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Research Article

Global Stabilization of Nonholonomic Chained Form Systems with Input Delay

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This paper investigates the global stabilization problem for a class of nonholonomic systems in chained form with input delay. A particular transformation is introduced to convert the original time-delay system into a delay-free form. Then, by using input-state-scaling technique and the method of sliding mode control, a constructive design procedure for state feedback control is given, which can guarantee that all the system states globally asymptotically converge to the origin. An illustrative example is also provided to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Nonholonomic systems, which can model many classes of mechanical systems such as mobile robots and wheeled vehicles, have attracted intensive attention over the past decades. However, due to the limitation imposed by Brockett's condition [1], this class of nonlinear systems cannot be stabilized by stationary continuous state feedback, although it is controllable. As a consequence, the well-developed smooth nonlinear control theory and methodology cannot be directly used to such systems. To overcome this obstruction, a novel array of approaches have been generated; see [2–9]. In literatures, three methods are adopted for stabilization of nonholonomic systems, that is, discontinuous time-invariant stabilization [5, 9], smooth time-varying stabilization [3, 6, 7], and hybrid stabilization [4, 7, 8]. By using these methods, the stabilization problem for several classes of nonholonomic systems is solvable [10–17].

On the other hand, sliding mode control (SMC), also known as variable structure control (VSC), in essence, is a special nonlinear control, and its nonlinearity is reflected in the noncontinuity of control. Since the properties and parameters of the SMC just depend on the design of the switching hyperplane and have nothing with the external interferences, the SMC has many advantages such as simple algorithm, fast response, and robustness to external noise and parameter perturbation. During the past few years, the SMC strategy has been also applied to the nonholonomic system control [18–21]. However, it should be noted that the aforementioned results do not consider the effect of input delay. In practice, the delay in the input is often unavoidable due to sensors, calculation, information processing, or transport. Hence, the problem of global feedback stabilization of nonholonomic systems with delay in the input is interesting.

In this paper, we introduce a new class of nonholonomic chained systems with input delay and then study the problem of robust state feedback stabilization for the concerned systems. Since the nonholonomic system considered in this paper contains input delay, therefore it cannot be handled by general existing methods. By composing linear transformation and input-state-scaling techniques with the SMC strategy, a state feedback controller is constructed to guarantee that the states of the closed-loop systems are asymptotically regulated to the origin.

The rest of this paper is organized as follows. In Section 2, preliminary knowledge and the problem formulation are given. Section 3 presents the linear transformation, input-state-scaling technique, and the main results. Section 4 gives a simulation example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 5.

Notations. The following notations are to be used throughout the paper. R^+ denotes the set of all nonnegative real numbers and R^n denotes the real n-dimensional space. For a given vector or matrix X, X^T denotes its transpose and $\|X\|$ is the Euclidean norm of a vector X. To simplify the deduction procedure, sometimes the arguments of the functions will be omitted, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$ or f.

2. Problem Formulation and Preliminaries

Since many mechanical systems with nonholonomic constraints, such as wheeled mobile robot, can be transformed to a kind of nonholonomic systems in the so-called chained form [3], this paper considers the following class of chained systems with input delay:

$$\begin{aligned} \dot{x}_0 &= u_0(t-\tau), \\ \dot{x}_i &= u_0(t-\tau)\, x_{i+1}, \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= u_1(t-\tau), \end{aligned} \tag{1}$$

where $(x_0, x)^T = (x_0, x_1, \ldots, x_n)^T \in R^{n+1}$ and $u = (u_0, u_1)^T \in R^2$ are the system state and control input, respectively, and $\tau \in R^+$ is time delay of the input.

The control objective is to find a state feedback controller which makes the closed-loop system be globally asymptotically regulated at origin.

Before the analysis of system (1), we first introduce the following technical linear transformation, which will be the base of the coming control design and performance analysis.

Consider the following linear system with input delay:

$$\dot{x} = Ax + Bu(t-\tau), \tag{2}$$

where $x \in R^n$ and $u \in R^m$ are the state vector and the control input, respectively, τ is bounded constant delay, and A and B are system matrices with appropriate dimensions.

For system (2) containing the input delay, now we make some transformation that the system with delayed input is transformed into a non-input-delayed system.

