Research Article

Common Fixed Points for Weak $\psi$-Contractive Mappings in Ordered Metric Spaces with Applications

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1. Introduction and Preliminaries

Fixed point and common fixed point theorems for different types of nonlinear contractive mappings have been investigated extensively by various researchers (see [1–41]). Fixed point problems involving weak contractions and the mappings satisfying weak contractive type inequalities have been studied by many authors (see [10–20] and references cited therein).

Recently, many researchers have obtained fixed point, common fixed point, coupled fixed point, and coupled common fixed point results in partially ordered metric spaces (see [3, 6–8, 10–12, 29, 30, 32, 36]) and other spaces (see [5, 15, 31, 35, 38, 40, 41]).

Let $(X, \leq)$ be a partially ordered set. A mapping $f : X \to X$ is called a weak annihilator of a mapping $g : X \to X$ if $fgx \leq x$ for all $x \in X$.

Example 2 (see [3]). Let $X = [0, 1]$ be endowed with usual ordering and $f, g : X \to X$ be two mappings defined by $f(x) = x^2$ and $g(x) = x^3$. It is clear that $fgx = x^6 \leq x$ for $x \in X$ implies that $f$ is a weak annihilator of $g$.

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Let $M$ be a nonempty subset of a metric space $(X, d)$. Let $S$ and $T$ be mappings from a metric space $(X, d)$ into itself. A point $x \in M$ is a common fixed (resp., coincidence) point of $S$ and $T$ if $x = Sx = Tx$ (resp., $Sx = Tx$). The set of fixed points increasing. But $gx = \sqrt{x} \not\leq x = fx$ for any $x \in (0, 1)$ implies that the pair $(g, f)$ is not partially weakly increasing.

Let $(X, \leq)$ be a partially ordered set. A mapping $f : X \to X$ is called a weak annihilator of a mapping $g : X \to X$ if $fgx \leq x$ for all $x \in X$.

Example 2 (see [3]). Let $X = [0, 1]$ be endowed with usual ordering and $f, g : X \to X$ be two mappings given by $fx = x^2$ and $gx = x^3$. It is clear that $fgx = x^6 \leq x$ for $x \in X$ implies that $f$ is a weak annihilator of $g$.

Let $(X, \leq)$ be a partially ordered set. A mapping $f$ is called a dominating if $x \leq fx$ for any $x \in X$.

Example 3 (see [3]). Let $X = [0, 1]$ be endowed with usual ordering and $f : X \to X$ a mapping defined by $fx = x^{1/3}$, since $x \leq x^{1/3} = fx$ for $x \in X$ implies that $f$ is a dominating mapping.

A subset $W$ of a partially ordered set $X$ is said to be well ordered if every two elements of $W$ are comparable.

Let $M$ be a nonempty subset of a metric space $(X, d)$. Let $S$ and $T$ be mappings from a metric space $(X, d)$ into itself. A point $x \in M$ is a common fixed (resp., coincidence) point of $S$ and $T$ if $x = Sx = Tx$ (resp., $Sx = Tx$). The set of fixed points...
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(resp., coincidence points) of \( S \) and \( T \) is denoted by \( F(S,T) \) (resp., \( C(S,T) \)).

In 1986, Jungck [24] introduced the more generalized commuting mappings in metric spaces, called compatible mappings, which also are more general than the concept of weakly commuting mappings (that is, the mappings \( S,T : X \rightarrow X \) are said to be weakly commuting if \( d(STx,TSx) \leq d(Sx,Tx) \) for all \( x \in X \)) introduced by Sessa [34] as follows.

**Definition 4.** Let \( S \) and \( T \) be mappings from a metric space \((X,d)\) into itself. The mappings \( S \) and \( T \) are said to be compatible if

\[
\lim_{n \to \infty} d(STx_n,TSx_n) = 0,
\]

whenever \( \{x_n\} \) is a sequence in \( X \) such that \( \lim_{n \to \infty} x_n = z \) for some \( z \in X \).

In general, commuting mappings are weakly commuting and weakly commuting mappings are compatible, but the converses are not necessarily true and some examples can be found in [24–26].

