Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 693032, 4 pages
http://dx.doi.org/10.1155/2013/693032

Research Article

Orbital Shadowing for C^1-Generic Volume-Preserving Diffeomorphisms

Manseob Lee

Department of Mathematics, Mokwon University, Dajeon 302-729, Republic of Korea

Correspondence should be addressed to Manseob Lee; lmsds@mokwon.ac.kr

Received 13 June 2013; Revised 29 August 2013; Accepted 1 September 2013

Academic Editor: Chun-Gang Zhu

We show that C^1-generically, if a volume-preserving diffeomorphism has the orbital shadowing property, then the diffeomorphism is Anosov.

1. Introduction

In the differentiable dynamical systems, the shadowing theory is a very useful notion for the investigation of the stability condition. In fact, Robinson [1] and Sakai [2] proved that a diffeomorphism belongs to the C^1-interior of the set of diffeomorphisms having the shadowing property coincides the structurally stable; that is, the diffeomorphism satisfies both Axiom A and the strong transversality condition. In general, if a diffeomorphism is Ω-stable, that is, a diffeomorphism satisfies both Axiom A and the no-cycle condition, then there is a diffeomorphism which does not have the shadowing property. Indeed, let a diffeomorphism f of the two-dimensional torus $\mathbb{T}^2$. The nonwandering set $\Omega(f)$ consists of 4 hyperbolic fixed points, $\Omega(f) = \{p_1, p_2, p_3, p_4\}$, where p_1 is a sink, p_4 is a source, and p_2 and p_3 are saddles such that $W^s(p_2) \cup \{p_3\} = W^u(p_3) \cup \{p_2\}$. It is assumed that the eigenvalues of $D_{p_2}f$ are $-\mu, \nu$ with $\mu > 1$, $0 < \nu < 1$ and the eigenvalues of $D_{p_3}f$ are $-\lambda, \kappa$ with $\kappa > 1$, $0 < \lambda < 1$. Then f does not have the shadowing property. But it has the orbital shadowing property (see [3]).

In this paper, we study the orbital shadowing property in which it is clear that if a diffeomorphism has the shadowing property, then it has the orbital shadowing property, but the converse is not true. In fact, an irrational rotation map does not have the shadowing property, but it has the orbital shadowing property.

The orbital shadowing property was introduced by Pilyugin et al. [3]. They showed that a diffeomorphism belongs to the C^1-interior of the set of all diffeomorphisms having the orbital shadowing property if and only if the diffeomorphism is structurally stable.

For a conservative diffeomorphism, Bessa and Rocha proved in [4] that if a conservative diffeomorphism belongs to the C^1-interior of the set of all topologically stable conservative diffeomorphisms, then it is Anosov. In [5], Bessa proved that a conservative diffeomorphism is in the C^1-interior of the set of all conservative diffeomorphisms having the shadowing property if and only if it is Anosov. K. Lee and M. Lee [6] proved that a conservative diffeomorphism is in the C^1-interior of the set of all conservative diffeomorphisms having the orbital shadowing property if and only if it is Anosov. Our result is a generalization of the result in [7].

Let M be a closed C^∞ Riemannian manifold endowed with a volume form ω. Let μ denote the Lebesgue measure associated to ω, and let d denote the metric induced on M by the Riemannian structure. Denote by $\mathrm{Diff}_\mu(M)$ the set of diffeomorphisms which preserves the Lebesgue measure μ endowed with the C^1 Whitney topology. We know that every volume preserving diffeomorphism satisfying Axiom A is Anosov (for more details, see [8]).

For $\delta > 0$, a sequence of points $\{x_i\}_{i=a}^{b}(-\infty \le a < b \le \infty)$ in M is called a *δ-pseudo-orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $a \le i \le b-1$. We say that f has the *shadowing property* if, for every $\epsilon > 0$, there is $\delta > 0$, such that, for any δ-pseudo-orbit $\{x_i\}_{i\in\mathbb{Z}}$ of f, there is a point $y \in M$, such that, $d(f^i(y), x_i) < \epsilon$ for all $i \in \mathbb{Z}$. It is easy to see that f has the shadowing property

if and only if f^n has the shadowing property for $n \in \mathbb{Z} \setminus \{0\}$. For each $x \in M$, let $\mathcal{O}_f(x)$ be the orbit of f through x; that is,

