Research Article

Global Strong Solution to the Density-Dependent 2-D Liquid Crystal Flows

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The initial-boundary value problem for the density-dependent flow of nematic crystals is studied in a 2-D bounded smooth domain. For the initial density away from vacuum, the existence and uniqueness is proved for the global strong solution with the large initial velocity u_0 and small ∇d_0 . We also give a regularity criterion $\nabla d \in L^p(0, T; L^q(\Omega))$ $((2/q) + (2/p) = 1, 2 < q \leq \infty)$ of the problem with the Dirichlet boundary condition $u = 0, d = d_0$ on $\partial\Omega$.

1. Introduction and Main Results

Let $\Omega \subseteq \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\Omega$, and ν is the unit outward normal vector on $\partial\Omega$. We consider the global strong solution to the density-dependent incompressible liquid crystal flow [1–4] as follows:

$$\operatorname{div} u = 0, \quad (1)$$

$$\partial_t \rho + \operatorname{div}(\rho u) = 0, \quad (2)$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla \pi - \Delta u = -\nabla \cdot (\nabla d \odot \nabla d), \quad (3)$$

$$\partial_t d + u \cdot \nabla d - \Delta d = |\nabla d|^2 d, \quad (4)$$

in $(0, \infty) \times \Omega$ with initial and boundary conditions

$$(\rho, u, d)(\cdot, 0) = (\rho_0, u_0, d_0) \quad \text{in } \Omega, \quad (5)$$

$$u = 0, \quad \partial_\nu d = 0 \quad \text{on } \partial\Omega, \quad (6)$$

where ρ denotes the density, u the velocity, d the unit vector field that represents the macroscopic molecular orientations, and π the pressure. The symbol $\nabla d \odot \nabla d$ denotes a matrix whose (i, j) th entry is $\partial_i d \partial_j d$, and it is easy to find that $\nabla d \odot \nabla d = \nabla d^T \nabla d$.

When d is a given constant unit vector, then (1), (2), and (3) represent the well-known density-dependent Navier-Stokes system, which has received many studies; see [5–7] and references therein.

When $\rho \equiv 1$ and $\Omega := \mathbb{R}^2$, Xu and Zhang [8] proved global existence of weak solutions to the problem if $u_0 \in L^2, \nabla d_0 \in L^2, |d_0| = 1$, and

$$\exp\left(216\left(\|u_0\|_{L^2}^2 + \frac{1}{16}\right)\right) \|\nabla d_0\|_{L^2}^2 < \frac{1}{16}. \quad (7)$$

When $\rho \equiv 1$ and (6) is replaced by

$$u = 0, \quad d = d_0 \quad \text{on } \partial\Omega. \quad (8)$$

Lin et al. [9] proved the global existence of weak solutions to the system (1)–(5) and (8), which are smooth away from at most finitely many singular times, and they also prove a regularity criterion

$$d \in L^2(0, T; H^2(\Omega)). \quad (9)$$

When $\rho = 1$ and the term $|\nabla d|^2$ in (4) is replaced by $(1 - |d|^2)d$, then the problem has been studied in [10–15].

Very recently, Wen and Ding [16] proved the global existence and uniqueness of strong solutions to the problem (1)–(6) with small u_0 and ∇d_0 and the local strong solutions with large initial data when $\Omega \subseteq \mathbb{R}^2$ is a smooth bounded domain.

Fan et al. [17] studied the regularity criterion of the Cauchy problem (1)–(5) when $\Omega := \mathbb{R}^2$.

We will prove the following.

Theorem 1. *Let $0 < m \leq \rho_0 \leq M < \infty$, $\rho_0 \in W^{1,r}$ for some $r \in (2, \infty)$, $u_0 \in H_0^1 \cap H^2$, and $d_0 \in H^3$ with $\operatorname{div} u_0 = 0$, and $|d_0| = 1$ in Ω . If*

$$\|\nabla d_0\|_{L^2}^2 \exp \left[216 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \frac{1}{8C_0^2} \right)^2 \right] \leq \frac{1}{8C_0^2}, \quad (10)$$

with an absolute constant C_0 in (22), then the problem (1)–(6) has a unique global-in-time strong solution (ρ, u, d) satisfying

$$\begin{aligned} \|\rho\|_{L^\infty(0,T;W^{1,r})} &\leq C, \quad \|\rho_t\|_{L^\infty(0,T;L^r)} \leq C, \\ \|u\|_{L^\infty(0,T;H^2) \cap L^2(0,T;W^{2,s})} &\leq C, \quad \text{for some } s > 2, \\ \|d\|_{L^\infty(0,T;H^3)} &\leq C. \end{aligned} \quad (11)$$

Remark 2. When $\Omega := \mathbb{R}^2$, Theorem 1 is also correct, thus improving the result in [18], where u_0 and ∇d_0 are assumed to be small.