In [27], Jungck and Rhoades introduced the concept of weakly compatible mappings and proved some common fixed point theorems for these mappings.

**Definition 5.** The mappings \( S \) and \( T \) are said to be weakly compatible if they commute at coincidence points of \( S \) and \( T \).

In Djoudi and Nisse [21], we can find an example to show that there exists weakly compatible mappings which are not compatible mappings in metric spaces.

Let \( \Psi \) denote the set of all functions \( \psi : [0,\infty)^5 \rightarrow [0,\infty) \) such that

\(\psi\) is continuous;

(\(b\)\) \(\psi\) is strictly increasing in all the variables;

(\(c\)\) for all \( t \in (0,\infty) \setminus \{0\} \)

\[
\psi(t,t,t,0,2t) < t, \quad \psi(t,t,t,2t,0) < t,
\]

\[
\psi(0,0,t,t,0) < t, \quad \psi(0,t,0,0,t) < t,
\]

\[\psi(t,0,0,t,t) < t.\]

It is easy to verify that the following functions are from the class \( \Psi \), see [18]:

\[
\psi(t_1,t_2,t_3,t_4,t_5) = k \max\left\{ t_1,t_2,t_3, t_4, t_5 \right\},
\]

for \( k \in (0,1) \);  \[ (3) \]

\[
\psi(t_1,t_2,t_3,t_4,t_5) = k \max\left\{ t_1,t_2,t_3, \frac{t_4+t_5}{2} \right\},
\]

for \( k \in (0,1) \).

**Definition 6** (see [18]). Let \((X,\leq)\) be a partially ordered set and suppose that there exists a metric \( d \) in \( X \) such that \((X,d)\) is a metric space. The mapping \( f : X \rightarrow X \) is said to be a \( \psi \)-contractive mapping, if

\[
d(fx, fy) \leq \psi(d(Sx,Ty), d(Sx,fx), d(Ty,gy), d(x, fy), d(y, fx)),
\]

for \( x \geq y \).

Recently, Chen introduced \( \psi \)-contractive mappings. The purpose of this paper is to extend the results of Chen for four mappings, in the framework of ordered metric spaces.

2. Main Results

Now, we give the main results in this paper.

**Theorem 7.** Let \((X,\leq)\) be a partially ordered set, and suppose that there exists a metric \( d \) on \( X \) such that \((X,d)\) is a complete metric space. Suppose that \( T, f, g, S \) are self-mappings on \( X \), the pairs \((T,f)\) and \((S,g)\) are partially weakly increasing with \( f(X) \subseteq T(X) \) and \( g(X) \subseteq S(X) \), and the dominating mappings \( f \) and \( g \) are weak annihilators of \( T \) and \( S \), respectively. Further, suppose that for any two comparable elements \( x, y \in X \), and \( \psi \in \Psi \),

\[
d(fx, fy) \leq \psi(d(Sx,Ty), d(Sx,fx), d(Ty,gy), d(Sx, gy), d(Ty, fx)),
\]

holds. If, for a nondecreasing sequence \( \{x_n\} \) with \( x_n \leq y_n \) for all \( n \geq 1 \), \( y_n \nrightarrow u \) implies that \( x_n \nrightarrow u \) and either

(a) \( f \) and \( S \) are compatible, \( f \) or \( S \) is continuous, and \( g, T \) are weakly compatible or

(b) \( g \) and \( T \) are compatible, \( g \) or \( T \) is continuous, and \( S \) is compatible,

then \( f, g, S,\) and \( T \) have a common fixed point in \( X \). Moreover, the set of common fixed points of \( f, g, S, \) and \( T \) is well ordered if and only if \( f, g, S, \) and \( T \) have one and only one common fixed point in \( X \).

