k+1} \in A$. From (5), (6), and the above equality, we get the desired equality:

$$\begin{aligned} & \sum_{i_1=k}^{n-1} \sum_{i_2=k-1}^{i_1-1} \dots \sum_{i_k=1}^{i_{k-1}-1} \binom{n}{i_1} \binom{i_1}{i_2} \dots \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1^{n-i_1} x_2^{i_1-i_2} \dots x_{k+1}^{i_k}) \\ &= \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \sum_{i_k=0}^{i_{k-1}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} \\ & \quad \times x_k^{i_{k-1}-i_k} x_{k+1}^{i_k}) \\ & \quad - \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_{k-1}} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} x_{k+1}^{i_{k-1}}) \\ & \quad - \sum_{i_1=k-1}^{n-1} \dots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \dots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{0} \\ & \quad \times f(x_1^{n-i_1} \dots x_{k-1}^{i_{k-2}-i_{k-1}} x_k^{i_{k-1}}) \end{aligned}$$

$$\begin{aligned}
 &= \sum_{i_1=k-1}^{n-1} \cdots \sum_{i_{k-1}=1}^{i_{k-2}-1} \sum_{i_k=0}^{i_{k-1}-1} \binom{n}{i_1} \cdots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_k} \\
 &\quad \times f(x_1)^{n-i_1} \cdots f(x_k)^{i_{k-1}-i_k} \\
 &\quad \times f(x_{k+1})^{i_k} \\
 &- \sum_{i_1=k-1}^{n-1} \cdots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \cdots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{i_{k-1}} \\
 &\quad \times f(x_1)^{n-i_1} \cdots f(x_{k-1})^{i_{k-2}-i_{k-1}} \\
 &\quad \times f(x_{k+1})^{i_{k-1}} \\
 &- \sum_{i_1=k-1}^{n-1} \cdots \sum_{i_{k-1}=1}^{i_{k-2}-1} \binom{n}{i_1} \cdots \binom{i_{k-2}}{i_{k-1}} \binom{i_{k-1}}{0} \\
 &\quad \times f(x_1)^{n-i_1} \cdots f(x_{k-1})^{i_{k-2}-i_{k-1}} \\
 &\quad \times f(x_k)^{i_{k-1}} \\
 &= \sum_{i_1=k}^{n-1} \sum_{i_2=k-1}^{i_1-1} \cdots \sum_{i_k=1}^{i_{k-1}-1} \binom{n}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{k-1}}{i_k} \\
 &\quad \times f(x_1)^{n-i_1} f(x_2)^{i_1-i_2} \cdots f(x_{k+1})^{i_k}, \quad (9)
 \end{aligned}$$

for all $x_1, x_2, x_3, \dots, x_k, x_{k+1} \in A$. $\square$

The following theorem is the generalization of Theorem 1.

Theorem 4. Let A, B be two commutative algebras, and let $f : A \rightarrow B$ be an n -Jordan homomorphism. Then f is an n -ring homomorphism.

Proof. Since f is an n -Jordan homomorphism, together with the additivity of f , we get

$$\begin{aligned}
 \sum_{i=0}^n \binom{n}{i} f(x_1^{n-i} x_2^i) &= f((x_1 + x_2)^n) = f(x_1 + x_2)^n \\
 &= (f(x_1) + f(x_2))^n \\
 &= \sum_{i=0}^n \binom{n}{i} f(x_1)^{n-i} f(x_2)^i, \quad (10)
 \end{aligned}$$

for all $x_1, x_2 \in A$. It is clear that $f(x_1^n) = f(x_1)^n$ and $f(x_2^n) = f(x_2)^n$, so we obtain

$$\sum_{i=1}^{n-1} \binom{n}{i} f(x_1^{n-i} x_2^i) = \sum_{i=1}^{n-1} \binom{n}{i} f(x_1)^{n-i} f(x_2)^i, \quad (11)$$

for all $x_1, x_2 \in A$. If $n = 2$, then by (11) we have $f(x_1 x_2) = f(x_1) f(x_2)$. Now let $n > 2$. Together with Lemma 3 and (11), we can say that the equality (4) holds for $k = n - 1$; that is,

