Finite-Time Control for Markovian Jump Systems with Polytopic Uncertain Transition Description and Actuator Saturation

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The problem of finite-time $L_2$-$L_\infty$ control for Markovian jump systems (MJS) is investigated. The systems considered time-varying delays, actuator saturation, and polytopic uncertain transition description. The purpose of this paper is to design a state feedback controller such that the system is finite-time bounded (FTB) and a prescribed $L_2$-$L_\infty$ disturbance attenuation level during a specified time interval is guaranteed. Based on the Lyapunov method, a linear matrix inequality (LMI) optimization problem is formulated to design the delayed feedback controller which satisfies the given attenuation level. Finally, illustrative examples show that the proposed conditions are effective for the design of robust state feedback controller.

1. Introduction

In the aspect of modeling practical systems with abrupt random changes, such as manufacturing system, telecommunication, and economic systems, MJS have powerful ability. MJS have been extensively studied during the past decades and many systematic results have been obtained [1–3]. The peak-to-peak filtering problem was studied for a class of Markov jump systems with uncertain parameters in [4]. A robust $H_2$ state feedback controller for continuous-time Markov jump linear systems subject to polytopic-type parameter uncertainty was designed in [5]. In [6], the authors address the stabilization problem for single-input Markov jump linear systems via mode-dependent quantized state feedback for control.

Actuator saturation which can lead to poor performance of the closed-loop system is another active research area. In practical situations, it may be encountered sometimes. How to preserve the closed-loop system performance in the case of actuator saturation would be more meaningful. In [7], the $H_\infty$ control problem for discrete-time singular Markov jump systems with actuator saturation was considered. In [8] the stochastic stabilization problem for a class of Markov jump linear systems subject to actuator saturation was considered.

In some practical applications, the behavior of the system over a finite-time interval is mainly considered. Finite-time stable (FTS) and Lyapunov asymptotic stability are independent concepts. The concept of FTS was first introduced in [9]. A system is said to be finite-time stable if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval. FTS of linear time-varying systems was considered in [10]. Sufficient conditions for the solvability of both the state and the output feedback problems are stated. Amato [11] provided a necessary and sufficient condition for the FTS of linear-varying systems with jumps. Recently, robust finite-time $H_\infty$ control of jump systems was dealt with in [12–14]. In [15], the problems of finite-time stability analysis were investigated for a class of Markovian switching stochastic systems. To the best of authors’ knowledge, however, the problem of finite-time $L_2$-$L_\infty$ performance for discrete-time MJS with imprecise transition probabilities and time-varying delays has not been well addressed, which motivates our work.

This paper deals with this problem. More specifically, the actuator is saturation. By using the Lyapunov-Krasovskii functional, a new sufficient condition for stochastic asymptotic stability with finite-time $L_2$-$L_\infty$ performance is derived in terms of LMI. Based on this, the existence condition of
the desired performance which guarantees finite-time stability and an $L_2-L_{\infty}$ performance of the MJS is presented. A numerical example is provided to show the effectiveness of the proposed results.

Throughout the paper, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $W > (\geq, <, \leq) 0$ is used to denote a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix. $\lambda_{\text{min}}(\cdot)$ and $\lambda_{\text{max}}(\cdot)$ represent the minimum and maximum eigenvalues of the corresponding matrix, respectively. $I$ is the identity matrix with compatible dimensions. The notation $\| \cdot \|$ refers to the Euclidean norm of vectors and $E[\cdot]$ stands for the mathematical expectation. For a symmetric block matrix, “*” is used as an ellipsis for the terms that are obtained by symmetry.