**Proof.** Let \( x_0 \in X \) be an arbitrary point. Since \( f(X) \subseteq T(X) \) and \( g(X) \subseteq S(X) \), we can construct the sequences \( \{x_n\} \) and \( \{y_n\} \) in \( X \) such that

\[
y_{2n-1} = fx_{2n-2} = Tx_{2n-1}, \quad y_{2n} = gx_{2n-1} = Sx_{2n},
\]

for each \( n \geq 1 \). By assumptions, we have

\[
x_{2n-2} \leq fx_{2n-2} = Tx_{2n-1} \leq fTx_{2n-1} \leq x_{2n-1},
\]

\[
x_{2n-1} \leq gx_{2n-1} = Sx_{2n} \leq gSx_{2n} \leq x_{2n},
\]

for each \( n \geq 1 \). Thus, for each \( n \geq 1 \), we have \( x_n \leq x_{n+1} \). Without loss of generality, we assume that \( y_{2n} \neq y_{2n+1} \) for each \( n \geq 1 \). Now, we claim that for all \( n \in \mathbb{N} \), we have

\[
d(y_{n+1}, y_{n+2}) < d(y_n, y_{n+1}).
\]
Suppose to the contrary that \( d(y_{2n}, y_{2n+1}) \leq d(y_{2n+1}, y_{2n+2}) \) for some \( n \in \mathbb{N} \). Since \( y_{2n} \) and \( y_{2n+1} \) are comparable, from (5), we have

\[
d(y_{2n+1}, y_{2n+2}) = d(fx_{2n}, gx_{2n+1}) \leq \psi \left( d(Sx_{2n}, Tx_{2n+1}), d(Sx_{2n}, fx_{2n}), d(Tx_{2n+1}, gx_{2n+1}) \right) \leq \psi \left( d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2}) \right) = \psi \left( d(y_{2n}, y_{2n+1}), d(y_{2n+1}, y_{2n+2}) \right).
\]

Let us denote \( c_n = d(y_{2n+1}, y_{2n}) \). Then, from (8), \( c_n \) is a nonincreasing sequence and bounded below. Thus, it must converge to some \( c \geq 0 \). If \( c > 0 \), then by the above inequalities, we have \( c \leq c_{n+1} \leq \psi(c_n, c_n, c_n, 2c_n, 0) \). Taking the limit, as \( n \to \infty \), we have \( c \leq c \leq \psi(c, c, c, 2c, 0) < c \), which is a contradiction. Hence,

\[
d(y_{n+1}, y_n) \to 0.
\]

Now, we show that \( \{y_n\} \) is a Cauchy sequence. Suppose that \( \{y_n\} \) is not a Cauchy sequence. Then, there exists \( \epsilon > 0 \) for which we can find two sequences of natural numbers \( \{m(k)\} \) and \( \{n(k)\} \) with \( n(k) > m(k) > k \) such that

\[
d(y_{m(k)-1}, y_{n(k)-1}) \geq \epsilon, \quad d(y_{m(k)}, y_{n(k)-1}) < \epsilon.
\]

From (11), it follows that

\[
e \leq d(y_{m(k)}, y_{n(k)}) \leq d(y_{m(k)}, y_{n(k)-1}) + d(y_{n(k)-1}, y_{n(k)}) \leq \epsilon + d(y_{n(k)-1}, y_{n(k)}).
\]

Letting \( k \to \infty \) and using (10), we have

\[
\lim_{k \to \infty} d(y_{m(k)}, y_{n(k)}) = \epsilon.
\]

Again,

\[
d(y_{n(k)-1}, y_{n(k)}) \leq d(y_{m(k)-1}, y_{n(k)}) + d(y_{m(k)}, y_{n(k)}) + d(y_{m(k)+1}, y_{n(k)}) + d(y_{n(k)}, y_{m(k)+1}).
\]

Letting \( k \to \infty \) in the above inequalities and using (10) and (13), we have

\[
\lim_{k \to \infty} d(y_{m(k)-1}, y_{n(k)}) = \epsilon.
\]

Similarly, we have

\[
\lim_{k \to \infty} d(y_{m(k)-1}, y_{n(k)}) = \epsilon.
\]

Also, again from (10), (15), and the inequality

\[
d(y_{m(k)-1}, y_{n(k)-1}) - d(y_{m(k)-1}, y_{n(k)}) \leq d(y_{n(k)}, y_{n(k)+1}),
\]

it follows that

\[
\lim_{k \to \infty} d(y_{m(k)-1}, y_{n(k)+1}) = \epsilon.
\]