Next, we consider (1)–(4) with $\rho \equiv 1$ as follows:

$$\operatorname{div} u = 0, \quad (12)$$

$$\partial_t u + u \cdot \nabla u + \nabla \pi - \Delta u = -\nabla \cdot (\nabla d \odot \nabla d), \quad (13)$$

$$\partial_t d + u \cdot \nabla d - \Delta d = |\nabla d|^2 d, \quad (14)$$

$$u = 0, \quad d = d_0 \quad \text{on } \partial\Omega, \quad (15)$$

$$(u, d)(\cdot, 0) = (u_0, d_0) \quad \text{in } \Omega. \quad (16)$$

We will prove the following.

Theorem 3. *Let $u_0 \in L^2$ and $d_0 \in H^1$ with $\operatorname{div} u_0 = 0$ and $|d_0| = 1$ in Ω and $d_0 \in C^{2,\beta}(\partial\Omega)$ for some $\beta \in (0, 1)$. If d satisfies*

$$\nabla d \in L^q(0, T; L^p), \quad \frac{2}{q} + \frac{2}{p} = 1, \quad 2 < p \leq \infty, \quad (17)$$

then the strong solution (u, d) can be extended beyond $T > 0$.

Remark 4. In [9], the authors prove the regularity criterion (9) for the problem (12)–(16), and our condition (17) is weaker than (9). Moreover, (17) is scaling invariant for (12)–(14).

2. Proof of Theorem 1

This section is devoted to the proof of Theorem 1. Since the local-in-time well-posedness has been proved in [16], we only need to establish a priori estimates. Also, by the local well-posedness result in [16], we note that ∇d is absolutely continuous on $[0, T]$ for any given $T > 0$.

By the maximum principle, it follows from (1) and (2) that

$$0 < m \leq \rho \leq M < \infty. \quad (18)$$

Testing (3) by u and using (1) and (2), we see that

$$\frac{1}{2} \frac{d}{dt} \int \rho u^2 dx + \int |\nabla u|^2 dx = - \int (u \cdot \nabla) d \cdot \Delta d dx. \quad (19)$$

Testing (4) by $-\Delta d - |\nabla d|^2 d$, using $|d| = 1$, we find that

$$\frac{1}{2} \frac{d}{dt} \int |\nabla d|^2 dx + \int |\Delta d + |\nabla d|^2 d|^2 dx = \int (u \cdot \nabla) d \cdot \Delta d dx. \quad (20)$$

Summing up (19) and (20) and integrating over $(0, T)$, we get

$$\begin{aligned} \int (\rho u^2 + |\nabla d|^2) dx + 2 \int_0^T \int (|\nabla u|^2 + |\Delta d + |\nabla d|^2 d|) dx dt \\ \leq \int (\rho_0 u_0^2 + |\nabla d_0|^2) dx. \end{aligned} \quad (21)$$

Since $\partial_\nu d = 0$ on $(0, \infty) \times \partial\Omega$, we have the following Gagliardo-Nirenberg inequality:

$$\|\nabla d\|_{L^4}^2 \leq C_0 \|\nabla d\|_{L^2} \|\Delta d\|_{L^2}. \quad (22)$$

By (20) and the Ladyzhenskaya inequality in 2D, we derive

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} \int |\nabla d|^2 dx + \int |\Delta d + |\nabla d|^2 d|^2 dx \\ \leq \|u\|_{L^4} \|\nabla d\|_{L^4} \|\Delta d\|_{L^2} \\ \leq \sqrt{2} \|u\|_{L^2}^{1/2} \|\nabla u\|_{L^2}^{1/2} \cdot \sqrt{C_0} \|\nabla d\|_{L^2}^{1/2} \|\Delta d\|_{L^2}^{3/2} \\ \leq \frac{\|\Delta d\|_{L^2}^2}{8} + 216 C_0^2 \|u\|_{L^2}^2 \|\nabla u\|_{L^2}^2 \|\nabla d\|_{L^2}^2 \\ \leq \frac{\|\Delta d\|_{L^2}^2}{8} + 216 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \|\nabla d_0\|_{L^2}^2 \right) \|\nabla u\|_{L^2}^2 \|\nabla d\|_{L^2}^2. \end{aligned} \quad (23)$$