Let

$$z = x + \int_{t-\tau}^{t} e^{A(t-\tau-\theta)} Bu(\theta)\, d\theta. \tag{3}$$

Taking the derivative of (3) with respect to time t, we obtain

$$\dot{z} = \dot{x} + A\int_{t-\tau}^{t} e^{A(t-\tau-\theta)} Bu(\theta)\, d\theta + e^{-A\tau} Bu - Bu(t-\tau). \tag{4}$$

Substituting (2) into (4) yields

$$\dot{z} = \overline{A}z + \overline{B}u, \tag{5}$$

where $\overline{A} = A$ and $\overline{B} = e^{-A\tau}B$. If (A, B) is completely controllable, it can be proved that $(\overline{A}, \overline{B})$ is also completely controllable. So the following lemma is obtained.

Lemma 1. *If there exists a state feedback controller in the form $u = Kz$ such that system* (5) *is asymptotically stable, then system* (2) *is also asymptotically stable.*

Proof. From the linear transformation (3), we have

$$\begin{aligned} \|x(t)\| &= \left\| z(t) - \int_{t-\tau}^{t} e^{A(t-\tau-\theta)} Bu(\theta)\, d\theta \right\| \\ &\le \|z(t)\| + \left\| \int_{t-\tau}^{t} e^{A(t-\tau-\theta)} Bu(\theta)\, d\theta \right\| \\ &\le \|z(t)\| + \tau \max_{-\tau\le\theta\le 0} \left\| e^{A\theta} \right\| \|B\| \|u(t+\theta)\| \\ &\le \|z(t)\| + \tau \max_{-\tau\le\theta\le 0} \left\| e^{A\theta} \right\| \|B\| \|K\| \|z(t+\theta)\|. \end{aligned} \tag{6}$$

Since system (5) is asymptotically stable, it follows that

$$\lim_{t\to\infty} z(t) = 0. \tag{7}$$

Putting together (6) and (7), we have

$$\lim_{t\to\infty} x(t) = 0 \tag{8}$$

which means that system (2) is asymptotically stable. This completes the proof of Lemma 1. □

3. Robust Controller Design

In this section, we present a systematic controller design procedure for the system (1). For clarity, the case that $t \in [0, \tau)$ is considered first, while the case that $t \ge \tau$ is dealt with later.

3.1. The Case That $t \in [0,\tau)$. Consider the control input u_0 as

$$u_0 = \lambda_0 \operatorname{sgn}(x_0(0)) + u_0^*, \tag{9}$$

where λ_0 and u_0^* are positive design constants satisfying $\lambda_0 > u_0^*$.

Remark 2. Under the control law (9), the solution of x_0-subsystem can be expressed as $x(t) = (\lambda_0 \operatorname{sgn}(x_0(0)) + u_0^*)t$. By the fact that $\lambda_0 > u_0^*$, we obtain that $x_0(\tau) \ne 0$ and $x_0(t)$ not crossing zero for all $t \in (0, \tau)$ are guaranteed irregardless of the initial value of $x_0(0)$.

Under the control law (9), the x-subsystem is transformed into

$$\begin{aligned} \dot{x}_i &= d_i x_{i+1}, \quad i = 1, \ldots, n-1, \\ \dot{x}_n &= u_1(t-\tau), \end{aligned} \tag{10}$$

where $d_i = \lambda_0 \operatorname{sgn}(x_0(0)) + u_0^*$.

Since the time-delay nonlinear system (10) is the absence of the disturbances, it can be dealt with by many methods. For instance, we can simply choose $u_1 = u_1^*$, where u_1^* is a positive design constant.

3.2. The Case That $t \ge \tau$. The inherent structure of system (1) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

3.2.1. Design u_0 for x_0-Subsystem. For x_0-subsystem, we introduce linear transformation

$$z_0(t) = x_0(t) + \int_{t-\tau}^{t} e^{t-\tau-\theta} u_0(\theta)\, d\theta. \tag{11}$$

So the x_0-subsystem is transformed into

$$\dot{z}_0(t) = e^{-\tau} u_0(t). \tag{12}$$

Obviously, the control input u_0 can be chosen as

$$u_0(t) = -k_0 z_0(t), \tag{13}$$

where k_0 is a positive design constant.

As a result, the following lemma can be easily established by direct calculation.