$$\begin{aligned}
 &\sum_{i_1=n-1}^{n-1} \sum_{i_2=n-2}^{i_1-1} \cdots \sum_{i_{n-1}=1}^{i_{n-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{n-2}}{i_{n-1}} \\
 &\quad \times f(x_1^{n-i_1} x_2^{i_1-i_2} x_3^{i_2-i_3} \cdots x_n^{i_{n-1}}) \\
 &= \sum_{i_1=n-1}^{n-1} \sum_{i_2=n-2}^{i_1-1} \cdots \sum_{i_{n-1}=1}^{i_{n-2}-1} \binom{n}{i_1} \binom{i_1}{i_2} \cdots \binom{i_{n-2}}{i_{n-1}} \\
 &\quad \times f(x_1)^{n-i_1} f(x_2)^{i_1-i_2} \cdots f(x_n)^{i_{n-1}}, \quad (12)
 \end{aligned}$$

holds for all $x_1, x_2, x_3, \dots, x_n \in A$. Notice that

$$n - 1 > i_1 > i_2 > \cdots > i_{n-2} > i_{n-1} \geq 1 \quad (13)$$

implies $i_1 = n - 1, i_2 = n - 2, \dots, i_{n-2} = 2, i_{n-1} = 1$ and so

$$n - i_1, i_1 - i_2, \dots, i_{n-1} - i_{n-2}, i_{n-1} = 1. \quad (14)$$

Therefore we get the desired equality:

$$f(x_1 x_2 x_3 \cdots x_n) = f(x_1) f(x_2) f(x_3) \cdots f(x_n), \quad (15)$$

for all $x_1, x_2, x_3, \dots, x_n \in A$. $\square$

3. Generalization of Theorem 2

We need the following lemmas to prove the generalization of Theorem 2.

Lemma 5 (see [9, Corollaries 2.5 and 3.5]). Let V be a normed space, and let W be a Banach space. Assume that $f, g, h : V \rightarrow W$ are mappings such that

$$\|f(x + y) - g(x) - h(y)\| \leq \varepsilon (\|x\|^p + \|y\|^p), \quad (16)$$

for all $x, y \in V \setminus \{0\}$, where $p \neq 1$ and $\varepsilon \geq 0$. Then there exists a unique additive mapping $T : V \rightarrow W$ such that

$$\|f(x) - T(x) - f(0)\| \leq \frac{|4(3 + 3^p)|\varepsilon}{|2^p(3 - 3^p)|} \|x\|^p, \quad (17)$$

for all $x \in V \setminus \{0\}$. In particular, T is given by

$$T(x) = \lim_{m \rightarrow \infty} \frac{f(3^m x) - f(0)}{3^{sm}}, \quad (18)$$

for all $x \in V \setminus \{0\}$, where $s := -\operatorname{sgn}(p - 1)$.

Lemma 6. Let $V, W, f, g, h, \varepsilon$ be as in Lemma 5. If $p < 0$ and $f(0) = 0$, then f is an additive mapping.

Proof. Let $T : V \rightarrow W$ be the additive mapping satisfying (17). Then we have

$$\begin{aligned}
 \|2f(x) - 2T(x)\| &\leq \|f(2(n+1)x) - T(2(n+1)x)\| \\
 &\quad + \|f(-2nx) - T(-2nx)\| \\
 &\quad + \|f(x) - g((n+1)x) - h(-nx)\| \\
 &\quad + \|f(x) - g(-nx) - h((n+1)x)\| \\
 &\quad + \|f(2(n+1)x) \\
 &\quad \quad - g((n+1)x) - h((n+1)x)\| \\
 &\quad + \|f(-2nx) - g(-nx) - h(-nx)\| \\
 &\leq \left(\frac{4(3+3^p)}{2^p(3-3^p)} + 4 \right) (n+1)^p \varepsilon \|x\|^p \\
 &\quad + \left(\frac{4(3+3^p)}{2^p(3-3^p)} + 4 \right) n^p \varepsilon \|x\|^p,
 \end{aligned} \tag{19}$$

for all $x \in V \setminus \{0\}$ and $n \in \mathbb{N}$. Taking the limit as $n \rightarrow \infty$, we get $f(x) = T(x)$ as desired. $\square$

The following result has already been proved in [7] (see also [8]). We show that it can also be derived from Lemma 6.