$$\mathcal{O}_f(x) = \{f^n(x) : n \in \mathbb{Z}\}. \tag{1}$$

We say that f has the *orbital shadowing property* if, for any $\epsilon > 0$, there exists $\delta > 0$, such that for any δ-pseudo-orbit $\xi = \{x_i\}_{i\in\mathbb{Z}}$, we can find a point $y \in M$ such that

$$\mathcal{O}_f(y) \subset B_\epsilon(\xi), \quad \xi \subset B_\epsilon(\mathcal{O}_f(y)), \tag{2}$$

where $B_\epsilon(A)$ denotes the ϵ-neighborhood of a set $A \subset M$. It is easy to see that f has the orbital shadowing property if and only if f^n has the orbital shadowing property for $n \in \mathbb{Z} \setminus \{0\}$. Let Λ be a closed f-invariant set. We say that Λ is *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exist constants $C > 0$ and $0 < \lambda < 1$, such that

$$\left\|D_x f^n|_{E^s_x}\right\| \le C\lambda^n, \qquad \left\|D_x f^{-n}|_{E^u_x}\right\| \le C\lambda^n, \tag{3}$$

for all $x \in \Lambda$ and $n \ge 0$.

In [3], the authors proved that the C^1-interior of the set of dissipative diffeomorphisms having the orbital shadowing property coincides with the set of structurally stable diffeomorphisms. Note that if a diffeomorphism satisfies structurally stable, then it is not Anosov in general, but the converse is true.

We say that a subset $\mathcal{G} \subset \mathrm{Diff}(M)$ is *residual* if $\mathcal{G}$ contains the intersection of a countable family of open and dense subsets of $\mathrm{Diff}(M)$; in this case, $\mathcal{G}$ is dense in $\mathrm{Diff}(M)$. A property "P" is said to be C^1*-generic* if "P" holds for all diffeomorphisms which belong to some residual subset of $\mathrm{Diff}(M)$. We use the terminology "for C^1-generic f" to express that "there is a residual subset $\mathcal{G} \subset \mathrm{Diff}(M)$ such that for any $f \in \mathcal{G} \cdots$." The following is the main result in this paper.

Theorem 1. *For C^1-generic f, if f has the orbital shadowing property, then f is Anosov.*

Let p be a periodic point of f with period $\pi(p)$. We say that p is an *elementary point* if $D_p f^{\pi(p)}$ eigenvalues are multiplicatively independent over $\mathbb{Z}$. Elementary points have simple spectrum, and none of $D_p f^{\pi(p)}$ eigenvalues are a root of unity or equal to 1. For a periodic point p of f, if we consider $\dim M = 2$, then we have three cases. Firstly, p is a hyperbolic saddle, that is, real eigenvalues λ_1, λ_2 with $\lambda_2 = \lambda_1^{-1}$. Secondly, p is an elliptic point; that is, nonreal eigenvalues are conjugated and of norm one. Finally, p is a parabolic point; that is, eigenvalues equal 1 or –1. Note that the first and second cases are robust under small perturbations. Elementary elliptic points are associated with an irrational rotation number. In [9], Robinson showed that if $\dim M = 2$, there is a residual set in $\mathrm{Diff}_\mu(M)$ such that any elementary in this residual displays all its elliptic points of elementary type. In [10, Theorem 1.3], Newhouse showed that C^1-generic volume-preserving diffeomorphisms in surfaces are Anosov, or else the elliptical points are dense. Actually, Newhouse's proof is strongly supported in the symplectic structure. By Newhouse [10] and Robinson [9], we give a problem as follows: For C^1 generic $f \in \mathrm{Diff}_\mu(M^2)$, if f has the orbital shadowing property, then is it Anosov?