Lemma 3. *For any initial $t_0 \geq \tau$, the corresponding solution $x_0(t)$ exists for each $t \geq t_0$ and satisfies $\lim_{t\to\infty} x_0(t) = 0$. Furthermore, the control u_0 given by* (13) *does not cross zero for all $t \in [t_0, \infty)$ and satisfies $\lim_{t\to\infty} u_0(t) = 0$.*

Proof. Substituting (13) into (12), we have

$$z_0(t) = z_0(t_0)\, e^{-k_0'(t-t_0)}, \tag{14}$$

where $k_0' = k_0 e^{-\tau}$. Obviously, $z_0(t)$ exponentially tends to zero as $t \to \infty$.

Furthermore, from (11) and (13), we can get

$$x_0(t) = z_0(t) + k_0 \int_{t-\tau}^{t} e^{t-\tau-\theta} z_0(\theta)\, d\theta \tag{15}$$

which together with (14) implies that $x_0(t)$ exists and satisfies

$$\lim_{t\to\infty} x_0(t) = 0. \tag{16}$$

Since (14) implies that $z_0(t)$ does not cross zero for all $t \in [t_0, \infty)$, from this and (13), we have that the u_0 does not cross zero for all $t \in [t_0, \infty)$ and satisfies $\lim_{t\to\infty} u_0(t) = 0$.

Hence, we can see that the $u_0(t-\tau)$ also does not cross zero for all $t \in [t_0 - \tau, \infty)$ and $\lim_{t\to\infty} u_0(t-\tau) = 0$ independent of the x-subsystem. □

3.2.2. Input-State-Scaling Transformation. The design in Section 3.2.1 can ensure that x_0-state in (1) can be globally regulated to zero via u_0 in (13) as $t \to \infty$. However, it is troublesome in controlling the x-subsystem via the control input u_1, because, in the limit (i.e., $u_0 = 0$), the x-subsystem is uncontrollable. This problem can be avoided by utilizing the following discontinuous input-state-scaling transformation:

$$y_i = \frac{x_i}{u_0^{n-i}(t-\tau)}, \quad 1 \leq i \leq n. \tag{17}$$

Under the new y-coordinates, the x-subsystem is transformed into

$$\begin{aligned} \dot{y}_i &= y_{i+1} + (n-i)\, k_0 e^{-\tau} y_i, \\ \dot{y}_n &= u_1(t-\tau). \end{aligned} \tag{18}$$

The differential equations (18) can be rewritten into the compact form

$$\dot{y} = Ay + Bu_1(t-\tau), \tag{19}$$

$$A = \begin{pmatrix} k_1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & k_2 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & k_3 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \vdots & \vdots & \ddots & k_{n-1} & 1 \\ 0 & 0 & \cdots & \cdots & 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \tag{20}$$

where $k_i = (n-i) k_0 e^{-\tau}$.

Obviously, (A, B) is completely controllable. Now, we introduce linear transformation

$$z = y + \int_{t-\tau}^{t} e^{A(t-\tau-\theta)} Bu_1(\theta)\, d\theta. \tag{21}$$

Putting together (19) and (21) yields

$$\dot{z} = \overline{A} z + \overline{B} u_1, \tag{22}$$

where $\overline{A} = A$ and $\overline{B} = e^{-A\tau} B$. Furthermore by introducing the linear transformation

$$\eta = e^{A\tau} z \tag{23}$$

we have

$$\dot{\eta} = \widetilde{A}\eta + \widetilde{B} u_1, \tag{24}$$

where

$$\widetilde{A} = e^{A\tau} A e^{-A\tau} = \begin{pmatrix} C & d \\ e & f \end{pmatrix}, \qquad \widetilde{B} = B. \tag{25}$$

Let $\eta_{[n-1]} = (\eta_1, \ldots, \eta_{n-1})$; thus the system (24) can be rewritten as

$$\begin{aligned} \dot{\eta}_{[n-1]} &= C\eta_{[n-1]} + d\eta_n, \\ \dot{\eta}_n &= e\eta_{[n-1]} + f\eta_n + u_1. \end{aligned} \tag{26}$$

In terms of Lemma 1 and the equivalence property of linear transformation, we obtain that the control problem for system (18) with delayed control is transformed into a control problem for delay-free system (26).