Lemma 7. Let V, W, ε be as in Lemma 5 and $p < 0$. If $f : V \rightarrow W$ is a mapping such that

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon (\|x\|^p + \|y\|^p) \quad \forall x, y \in V \setminus \{0\}, \tag{20}$$

then f is an additive mapping.

Proof. By Lemma 5, we can take an additive mapping $T : V \rightarrow W$ satisfying (17). Observe that

$$\begin{aligned}
 \|f(0)\| &\leq \|f(nx) - T(nx) - f(0)\| \\
 &\quad + \|f(-nx) - T(-nx) - f(0)\| \\
 &\quad + \|f(0) - f(nx) - f(-nx)\| \\
 &\leq \left(\frac{8(3+3^p)}{2^p(3-3^p)} + 2 \right) n^p \varepsilon \|x\|^p,
 \end{aligned} \tag{21}$$

for all $x \in V \setminus \{0\}$ and for all $n \in \mathbb{N}$. Taking the limit as $n \rightarrow \infty$, we get $f(0) = 0$. By Lemma 6, f is an additive mapping. $\square$

Now we can prove the following theorem which is the generalization of Theorem 2.

Theorem 8. Let A be a commutative normed algebra and B a commutative Banach algebra. Assume that $f, g, h : A \rightarrow B$ satisfy (16) and

$$\|f(x^n) - f(x)^n\| \leq \delta \|x\|^{nq}, \tag{22}$$

for all $x \in A \setminus \{0\}$, where $\delta \geq 0$ and $(p-1)(q-1) > 0$. If $f(0) = 0$, then there exists a unique n -ring homomorphism $T : A \rightarrow B$ satisfying (17).

Proof. By Lemma 5, there exists a unique additive mapping T satisfying (17). By Theorem 4, it suffices to show that $T(x^n) = T(x)^n$. Put $s := -\operatorname{sgn}(q-1)$. From the equality below (17) in Lemma 5, we have

$$T(x) = \lim_{m \rightarrow \infty} \frac{f(3^{sm}x)}{3^{sm}}, \tag{23}$$

for all $x \in A \setminus \{0\}$. It follows from (22) that

$$\begin{aligned}
 \|T(x^n) - T(x)^n\| &= \lim_{m \rightarrow \infty} \frac{1}{3^{smn}} \{ \|f((3^{sm}x)^n) - (f(3^{sm}x))^n\| \} \\
 &\leq \lim_{m \rightarrow \infty} \frac{\delta}{3^{smn}} \|3^{sm}x\|^{nq} \\
 &= \lim_{m \rightarrow \infty} (3^{smn(q-1)}) \delta \|x\|^{nq} = 0,
 \end{aligned} \tag{24}$$

for all $x \in A \setminus \{0\}$. Hence T is an n -Jordan homomorphism. By Theorem 4, T is an n -ring homomorphism. $\square$

The following two corollaries give results on the hyperstability of n -ring homomorphisms between Banach algebras.

Corollary 9. Let A, B, q, δ, f, g, h be as in Theorem 8. If $f(0) = 0$ and $p < 0$, then f is an n -ring homomorphism.

Proof. Let T be the unique n -ring homomorphism satisfying (17) in Theorem 8. By Lemma 6, f is the unique additive mapping satisfying (17). So f is the unique n -ring homomorphism. $\square$

Corollary 10. Let A, B, p, q be as in Corollary 9. Assume that $f : A \rightarrow B$ satisfies the system of functional inequalities (20) and (22) for all $x \in A \setminus \{0\}$. Then f is an n -ring homomorphism.

Proof. The proof is analogous as for Corollary 9, with Lemma 6 replaced by Lemma 7. $\square$

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