2. Proof of Theorems 1

Let M be as before, and let $f \in \mathrm{Diff}_\mu(M)$. Let Λ be a closed f-invariant set. We say that Λ is a *transitive set* if there is a point $x \in \Lambda$ such that $x \in \omega_f(x)$, where $\omega_f(x)$ is the omega-limit set. If $\Lambda = M$, then we called that f is transitive. We say that $p \in P(f)$ is a *hyperbolic* if $D_p f^{\pi(p)} : T_p M \to T_p M$ has no eigenvalues of absolute value one. It is well known that if p is a hyperbolic periodic point of f with period k, then

$$\begin{aligned} W^s(p) &= \left\{x \in M : f^{kn}(x) \longrightarrow p \text{ as } n \longrightarrow \infty\right\}, \\ W^u(p) &= \left\{x \in M : f^{kn}(x) \longrightarrow p \text{ as } n \longrightarrow -\infty\right\} \end{aligned} \tag{4}$$

are C^1-injectively immersed submanifolds of M. Let q be a hyperbolic periodic point of f. We say that p and q are *homoclinically related* if

$$W^s(p) \pitchfork W^u(q) \neq \emptyset, \qquad W^u(p) \pitchfork W^s(q) \neq \emptyset. \tag{5}$$

For given hyperbolic periodic points p and q of f, we write $p \sim q$ if p and q are homoclinically related. It is clear that if $p \sim q$, then $\mathrm{index}(p) = \mathrm{index}(q)$. The following result is very useful to prove Theorem 1. Let p be a hyperbolic periodic point. For $x \in M$, we say that x is a *homoclinic point* if $W^s(p) \pitchfork W^u(p)$. Denote by $H(p, f) = \overline{W^s(p) \pitchfork W^u(p)}$.

Theorem 2 (see [11, Theorem 1.3]). *There is a residual set $\mathcal{G}_1 \subset \mathrm{Diff}_\mu(M)$ such that, for any $f \in \mathcal{G}_1$, f is transitive. Moreover, M is a unique homoclinic class.*

We denote $\mathcal{F}_\mu(M)$ by the set of diffeomorphisms $f \in \mathrm{Diff}_\mu(M)$ which has a C^1-neighborhood $\mathcal{U}(f) \subset \mathrm{Diff}_\mu(M)$ such that for any $g \in \mathcal{U}(f)$, every periodic point of g is hyperbolic.

Very recently, Arbieto and Catalan [8] proved that every volume preserving diffeomorphism in $\mathcal{F}_\mu(M)$ is Anosov.

Theorem 3 (see [8, Theorem 1]). *Every volume preserving diffeomorphism in $\mathcal{F}_\mu(M)$ is Anosov.*

To prove Theorem 1, it is enough to show that $f \in \mathcal{F}_\mu(M)$. Let p be a hyperbolic periodic point of f; there exists $\epsilon(p) > 0$ such that, for any $x \in W^s_{\epsilon(p)}(p)$ and $x \in W^u_{\epsilon(p)}(p)$, we know that

$$d\left(f^i(x), f^i(p)\right) \le \epsilon(p), \qquad d\left(f^{-i}(x), f^{-i}(p)\right) \le \epsilon(p), \tag{6}$$

for all $i \ge 0$. Then $W^s_{\epsilon(p)}(p)$ is called the local stable manifold of p, and $W^u_{\epsilon(p)}(p)$ is the local unstable manifold of p. It is clear that $W^s_{\epsilon(p)}(p) \subset W^s(p)$, and $W^u_{\epsilon(p)}(p) \subset W^u(p)$.

Lemma 4. *Let $f \in \mathcal{G}_1$, and let $p, q \in P_h(f)$. If f has the orbital shadowing property, then*

$$W^s(p) \cap W^u(q) \neq \emptyset, \qquad W^u(p) \cap W^s(q) \neq \emptyset, \tag{7}$$

where $P_h(f)$ is the set of all hyperbolic periodic points of f.

Proof. Let $f \in \mathcal{G}_1$, and let $p, q \in P_h(f)$ be hyperbolic periodic points of f. Take $\epsilon(p) > 0$ and $\epsilon(q) > 0$ as before with respect to p and q. For simplicity, we may assume that $f(p) = p$ and $f(q) = q$. Take $\epsilon = \min\{\epsilon(p), \epsilon(q)\}$. Let $0 < \delta = \delta(\epsilon) < \epsilon$ be the number of the orbital shadowing property of f for ϵ. Since f is transitive, there exists $x \in M$ such that $\omega(x) = M$. Then there exist $l_1 > 0$ and $l_2 > 0$ such that $d(f^{l_1}(x), p) < \delta$ and $d(f^{l_2}(x), q) < \delta$. We may assume that $l_2 = l_1 + k$ for some $k > 0$. Then we get a finite δ-pseudo-orbit $\{p, f^{l_1}(x), f^{l_1+1}(x), \ldots, f^{l_2-1}(x), q\}$. Now we construct a δ-pseudo orbit as follows: put (i) $f^i(p) = x_i$ for $i \leq 0$, (ii) $f^{l_1+i}(x) = x_{1+i}$ for $0 \leq i \leq k-1$, and (iii) $f^{l_2+i}(q) = x_{k+i}$ for $i \geq 0$. Then