3.2.3. Design u_1 by Using SMC Technique. To fulfill the controller design of u_1, we choose the switching function as

$$s = \eta_n. \tag{27}$$

Control input u_1 in system (26) should be appropriately designed such that the state can be driven to the sliding surface. The SMC law in this paper is derived as follows:

$$u_1 = -e\eta_{[n-1]} - f\eta_n - \beta, \tag{28}$$

where $\beta > 0$ is a constant.

The above proposed control scheme will drive the state to approach the sliding mode surface $s = 0$ in a finite time, and it is stated in the following lemma.

Lemma 4. *Under the control law* (28)*, the trajectories of the system* (26) *converge to the sliding surface* $s = 0$ *in a finite time.*

Proof. From (26), (27), and (28), we have

$$\dot{s} = -\beta. \tag{29}$$

Letting $V_1 = s^2/2$, it follows from (29) that

$$\dot{V}_1 = s\dot{s} = -\beta s \tag{30}$$

from which we prove the finite time convergence of system (26) toward the surface $s = 0$. Thus, the proof is completed. □

Substituting (28) into the system (26), the sliding mode dynamics can be obtained as follows:

$$\dot{\eta}_{[n-1]} = C\eta_{[n-1]}. \tag{31}$$

In the following lemma, the sufficient condition for the asymptotic stability of the system (31) is given.

Lemma 5. *If there exists a positive definite matrix* P *such that the following inequality holds:*

$$C^T P + PC < 0, \tag{32}$$

then system (26) *and* (28) *is globally asymptotically stable.*

Proof. It is straightforward and thus is omitted here. □

Based on the input-state-scaling transformation and Lemmas 1–5, the main theorem of our paper can be summarized here.

Theorem 6. *If there exists a positive definite matrix* P *such that* (32) *holds, then system* (1) *is globally asymptotically regulated at origin by the proposed control design procedure together with the above switching control strategy.*

Considering the robustness of SMC, the following result is a slight extension of Theorem 6.

Corollary 7. *For the uncertain chained system with input delay*

$$\begin{aligned} \dot{x}_0 &= u_0(t-\tau), \\ \dot{x}_i &= u_0(t-\tau)\, x_{i+1}, \\ \dot{x}_n &= u_1(t-\tau) + g, \\ i &= 1, \ldots, n-1 \end{aligned} \tag{33}$$

there is a continuous function $\overline{g} > 0$ *such that* $|g| \le \overline{g}$*. Then, global asymptotic stabilization of* (33) *is achievable by the state feedback controllers of the form the* (9), (13), *and* $u_1 = -e\eta_{[n-1]} - f\eta_n - \overline{g} - \beta$.

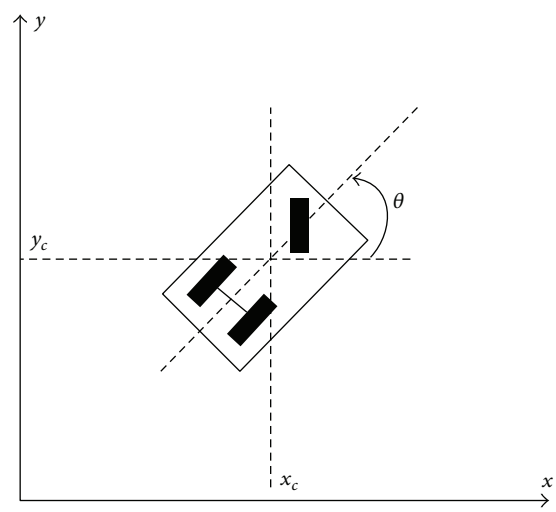


Figure 1: Schematic of the mobile robot.

4. Simulation Example

Consider a car-like mobile robot as shown in Figure 1. The kinematic model of the mobile robot can be written as

$$\begin{aligned} \dot{x}_c &= v\cos\theta, \\ \dot{y}_c &= v\sin\theta, \\ \dot{\theta} &= \omega, \end{aligned} \tag{34}$$

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity and ω is the angular velocity, of the robot.