On the other hand, since $(a + b)^2 \geq (a^2/2) - b^2$, we have

$$\begin{aligned} \int |\Delta d + |\nabla d|^2 d|^2 dx &\geq \frac{\|\Delta d\|_{L^2}^2}{2} - \|\nabla d\|_{L^4}^4 \\ &\geq \frac{\|\Delta d\|_{L^2}^2}{2} - C_0^2 \|\nabla d\|_{L^2}^2 \|\Delta d\|_{L^2}^2. \end{aligned} \quad (24)$$

If the initial data $\|\nabla d_0\|_{L^2}^2 < (1/C_0^2)(1/8)$, then there exists $T_1 > 0$ such that for any $t \in [0, T_1]$,

$$\|\nabla d(t)\|_{L^2}^2 \leq \frac{1}{C_0^2} \cdot \frac{1}{4}. \quad (25)$$

We denote by T_1^* the maximal time such that (25) holds on $[0, T_1^*]$. Therefore, by (23), (24), and (25), it follows that for any $t \in [0, T_1^*]$,

$$\begin{aligned} & \frac{d}{dt} \int |\nabla d|^2 dx + \frac{1}{4} \|\Delta d\|_{L^2}^2 \\ & \leq 432 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \|\nabla d_0\|_{L^2}^2 \right) \|\nabla u\|_{L^2}^2 \|\nabla d\|_{L^2}^2 \quad (26) \\ & \leq 432 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \frac{1}{8C_0^2} \right) \|\nabla u\|_{L^2}^2 \|\nabla d\|_{L^2}^2, \end{aligned}$$

which gives

$$\begin{aligned} & \|\nabla d(t)\|_{L^2}^2 + \frac{1}{4} \int_0^t \|\Delta d(\tau)\|_{L^2}^2 d\tau \\ & \leq \|\nabla d_0\|_{L^2}^2 \exp \left[432 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \frac{1}{8C_0^2} \right) \right. \\ & \quad \left. \times \int_0^{T_1^*} \|\nabla u\|_{L^2}^2 d\tau \right] \quad (27) \\ & \leq \|\nabla d_0\|_{L^2}^2 \exp \left[216 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \frac{1}{8C_0^2} \right)^2 \right] \\ & \leq \frac{1}{8C_0^2}, \end{aligned}$$

which implies that $T_1^* = T$ if the initial data satisfies

$$\|\nabla d_0\|_{L^2}^2 \exp \left[216 \frac{C_0^2}{m} \left(\|\sqrt{\rho_0} u_0\|_{L^2}^2 + \frac{1}{8C_0^2} \right)^2 \right] \leq \frac{1}{8C_0^2}. \quad (28)$$

Let T^* be a maximal existence time for the solution (ρ, u, d) . Then, (18), (21), and (27) ensure that $T^* = \infty$ by continuity argument.

Testing (3) by u_t , using (1), (18), (21), (22), $|d| = 1$, and the Gagliardo-Nirenberg inequalities, we obtain

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} \int |\nabla u|^2 dx + \int \rho u_t^2 dx \\ & = - \int \rho u \cdot \nabla u \cdot u_t dx - \int u_t \cdot \nabla d \cdot \Delta d dx \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2} (\|u\|_{L^4} \|\nabla u\|_{L^4} + \|\nabla d\|_{L^4} \|\Delta d\|_{L^4}) \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2} \left[\|u\|_{L^2}^{1/2} \|\nabla u\|_{L^2} \left(\|\Delta u\|_{L^2}^{1/2} + \|u\|_{L^2}^{1/2} \right) \right. \\ & \quad \left. + \|\nabla d\|_{L^2}^{1/2} \|\Delta d\|_{L^2} \left(\|\nabla \Delta d\|_{L^2}^{1/2} + \|\Delta d\|_{L^2}^{1/2} \right) \right] \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2} \left(\|\nabla u\|_{L^2} \|\Delta u\|_{L^2}^{1/2} + \|\nabla u\|_{L^2} + \|\Delta d\|_{L^2} \right. \\ & \quad \left. \times \|\nabla \Delta d\|_{L^2}^{1/2} + \|\Delta d\|_{L^2} \right). \quad (29) \end{aligned}$$