2. Problem Statement and Preliminaries

Consider a discrete-time MJS with actuator saturation and delay in the state. Let the system dynamics be described by the following:

$$x(k + 1) = A_{\text{q}}(r_k)x(k) + A_{\text{q2}}(r_k)x(k - d) + B_{\text{b1}}(r_k)\sigma(u_k) + B_{\text{q2}}(r_k)w_k,$$

$$z(k) = C_{\text{q1}}(r_k)x(k) + C_{\text{q2}}(r_k)x(k - d) + D_{\text{q1}}(r_k)w_k,$$

(1)

where $x_k \in \mathbb{R}^n$ is the system state, $z_k \in \mathbb{R}^n$ is the system output, $u_k \in \mathbb{R}^m$ is the control input, $w_k \in \mathbb{R}^l$ is the disturbance input which belongs to $L_2[0, \infty)$ and $\sum_{k=0}^{\infty} w_k^T w_k < \kappa^2$, and $\kappa$ is a given positive scalar. $A_{\text{q}}(r_k), A_{\text{q2}}(r_k), B_{\text{b1}}(r_k), B_{\text{q2}}(r_k), C_{\text{q1}}(r_k), C_{\text{q2}}(r_k), D_{\text{q1}}(r_k),$ and $D_{\text{q2}}(r_k)$ are appropriately dimensioned real-valued matrices, which belong to the part of convex polyhedron $\Phi(r_k)$:

$$\Phi(r_k) = \left\{ \sum_{l=1}^{L} \theta_l \begin{bmatrix} A_{\text{q1}}(r_k), A_{\text{q2}}(r_k), B_{\text{b1}}(r_k), B_{\text{q2}}(r_k), C_{\text{q1}}(r_k), C_{\text{q2}}(r_k), D_{\text{q1}}(r_k), D_{\text{q2}}(r_k) \end{bmatrix}, \sum_{l=1}^{L} \theta_l = 1, \theta_l \geq 0 \right\},$$

(2)

where $A_{\text{q1}}(r_k), A_{\text{q2}}(r_k), B_{\text{b1}}(r_k), B_{\text{q2}}(r_k), C_{\text{q1}}(r_k), C_{\text{q2}}(r_k), D_{\text{q1}}(r_k),$ and $D_{\text{q2}}(r_k)$ are matrix functions of the random jumping process $\{r_k\}$ (Figure 1), which is a discrete-time Markov chain taking values in a finite set $\Omega = \{1, 2, \ldots, S\}$ with transition probabilities:

$$P\{r_{k+1} = j \mid r_k = i\} = \pi_{ij},$$

(3)

Here $\pi_{ij} \geq 0$ and for any $i, j \in \Omega$, $\sum_{j=1}^{S} \pi_{ij} = 1$. Assuming that the transition probability $\pi_{ij}$ is not exactly known, a certain range can only be given

$$[\pi_{i}(i, 1), \pi_{i}(i, 2), \ldots, \pi_{i}(i, S)]$$

$$= \sum_{m=1}^{M} v_m [\pi_{m}(i, 1), \pi_{m}(i, 2), \ldots, \pi_{m}(i, S)],$$

(4)

where $v = [v_1 \cdots v_M]^T \in \mathbb{R}^M$ and $\sum_{m=1}^{M} v_m = 1$, and the transition probability belongs to the following convex polyhedron:

$$N(r_k = i) = \text{Co} \left\{ \begin{bmatrix} \pi_{i}(i, 1), \pi_{i}(i, 2), \ldots, \pi_{i}(i, N) \\ \pi_{M}(i, 1), \pi_{M}(i, 2), \ldots, \pi_{M}(i, N) \end{bmatrix}, \right. \left. \sum_{m=1}^{M} \pi_m = 1 \right\}.$$ 

(5)

When the system operates in the $i$th mode ($r_k = i$), for simplicity, the matrices $A_{\text{q}}(r_k), A_{\text{q2}}(r_k), B_{\text{b1}}(r_k), B_{\text{q2}}(r_k), C_{\text{q1}}(r_k), C_{\text{q2}}(r_k),$ and $D_{\text{q1}}(r_k)$ are denoted as $A_{\text{q1}}(r_k), A_{\text{q2}}(r_k), B_{\text{b1}}(r_k), B_{\text{q2}}(r_k), C_{\text{q1}}(r_k),$ and $D_{\text{q1}}(r_k)$, respectively, $d$ is a positive integer denoting the constant delay of the system state (Figures 2 and 3).