Using the same modeling method in [22], the first-order approximation of system (34) near the origin is given by

$$\begin{aligned} \dot{x}_l &= v, \\ \dot{y}_l &= v\theta_l, \\ \dot{\theta}_l &= \omega, \end{aligned} \tag{35}$$

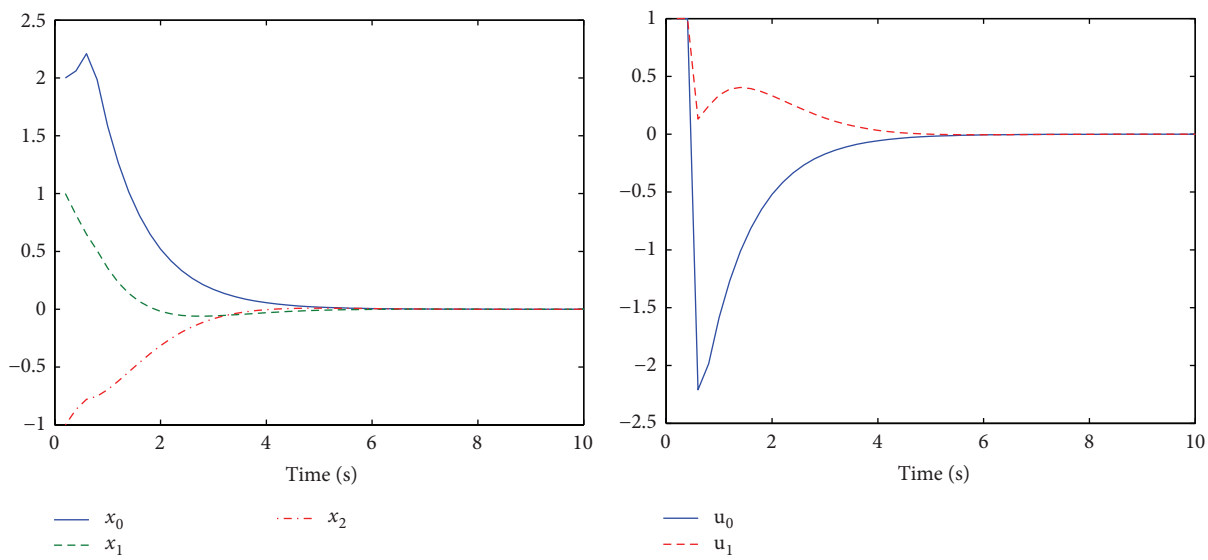
where (x_l, y_l, θ_l) stands for the state of the locally approximate model (35).

Considering that the delay in the input is often unavoidable due to sensors, calculation, information processing, or transport, here we assume that the forward velocity v and the angular velocity ω are subject to some time delay τ; therefore the above plant model is transformed into

$$\begin{aligned} \dot{x}_l &= v(t-\tau), \\ \dot{y}_l &= v(t-\tau)\,\theta_l, \\ \dot{\theta}_l &= \omega(t-\tau). \end{aligned} \tag{36}$$

By taking the following state and input transformation:

$$x_0 = x_l, \qquad x_1 = y_l, \qquad x_2 = \theta_l, \qquad u_0 = v, \qquad u_1 = \omega, \tag{37}$$



(a) System states (b) Control inputs

Figure 2: Transient responses of the closed system.

we obtain

$$\begin{aligned} \dot{x}_0 &= u_0(t-\tau), \\ \dot{x}_1 &= u_0(t-\tau)\, x_2, \\ \dot{x}_2 &= u_1(t-\tau). \end{aligned} \tag{38}$$

Clearly, system (38) is a simple form of (1). Hence our proposed control design procedure is straightforward to apply.

Assume that $\tau = 0.1$ and the design parameters are chosen as $\lambda_0 = k_1 = -1$, respectively. The simulation results for initial condition $(x_0(0), x_1(0), x_2(0)) = (2, 1, -1)$ are shown in Figure 2. From the figure, it is clear that all the closed-loop system states converge to zero, as well as the designed controller.

5. Conclusion

In this paper, we consider the global stabilization problem for a class of nonholonomic systems in chained form with input delay via state feedback. First, a particular linear transformation is introduced to convert the original time-delay system into a delay-free form. Then, by using input-state-scaling technique and the SMC approach to design control laws, global asymptotic regulation of the closed-loop system is guaranteed. Simulation results demonstrate the effectiveness of the proposed scheme.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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