Now, we have

\[
d(y_{m(k)}, y_{n(k)+1}) \leq \psi \left( d(y_{m(k)-1}, y_{n(k)}), d(y_{m(k)-1}, y_{n(k)}), d(y_{m(k)}, y_{n(k)+1}), d(y_{n(k)+1}, y_{n(k)}) \right).
\]

Letting \( k \to \infty \), we get

\[
e \leq \psi(\epsilon, 0, 0, \epsilon) < \epsilon,
\]

which is a contradiction. Thus \( \{y_n\} \) is a Cauchy sequence. Since \( X \) is a complete metric space, there exists \( z \in X \) such that

\[
\lim_{n \to \infty} y_n = z.
\]
that \( y_n \to z \). Therefore, we have
\[
\lim_{n \to \infty} y_{2n+1} = \lim_{n \to \infty} T x_{2n+1} = \lim_{n \to \infty} f x_{2n} = z,
\]
(23)
\[
\lim_{n \to \infty} y_{2n+2} = \lim_{n \to \infty} S x_{2n+2} = \lim_{n \to \infty} g x_{2n+1} = z.
\]
(24)
Assume that \( S \) is continuous. Since \( f \) and \( S \) are compatible, we have
\[
\lim_{n \to \infty} f S x_{2n+2} = \lim_{n \to \infty} S f x_{2n+2} = S z.
\]
(25)
Also,
\[
x_{2n+1} \leq g x_{2n+1} = S x_{2n+2}.
\]
Now, we have
\[
d(f S x_{2n}, g x_{2n+1})
\leq \psi(d(S^2 x_{2n}, T x_{2n+1}), d(T x_{2n+1}, f S x_{n+1}))
\leq \psi(d(S z, f z), d(T w, g w)),
\]
(27)
Letting \( n \to \infty \), we get
\[
d(z, g z) \leq \psi(d(z, g z), 0, 0, d(z, g z))<d(z, g z),
\]
(31)
which implies that \( g z = z \). Therefore, we have \( f z = g z = S z = T z = z \).

If \( f \) is continuous, then, following the similar arguments, also we get the result.

Similarly, the result follows when (b) holds.

Now, suppose that the set of common fixed points of \( T, S, f, \) and \( g \) is well ordered.

We claim that common fixed points of \( T, S, f, \) and \( g \) are unique.

Assume that \( T u = S u = f u = g u = u \) and \( T v = S v = f v = g v = v \), but \( u \neq v \). Then, from (5), we have
\[
d(u, v) = d(f u, g v)
\leq \psi(d(S u, T v), d(S u, f u), d(T v, g v))
\leq \psi(d(u, v), 0, 0, d(u, v))<d(u, v).
\]
(32)
This implies that \( d(u, v) = 0 \), and hence \( u = v \).

Conversely, if \( T, S, f, \) and \( g \) have only one common fixed point, then the set of common fixed point of \( f, g, S, \) and \( T \) being singleton is well ordered. This completes the proof. \( \square \)

**Example 8.** Consider \( X = [0, 1] \cup \{2, 3, 4, \ldots\} \) with usual ordering and
\[
d(x, y) = \begin{cases} |x - y| & \text{if } x, y \in [0, 1], x \neq y; \\ x + y & \text{if at least one of } x \text{ or } y \notin [0, 1], \\ x \neq y; \\ 0 & \text{if } x = y. \end{cases}
\]
(33)
Then \( (X, \leq, d) \) is a complete partially ordered metric space.
Let $f$, $g$, $S$, and $T$ be self-mappings on $X$ defined as

$$
f(x) = \begin{cases} 
0 & \text{if } x = 0; \\
\frac{1}{2} & \text{if } x \in \left(0, \frac{1}{2}\right]; \\
1 & \text{if } x \in \left[\frac{1}{2}, 1\right]; \\
x & \text{if } x \in \{2, 3, 4, \ldots\}; 
\end{cases}$$