In system (1), $\sigma(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is the vector-valued standard saturation function defined as follows:

$$\sigma(u) = [\sigma(u_1), \sigma(u_2), \ldots, \sigma(u_m)]^T,$$

(6)

where $\sigma(u_0) = \text{sign}(u_0) \min[1, \|u_0\|]$. It is assumed that system (1) is completely controllable. A mode-dependent controller is considered here with the following form:

$$\sigma(u(k)) = \sigma(K_{j}x(k)),$$

(7)

where $K_j \in \mathbb{R}^{m \times m}$ $(\forall r_k = i \in \Omega)$ is the controller gain to be determined.

Let $M$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. Suppose each element of $M$ is $M_{j}$, $j = 1, \ldots, 2^m$, and denote $M'_{j} = I - M_{j}$. Note that $M'_{j}$ is also an element of $M$ if $M_{j} \in M$. Let $h_{ij}$ be the $j$th row of the matrix $H_{ij}$, and define the symmetric polyhedron by $\Phi(H_{ij}) = \{x(t) \in \mathbb{R}^n : |f_{ij}x(t)| \leq 1, i = 1, 2, \ldots, m\}$. 

![Figure 1: Jumping mode.](image-url)
Lemma 1 (see [8]). Let $K_i, H_i \in \mathbb{R}^{m \times n}$ be given matrix. For $x(t) \in \mathbb{R}^n$, if $x(t) \in \varphi(H_i)$, then

$$\sigma(K_i x(t)) = \sum_{r=1}^{2^n} \zeta_r (M_r K_i + M_r^{-1} H_i) x(t),$$

where $0 \leq \zeta_r \leq 1$, $\sum_{r=1}^{2^n} \zeta_r = 1$.

By the connection of (6), (7) and (8), the following closed-loop MJS are obtained:

$$x(k+1) = \left( A_{\theta_1} (r_k) + B_{\theta_1} (r_k) \right) x(k)$$
$$+ \sum_{r=1}^{2^n} \zeta_r (M_r K_i + M_r^{-1} H_i) x(k)$$
$$+ A_{\theta_2} (r_k) x(k-d) + B_{\theta_2} (r_k) w_k.$$  (9)

Lemma 5. System (1) with $\sigma(u_k) \equiv 0$ is stochastic FTB with respect to $(h_1, h_2, T, R_i)$; if for scalars $\zeta \geq 1$, $h_1 > 0$, and $h_2 > 0$, there exist symmetric matrices $R_i > 0 (i \in \Omega)$

$$\|z(k)\|_\infty < \gamma \|w(k)\|_2$$  (13)

for all nonzero $w(k) \in L_2[0, \infty)$ subject to the zero-initial condition.

3. Main Results

In this section, firstly stochastic FTB analysis of nominal time-delay MJS (1) is provided. Then, these results will be extended to the MJS (1) with actuator saturation and uncertain transition probability. LMI conditions are established.

Lemma 5. System (1) with $\sigma(u_k) \equiv 0$ is stochastic FTB with respect to $(h_1, h_2, T, R_i)$; if for scalars $\zeta \geq 1$, $h_1 > 0$, and $h_2 > 0$, there exist symmetric matrices $R_i > 0 (i \in \Omega)$
and \( Q_i > 0 \) \((i \in \Omega)\), such that the following matrix inequalities hold:

\[
\Lambda = \begin{bmatrix}
A^T_{ii} \bar{P}_i A_{ii} - P_i + Q & * & * \\
A^T_{ij} \bar{P}_i A_{ji} - Q + A^T_{ij} \bar{P}_i A_{ji} & * \\
B^T_{ij} \bar{P}_i A_{ji} & B^T_{ij} \bar{P}_i A_{ji} & B^T_{ij} \bar{P}_i B_{ij} - I
\end{bmatrix} < 0
\]

\( \zeta^k [\lambda_{\max}(\bar{P}_r(0)) + \lambda_{\max}(Q) \cdot d] c_1 \leq c_2 \cdot \lambda_{\min}(\bar{P}_r) \),

where \( \bar{P}_i = \sum_{j=1}^{S} \pi_{ij} P_j \).