$$\begin{aligned}\xi &= \{\ldots, p, p, x_0 (= p), f^{l_1}(x), f^{l_1+1}(x), \ldots, \\ &\qquad f^{l_1+k-1}(x), f^{l_2}(x) (= q), q, q, \ldots\} \\ &= \{\ldots, x_{-2}, x_{-1}, x_0 (= p), x_1, x_2, \ldots, \\ &\qquad x_{k-1}, x_k (= q), x_{k+1}, \ldots, \}.\end{aligned} \tag{8}$$

Since f has the orbital shadowing property, there are points $y \in M$ and $j \geq 0, n > 0$ such that (i) $f^j(y) \in B_\epsilon(x_0)$ and $f^{-i}(f^j(y)) \in B_\epsilon(x_{-i})$ for all $i \geq 0$; (ii) $f^n(y) \in B_\epsilon(x_k)$ and $f^i(f^n(y)) \in B_\epsilon(x_{k+i})$ for all $i \geq 0$. Then

$$\mathcal{O}_f(y) \subset B_\epsilon(\xi), \quad \xi \subset B_\epsilon(\mathcal{O}_f(y)), \tag{9}$$

$f^j(y) \in W^u_\epsilon(p)$, and $f^n(y) \in W^s_\epsilon(q)$. Thus, $\mathcal{O}_f(y) \cap W^u(p) \cap W^s(q) \neq \emptyset$. □

The study of the Kupka-Smale systems within volume preserving maps was developed by Robinson (see [1]). We say that f is *Kupka-Smale* if every periodic point is hyperbolic and each invariant manifold has transverse intersections. Denote by $\mathcal{KS}(M)$ the set of Kupka-Smale volume-preserving diffeomorphisms. It is well-known that the $\mathcal{KS}(M)$ is residual in $\mathrm{Diff}_\mu(M)$.

Lemma 5. *There is a residual set $\mathcal{G}_2 \subset \mathrm{Diff}_\mu(M)$ such that, for any $f \in \mathcal{G}_2$, if f has the orbital shadowing property, then for any $p, q \in P_h(f)$,* index(p) = index(q).

Proof. Let $\mathcal{G}_2 = \mathcal{G}_1 \cap \mathcal{KS}(M)$, and let $f \in \mathcal{G}_2$. Suppose that f has the orbital shadowing property. Let p and q be hyperbolic periodic points of f. To derive a contradiction, we may assume that $\mathrm{index}(p) \neq \mathrm{index}(q)$. Then we know that $\dim W^s(p) + \dim W^u(q) < \dim M$ or $\dim W^u(p) + \dim W^s(q) < \dim M$. Assume that $\dim W^s(p) + \dim W^u(q) < \dim M$. Since f is Kupka-Smale, we have $W^s(p) \cap W^u(q) = \emptyset$. This is a contradiction by Lemma 4. □

Remark 6. In $\dim(M) = 2$, the index is always constant, and so these arguments cannot used in this low-dimensional case.

To prove our result, we use Franks' lemma which is proved in [12, Proposition 7.4].

Lemma 7. *Let $f \in \mathrm{Diff}^1_\mu(M)$, and let $\mathcal{U}(f)$ be a C^1-neighborhood of f in $\mathrm{Diff}^1_\mu(M)$. Then there exist a C^1-neighborhood $\mathcal{U}_0(f) \subset \mathcal{U}(f)$ of f and $\epsilon > 0$ such that if $g \in \mathcal{U}_0(f)$, any finite f-invariant set $E = \{x_1, \ldots, x_m\}$, any neighborhood U of E, and any volume-preserving linear maps $L_j : T_{x_j}M \to T_{g(x_j)}M$ with $\|L_j - D_{x_j}g\| \leq \epsilon$ for all $j = 1, \ldots, m$, there is a conservative diffeomorphism $g_1 \in \mathcal{U}(f)$ coinciding with f on E and out of U, and $D_{x_j}g_1 = L_j$ for all $j = 1, \ldots, m$.*

Denote by $P(f)$ the set of all periodic points of f. The following was proved by [7]. Since the paper is still not published yet, we give the proof for completeness.