$$
g(x) = \begin{cases} 
0 & \text{if } x = 0; \\
\frac{1}{2} & \text{if } x \in \left(0, \frac{1}{2}\right]; \\
x & \text{if } x \in \left[\frac{1}{2}, 1\right] \cup \{2, 3, 4, \ldots\}; 
\end{cases}$$

$$
T(x) = \begin{cases} 
0 & \text{if } x \leq \frac{1}{2}; \\
\frac{1}{2} & \text{if } x \in \left(\frac{1}{2}, 1\right]; \\
x-1 & \text{if } x \in \{2, 3, 4, \ldots\}; 
\end{cases}$$

$$
S(x) = \begin{cases} 
0 & \text{if } x \leq \frac{1}{2}; \\
2x-1 & \text{if } x \in \left(\frac{1}{2}, 1\right]; \\
x & \text{if } x \in \{2, 3, 4, \ldots\}. 
\end{cases}$$

(34)

Define function $\psi : [0, \infty)^5 \to [0, \infty)$ by the formula

$$
\psi(t_1, t_2, t_3, t_4, t_5) = \frac{6}{7} \max \left\{t_1, t_2, t_3, \frac{t_4 + t_5}{2}\right\}. 
$$

(35)

Note that $f$, $g$, $S$, and $T$ satisfy all the conditions given in Theorem 7. Moreover, 0 is a common fixed point of $f$, $g$, $S$, and $T$.

If $f = g$, then we have the following result.

**Corollary 9.** Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Suppose that $T$, $f$, and $S$ are self-mappings on $X$, the pairs $(T, f)$ and $(S, f)$ are partially weakly increasing with $f(X) \subseteq T(X)$ and $f(X) \subseteq S(X)$, and the dominating mapping $f$ is a weak annihilator of $T$ and $S$. Further, suppose that there exists the function $\psi \in \Psi$ such that, for any two comparable elements $x, y \in X$,

$$
d(fx, fy) \leq \psi(d(Sx, Ty), d(Sx, fx), d(Ty, fy), \\
d(Sx, fy), d(Ty, fx)) 
$$

(36)

holds. If, for a nondecreasing sequence $\{x_n\}$ with $x_n \leq y_n$ for all $n \geq 1$, $y_n \to u$ implies that $x_n \leq u$ and either $f, S$ are compatible, $f$ or $S$ is continuous, and $f, T$ are weakly compatible or

(b) $f, T$ are compatible, $f$ or $T$ is continuous, and $f, S$ are weakly compatible,

then $f$, $S$, and $T$ have a common fixed point in $X$. Moreover, the set of common fixed points of $f$, $S$, and $T$ is well ordered if and only if $f$, $S$, and $T$ have one and only one common fixed point in $X$.

**Corollary 10.** Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Suppose that $T$, $f$, and $S$ are self-mappings on $X$, the pairs $(T, f)$ and $(T, g)$ are partially weakly increasing with $f(X) \subseteq T(X)$ and $g(X) \subseteq T(X)$, and the dominating mappings $f$ and $g$ are weak annihilators of $T$. Further, suppose that there exists the function $\psi \in \Psi$ such that, for any two comparable elements $x, y \in X$,

$$
d(fx, gy) \leq \psi(d(Tx, Ty), d(Tx, fx), d(Ty, fy), \\
d(Tx, gy), d(Ty, fx)) 
$$

(37)

holds. If, for a nondecreasing sequence $\{x_n\}$ with $x_n \leq y_n$ for all $n \geq 1$, $y_n \to u$ implies that $x_n \leq u$ and either $f, T$ are compatible, $f$ or $T$ is continuous, and $f, S$ are weakly compatible or

(b) $g, T$ are compatible, $g$ or $T$ is continuous, and $f, T$ are weakly compatible,

then $f$, $g$, and $T$ have a common fixed point in $X$. Moreover, the set of common fixed points of $f$, $g$, and $T$ is well ordered if and only if $f$, $g$, and $T$ have one and only one common fixed point in $X$.