Proof. Choose the following Lyapunov functional:

\[
V(k) = x^T(k) P_i x(k) + \sum_{n=1}^{k-d} x^T(n) Q x(n).
\]

The proof of Lemma 5 is divided into two parts. In the first part, the following inequality is obtained:

\[
E[V(k)] < \zeta^k E[V(0)] + \zeta^k w^T(k) w(k).
\]

Then, we compute

\[
\Delta V(k) = E[V(k+1)] - V(k)
\]

\[
= \sum_{j=1}^{S} \pi_{ij} x^T(k+1) P_j x(k+1) - x^T(k) P_j x(k) + \sum_{j=1}^{S} \pi_{ij} x^T(k) Q x(k) - x^T(k-d) Q x(k-d)
\]

\[
= \zeta^T(k) \begin{bmatrix}
A^T_{ii} \bar{P}_i A_{ii} - P_i + Q & * & * \\
A^T_{ij} \bar{P}_i A_{ji} - Q + A^T_{ij} \bar{P}_i A_{ji} & * \\
B^T_{ij} \bar{P}_i A_{ji} & B^T_{ij} \bar{P}_i A_{ji} & B^T_{ij} \bar{P}_i B_{ij}
\end{bmatrix} \Theta(k)
\]

where \( \Theta(k) = [x(k) \ x(k-d) \ w(k)] \).

Note condition (14); it follows that

\[
E[V(k+1)] - V(k) < (\zeta - 1) V(k) + w^T(k) w(k), \quad \zeta \geq 1.
\]

Therefore, we obtain that

\[
E[V(k+1)] < \zeta V(k) + w^T(k) w(k).
\]

That is,

\[
E[V(x(1), r(1))] < \zeta V(x(0), r(0)) + w^T(k) w(k)
\]

\[
E[V(x(k+1), r(k+1))] < \zeta E[V(x(k), r(k))] + w^T(k) w(k).
\]

By recursive,

\[
E[V(k)] < \zeta^k E[V(0)] + \sum_{t=0}^{k-1} \zeta^{k-t-1} w^T(r) w(r).
\]

Then the inequality in (17) is obtained.
By (23) and (25), we know
\[
E \left\{ x^T(k)R_x(k) \right\} \\
\leq \frac{\zeta^k \left[ \{ \lambda_{\text{max}}(\tilde{P}_{(0)}) + \lambda_{\text{max}}(Q) \cdot d \} + 1 \right] h_1}{\lambda_{\text{min}}(\tilde{P}_1)} \\
\leq h_2.
\]

This completes the proof. \(\square\)

**Theorem 6.** System (1) is finite-time \(L_2-L_{\infty}\) control and satisfies the given lever \(\gamma\) with respect to \((h_1, h_2, d, R_i, N)\); if for scalars \(\zeta \geq 1, h_1 > 0, \) and \(h_2 > 0, \) there exist symmetric matrices \(R_i > 0 \) \((i \in \Omega)\) and \(Q > 0, \) such that the following matrix inequalities hold:

\[
\begin{align*}
\Theta_1 &= \\
&= \begin{bmatrix}
A_{11i}^T \tilde{P}_i A_{11i} - P_i + Q & * & * \\
A_{12i}^T \tilde{P}_i A_{12i} & -Q + A_{12i}^T \tilde{P}_i A_{12i} & * \\
B_{12i}^T \tilde{P}_i A_{12i} & B_{12i}^T \tilde{P}_i A_{12i} & B_{12i}^T \tilde{P}_i A_{12i} - I
\end{bmatrix} < 0, \\
\Theta_2 &= \\
&= \begin{bmatrix}
-P_i & * & * & * & * \\
0 & -Q & * & * & * \\
0 & 0 & 0 & 0 & 0 \\
C_{11i} & C_{12i} & D_{12i} & -\gamma^2 I
\end{bmatrix}
\end{align*}
\]

\[
\zeta^k \left[ \lambda_{\text{max}}(\tilde{P}_{(0)}) + \lambda_{\text{max}}(Q) \cdot d \right] c_1 \leq c_2 \cdot \lambda_{\text{min}}(\tilde{P}_1).
\]