On the other hand, (3) can be rewritten as

$$-\Delta u + \nabla \pi = f := -\rho u_t - \rho u \cdot \nabla u - \nabla \cdot (\nabla d \odot \nabla d). \quad (30)$$

By the H^2 -theory of Stokes system, we have

$$\begin{aligned} \|\Delta u\|_{L^2} & \leq C \|f\|_{L^2} \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2} + C \|u\|_{L^4} \|\nabla u\|_{L^4} + C \|\nabla d\|_{L^4} \|\Delta d\|_{L^4} \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2} + C \|\nabla u\|_{L^2} \|\Delta u\|_{L^2}^{1/2} + C \|\nabla u\|_{L^2} \\ & \quad + C \|\Delta d\|_{L^2} \|\nabla \Delta d\|_{L^2}^{1/2} + C \|\Delta d\|_{L^2}, \end{aligned} \quad (31)$$

which yields

$$\begin{aligned} \|\Delta u\|_{L^2} & \leq C \|\sqrt{\rho} u_t\|_{L^2} + C \|\nabla u\|_{L^2}^2 + C \\ & \quad + C \|\Delta d\|_{L^2} \|\nabla \Delta d\|_{L^2}^{1/2} + C \|\Delta d\|_{L^2}. \end{aligned} \quad (32)$$

Inserting (32) into (29), we deduce that

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} \int |\nabla u|^2 dx + \int \rho u_t^2 dx \\ & \leq C \|\sqrt{\rho} u_t\|_{L^2}^{3/2} \|\nabla u\|_{L^2} + C \|\sqrt{\rho} u_t\|_{L^2} \left(\|\nabla u\|_{L^2}^2 + \|\nabla u\|_{L^2} \right) \\ & \quad + C \|\sqrt{\rho} u_t\|_{L^2} \|\Delta d\|_{L^2} \|\nabla \Delta d\|_{L^2}^{1/2} + C \|\sqrt{\rho} u_t\|_{L^2} \|\Delta d\|_{L^2} \\ & \leq \frac{1}{8} \|\sqrt{\rho} u_t\|_{L^2}^2 + C \|\nabla u\|_{L^2}^4 + C + \frac{1}{8} \|\nabla \Delta d\|_{L^2}^2 + C \|\Delta d\|_{L^2}^4. \end{aligned} \quad (33)$$

Applying Δ to (4), testing by Δd , using $|d| = 1$, (21) and (22), and the Gagliardo-Nirenberg inequalities, we have

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} \int |\Delta d|^2 dx + \int |\nabla \Delta d|^2 dx \\ & \leq \int |\nabla (|\nabla d|^2 d)| |\nabla \Delta d| dx + \int |\nabla (u \cdot \nabla d)| |\nabla \Delta d| dx \\ & \leq C \left(\|\nabla d\|_{L^6}^3 + \|\nabla d\|_{L^4} \|\Delta d\|_{L^4} + \|u\|_{L^4} \|\Delta d\|_{L^4} \right. \\ & \quad \left. + \|\nabla u\|_{L^2} \|\nabla \Delta d\|_{L^2} \right) \|\nabla \Delta d\|_{L^2} \\ & \leq C \left(\|\nabla d\|_{L^2} \|\Delta d\|_{L^2}^2 + \|\Delta d\|_{L^2} \|\nabla \Delta d\|_{L^2}^{1/2} + \|\Delta d\|_{L^2} \right. \\ & \quad \left. + \|\nabla u\|_{L^2}^{1/2} \|\Delta d\|_{L^2}^{1/2} \|\nabla \Delta d\|_{L^2}^{1/2} \right. \\ & \quad \left. + \|\nabla u\|_{L^2}^{1/2} \|\Delta d\|_{L^2}^{1/2} + \|\nabla u\|_{L^2} \right. \\ & \quad \left. \times \|\nabla d\|_{L^2}^{1/2} \|\nabla \Delta d\|_{L^2}^{1/2} \right) \|\nabla \Delta d\|_{L^2} \\ & \leq \frac{1}{8} \|\nabla \Delta d\|_{L^2}^2 + C \|\Delta d\|_{L^2}^4 + C + C \|\nabla u\|_{L^2}^4. \end{aligned} \quad (34)$$