**Corollary 11.** Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Suppose that $T$ and $f$ are self-mappings on $X$, the pair $(T, f)$ is partially weakly increasing with $f(X) \subseteq T(X)$, and the dominating mapping $f$ is a weak annihilator of $T$. Further, suppose that there exists the function $\psi \in \Psi$ such that, for any two comparable elements $x, y \in X$,

$$
d(fx, fy) \leq \psi(d(Tx, Ty), d(Tx, fx), d(Ty, fy), \\
d(Tx, fy), d(Ty, fx)) 
$$

(38)

holds. If, for a nondecreasing sequence $\{x_n\}$ with $x_n \leq y_n$ for all $n \geq 1$, $y_n \to u$ implies that $x_n \leq u$ and further, $f, T$ are compatible, $f$ or $T$ is continuous, and $f, T$ are weakly compatible, then $f$ and $T$ have a common fixed point in X. Moreover, the set of common fixed points of $f$ and $T$ is well ordered if and only if $f$ and $T$ have one and only one common fixed point in $X$.

### 3. Applications

The aim of the section is to apply our new results to mappings involving contractions of integral type. For this purpose, denote by $\Lambda$ the set of functions $\mu : [0, \infty) \to [0, \infty)$ satisfying the following hypotheses:
(h1) $\mu$ is a Lebesgue-integrable mapping on each compact of $[0, \infty)$;
(h2) for any $\epsilon > 0$, we have $\int_0^\infty \mu(t) > 0$.

**Corollary 12.** Let $(X, \preceq)$ be a partially ordered set, and suppose that there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Suppose that $T, f, g, S$ and $T$ are self-mappings on $X$, the pairs $(T, f)$ and $(S, g)$ are partially weakly increasing with $f(X) \subseteq T(X)$ and $g(X) \subseteq S(X)$, and the dominating mappings $f$ and $g$ are weak annihilators of $T$ and $S$, respectively. Further, suppose that there exists the function $\psi \in \Psi$ such that, for any two comparable elements $x, y \in X$,

$$
\int_0^{d(fx, gy)} \alpha(s) \, ds
\leq \int_0^{\psi(d(Tx, Ty), d(Tx, fx), d(Ty, fy), d(Tx, fy), d(Ty, fx))} \alpha(s) \, ds
$$

(39)

holds, where $\alpha \in \Lambda$. If, for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all $n \geq 1$, $y_n \rightarrow u$ implies that $x_n \preceq u$ and either

(a) $f, S$ are compatible, $f$ or $S$ is continuous, and $g, T$ are weakly compatible or

(b) $g, T$ are compatible, $g$ or $T$ is continuous, and $f, S$ are weakly compatible,

then $f$, $g$, $S$, and $T$ have a common fixed point in $X$. Moreover, the set of common fixed points of $f$, $g$, $S$, and $T$ is well ordered if and only if $f$, $g$, $S$, and $T$ have one and only one common fixed point in $X$.

**Corollary 13.** Let $(X, \preceq)$ be a partially ordered set, and suppose that there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Suppose that $T, f, g$ and $S$ are self-mappings on $X$, the pairs $(T, f)$ and $(T, g)$ are partially weakly increasing with $f(X) \subseteq T(X)$ and $g(X) \subseteq T(X)$, and the dominating mappings $f$ and $g$ are weak annihilators of $T$. Further, suppose that there exists the function $\psi \in \Psi$ such that, for any two comparable elements $x, y \in X$,

$$
\int_0^{d(fx, gy)} \alpha(s) \, ds
\leq \int_0^{\psi(d(Tx, Ty), d(Tx, fx), d(Ty, fy), d(Tx, fy), d(Ty, fx))} \alpha(s) \, ds
$$

(40)

holds, where $\alpha \in \Lambda$. If, for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all $n \geq 1$, $y_n \rightarrow u$ implies that $x_n \preceq u$ and either

(a) $f, T$ are compatible, $f$ or $T$ is continuous, and $g, T$ are weakly compatible or

(b) $g, T$ are compatible, $g$ or $T$ is continuous, and $f, T$ are weakly compatible,

then $f$, $g$, and $T$ have a common fixed point in $X$. Moreover, the set of common fixed points of $f$, $g$, and $T$ is well ordered if and only if $f$, $g$, and $T$ have one and only one common fixed point in $X$.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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