Lemma 8. *Let $\dim M \geq 3$, and let $\mathcal{U}(f)$ be a C^1-neighborhood of f. If $p \in P(f)$ is not hyperbolic, then there is $g \in \mathcal{U}(f)$ such that g has two periodic points $p_1, p_2 \in P_h(f)$ with different indices.*

Proof. Let $p \in P(f)$ be the nonhyperbolic periodic orbit of period π and $\epsilon > 0$. By Pugh-Robinson's closing lemma [13] there is $f_1 \in \mathrm{Diff}_\mu(M)$, such that f_1 is arbitrarily C^1-close to f, with $q_1 \in P(f_1)$ close to p by closing some recurrent orbit, since Poincaré recurrence almost every point is recurrent. Moreover, since hyperbolicity holds open and is densely even in the volume-preserving setting, p_2 can be chosen to be hyperbolic. Let $\mathrm{index}(p_2) = i$. After this perturbation (away from the orbit of p_2), we get $f_1 \in \mathrm{Diff}_\mu(M)$ such that f_1 has a periodic orbit p_1 close to p, with period π. We observe that p_1 may not be the analytic continuation of p and this is precisely the case when 1 is an eigenvalue of the tangent map $Df^\pi(p)$. If p_1 is not hyperbolic take $f_2 = f_1$. If p_1 is hyperbolic for $Df_1^\pi(p_1)$, then, since f_1 is arbitrarily C^1-close to f, the distance between the spectrum of $Df_1^\pi(p_1)$ and the unitary circle can be taken arbitrarily close to zero. This means that we are in the presence of a very weak hyperbolicity, that is, of a δ-weak eigenvalue thus in a position to apply [12, Proposition 7.4] to obtain $f_2 \in \mathrm{Diff}_\mu(M)$, such that p_1 is a non-hyperbolic periodic orbit. Moreover, this *local* perturbation can be done far from the periodic point p_2. Once again, we use [12, Proposition 7.4] in order to obtain $g \in \mathrm{Diff}_\mu(M)$, such that $p_1 \in P(g)$ is hyperbolic and $\mathrm{index}(p_1) \neq i$. □

The following is a volume-preserving diffeomorphism version of [14, Lemma 2.2].

Lemma 9. *There is a residual set $\mathcal{G}_3 \subset \mathrm{Diff}_\mu(M)$ such that, for any $f \in \mathcal{G}_3$, if for any C^1-neighborhood $\mathcal{U}(f)$ of f there is $g \in \mathcal{U}(f)$ such that g has two hyperbolic periodic points $p_g, q_g \in P_h(g)$ with different indices, then f has two hyperbolic periodic points $p, q \in P_h(f)$ with different indices.*

For any $\delta > 0$, we say that $p \in P(f)$ has a *δ-weak eigenvalue* if there is an eigenvalue λ of $Df^{\pi(p)}(p)$, such that, $(1-\delta)^{\pi(p)} < |\lambda| < (1+\delta)^{\pi(p)}$, where $\pi(p)$ is the period of p.

Lemma 10. *There is a residual set $\mathcal{G}_4 \subset \mathrm{Diff}_\mu(M)$ such that for any $f \in \mathcal{G}_4$, if f has the orbital shadowing property then there is $\delta > 0$ such that for any $p \in P(f)$, p does not have a δ-weak eigenvalue.*

Proof. Let $\mathcal{G}_4 = \mathcal{G}_2 \cap \mathcal{G}_3$, and let $f \in \mathcal{G}_4$ have the orbital shadowing property. To derive a contradiction, we may assume that there is $p \in P(f)$ such that, for any $\delta > 0$, p has a δ-weak eigenvalue. Then by Lemma 7, we can find h C^1-close to f, such that p_h is not hyperbolic, where p_h is the continuation of p. By Lemma 8, again using the Lemma 7, we take g C^1-close to h and also C^1-close to f such that g has two hyperbolic periodic points $p_g, q_g \in P_h(g)$ with $\mathrm{index}(p_g) \neq \mathrm{index}(q_g)$. Since $f \in \mathcal{G}_6$, by Lemma 9, f has two hyperbolic periodic points, p, q, with $\mathrm{index}(p) \neq \mathrm{index}(q)$. This is a contradiction by Lemma 5. □