Here, we have used the Gagliardo-Nirenberg inequalities

$$\begin{aligned} \|\nabla d\|_{L^6}^3 & \leq C \|\nabla d\|_{L^2} \|\Delta d\|_{L^2}^2, \\ \|\nabla d\|_{L^\infty}^2 & \leq \|\nabla d\|_{L^2} \|\nabla \Delta d\|_{L^2}, \end{aligned} \quad (35)$$

$$\|\Delta d\|_{L^4}^2 \leq C \|\Delta d\|_{L^2} \|\nabla \Delta d\|_{L^2} + C \|\Delta d\|_{L^2}.$$

Combining (33) and (34) and using the Gronwall inequality, we have

$$\|u\|_{L^\infty(0,T;H^1)} + \|u\|_{L^2(0,T;H^2)} \leq C, \quad (36)$$

$$\|\sqrt{\rho}u_t\|_{L^2(0,T;L^2)} \leq C, \quad (37)$$

$$\|d\|_{L^\infty(0,T;H^2)} + \|d\|_{L^2(0,T;H^3)} \leq C. \quad (38)$$

Now, by the similar calculations as those in [17], we arrive at

$$\begin{aligned} \|(u_t, \nabla d_t)\|_{L^\infty(0,T;L^2) \cap L^2(0,T;H^1)} &\leq C, \\ \|(u, \nabla d)\|_{L^\infty(0,T;H^2)} &\leq C, \\ \|u\|_{L^2(0,T;W^{2,s})} &\leq C \quad \text{for some } s > 2, \\ \|\rho\|_{L^\infty(0,T;W^{1,r})} &\leq C, \quad \|\rho_t\|_{L^\infty(0,T;L^r)} \leq C. \end{aligned} \quad (39)$$

This completes the proof.

3. Proof of Theorem 3

This section is devoted to the proof of Theorem 3. By the results in [9], we only need to prove (9).

Similar to (21), we still have

$$\begin{aligned} &\int (u^2 + |\nabla d|^2) dx + 2 \int_0^T \int (|\nabla u|^2 + |\Delta d + |\nabla d|^2 d|) dx dt \\ &\leq \int (u_0^2 + |\nabla d_0|^2) dx. \end{aligned} \quad (40)$$

We will use the following Gagliardo-Nirenberg inequalities:

$$\|u\|_{L^{2p/(p-2)}} \leq C \|u\|_{L^2}^{1-(2/p)} \|\nabla u\|_{L^2}^{2/p}, \quad (41)$$

$$\|\nabla d\|_{L^{2p/(p-2)}} \leq C \|\nabla d\|_{L^2}^{1-(2/p)} \|\Delta d\|_{L^2}^{2/p} + C \|\nabla d\|_{L^2}. \quad (42)$$

Testing (14) by $-\Delta d$, using $|d| = 1$, (40), (41), and (42), we have

$$\begin{aligned} &\frac{1}{2} \frac{d}{dt} \int |\nabla d|^2 dx + \int |\Delta d|^2 dx \\ &= \int (u \cdot \nabla d - |\nabla d|^2 d) \Delta d dx \\ &\leq (\|u\|_{L^{2p/(p-2)}} \|\nabla d\|_{L^p} + \|\nabla d\|_{L^p} \|\nabla d\|_{L^{2p/(p-2)}}) \|\Delta d\|_{L^2} \\ &\leq C \|\nabla d\|_{L^p} (\|u\|_{L^2}^{1-(2/p)} \|\nabla u\|_{L^2}^{2/p} + \|\nabla d\|_{L^2} \\ &\quad + \|\nabla d\|_{L^2}^{1-(2/p)} \|\Delta d\|_{L^2}^{2/p}) \|\Delta d\|_{L^2} \\ &\leq C \|\nabla d\|_{L^p} (\|\nabla u\|_{L^2}^{2/p} + 1 + \|\Delta d\|_{L^2}^{2/p}) \|\Delta d\|_{L^2} \\ &\leq \frac{1}{4} \|\Delta d\|_{L^2}^2 + C \|\nabla d\|_{L^p}^2 (\|\nabla u\|_{L^2}^{4/p} + 1 + \|\Delta d\|_{L^2}^{4/p}) \\ &\leq \frac{1}{2} \|\Delta d\|_{L^2}^2 + \|\nabla u\|_{L^2}^2 + C \|\nabla d\|_{L^p}^{2p/(p-2)} + C, \end{aligned} \quad (43)$$

which gives (9).

This completes the proof.

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