Lemma 11 (see [15, Lemma 5.1]). *There is a residual set $\mathcal{G}_5 \subset \mathrm{Diff}_\mu(M)$ such that, for any $f \in \mathcal{G}_5$, for any $\delta > 0$, if for any C^1-neighborhood $\mathcal{U}(f)$, there is $g \in \mathcal{U}(f)$, such that for any $p_g \in P_h(g)$, p_g has a δ-weak eigenvalue, then $p \in P_h(f)$ has a 2δ-weak eigenvalue.*

Proof of Theorem 1. Let $\mathcal{G}_6 = \mathcal{G}_4 \cap \mathcal{G}_5$, and let $f \in \mathcal{G}_6$ have the orbital shadowing property. The proof is by a contradiction; we may assume that $f \notin \mathcal{F}(M)$. Then there is a non-hyperbolic periodic point p_g for some g C^1-nearby f, such that p_g is $\delta/2$-weak eigenvalue. Then by Lemma 11, $p \in P(f)$ has δ-weak eigenvalue. This is a contradiction by Lemma 10. □

Acknowledgments

The author wishes to express his deepest appreciation to the referee for his careful reading of the paper, critical comments, and valuable suggestions. This work is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology, Korea (no. 2011-0007649).

References

[1] C. Robinson, "Stability thereoms and hyperbolicity in dynamical systems," *Rocky Mountain Journal of Mathematics*, vol. 7, pp. 425–437, 1977.

[2] K. Sakai, "Pseudo-orbit tracing property and strong transversality of diffeomorphisms on closed manifolds," *Osaka Journal of Mathematics*, vol. 31, no. 2, pp. 373–386, 1994.

[3] S. Yu. Pilyugin, A. A. Rodionova, and K. Sakai, "Orbital and weak shadowing properties," *Discrete and Continuous Dynamical Systems A*, vol. 9, no. 2, pp. 287–308, 2003.

[4] M. Bessa and J. Rocha, "Topological stability for conservative systems," *Journal of Differential Equations*, vol. 250, no. 10, pp. 3960–3966, 2011.

[5] M. Bessa, "C^1-stable shadowable conservative diffeomorphisms areAnosov," http://arXiv:1112. 3466 .

[6] K. Lee and M. Lee, "Volume-preserving diffeomorphisms with orbital shadowing," *Journal of Inequalities and Applications*, vol. 2013, article 18, pp. 1–7, 2013.

[7] M. Bessa, M. Lee, and X. Wen, "Shadowing, expansiveness and specification for C^1-conservative systems".

[8] A. Arbieto and T. Catalan, "Hyperbolicity in the volume preservingsenario," *Ergodic Theory and Dynamical Systems*. In press.

[9] R. C. Robinson, "Generic properties of conservative systems," *American Journal of Mathematics*, vol. 92, pp. 562–603, 1970.

[10] S. E. Newhouse, "Quasi-elliptic periodic points in conservative dynamical systems," *American Journal of Mathematics*, vol. 99, no. 5, pp. 1061–1087, 1977.

[11] C. Bonatti and S. Crovisier, "Récurrence et généricité," *Inventiones Mathematicae*, vol. 158, no. 1, pp. 33–104, 2004.

[12] C. Bonatti, L. J. Díaz, and E. R. Pujals, "A C^1-generic dichotomy for diffeomorphisms: weak forms of hyperbolicity or infinitely many sinks or sources," *Annals of Mathematics*, vol. 158, no. 2, pp. 355–418, 2003.

[13] C. C. Pugh and C. Robinson, "The C^1 closing lemma, including Hamiltonians," *Ergodic Theory and Dynamical Systems*, vol. 3, no. 2, pp. 261–313, 1983.

[14] M. Lee and S. Lee, "Generic diffeomorphisms with robustly transitive sets," *Communications of the Korean Mathematical Society*, vol. 28, no. 3, pp. 581–587, 2013.

[15] A. Arbieto, "Periodic orbits and expansiveness," *Mathematische Zeitschrift*, vol. 269, no. 3-4, pp. 801–807, 2011.