**Proof.** System (1) with \(\sigma(u_k) \equiv 0\) is FTB according to Lemma 5 and inequality (27).

Subsequently, to establish the energy-to-peak performance for the system (1), assume that the initial values for the plant are zeros and consider the following function:

\[
\mathbb{N} := E \left\{ V(k) \right\} - \sum_{i=0}^{k-1} w_i^T w_i.
\]

For any nonzero \(w_k \in l_2(0, \infty)\) and \(k > 0, \) it follows from (18) that

\[
\mathbb{N} := E \left\{ \sum_{i=0}^{k-1} \Delta V(i) - \sum_{i=0}^{k-1} w_i^T w_i \right\}
= \Theta_1 \Theta_2 \epsilon(k).
\]

It follows from (27) that \(E[V(k)] < \sum_{i=0}^{k-1} w_i^T w_i.\)

For all the time instants \(k > 0, \) the expectation of the output can be evaluated as

\[
E \left\{ z_k^T z_k \right\} = E \left\{ \Theta_1 \Theta_2 \epsilon(k) \right\}
\]

**Theorem 7.** Consider the uncertain time-delay system (1); there exists a state feedback controller \(\sigma(K,S(t))\) such that the uncertain time-delay system (1) is finite-time \(L_2-L_{\infty}\) control with respect to \((h_1, h_2, d, R_i, N)\), if the following LMIs hold:

\[
\begin{align*}
\Lambda_1 &= \begin{bmatrix}
-X_i & 0 & \cdots & 0 & \epsilon_{i1} & \epsilon_{i2} \\
0 & \epsilon_{i2} & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
0 & 0 & \cdots & 0 & \epsilon_{i44} & 0 \\
\end{bmatrix} < 0, \\
\Lambda_2 &= \begin{bmatrix}
-P_i & * & * & * & * & * \\
0 & -Q & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 \\
C_{11i} & C_{12i} & D_{12i} & -\gamma^2 I
\end{bmatrix} < 0,
\end{align*}
\]

where \(\epsilon_{i4} = \sqrt{\tilde{P}_{11i}^T (A_{11i} + B_{11i}(M_Y + M_r Z_i)) \tilde{P}_{11i}}, \epsilon_{i44} = \sqrt{\tilde{P}_{11i}^T (A_{11i} + B_{11i}(M_Y + M_r Z_i))} , \epsilon_{26} = \sqrt{\tilde{P}_{26}^T R_{12i}},\)

\(\epsilon_{66} = \sqrt{\tilde{P}_{66}^T R_{12i}}, \) and \(\epsilon_{26} = \sqrt{\tilde{P}_{26}^T R_{12i}},\)

and \(\epsilon_{66} = \sqrt{\tilde{P}_{66}^T R_{12i}},\)

The state feedback controller is designed as \(\sigma(K_S(t)) = \sum_{i=1}^{2m} \zeta_i (M_i, K_i + M_r H_i) x(t).\)

**Proof.** Noting condition (27) and \(\tilde{P}_i = \Gamma_i \phi_i^T, \) where \(\kappa = \text{diag}(P_1, \ldots, P_5)\), \(\Gamma_i = [\sqrt{\tilde{P}_{11i}^T I}, \sqrt{\tilde{P}_{12i}^T I}]\) thus \(\Theta_1\) can be rewritten as

\[
\Theta_1 = \begin{bmatrix}
-P_i + Q & * & * & * & * & * \\
0 & -Q & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 \\
C_{11i} & C_{12i} & D_{12i} & -\gamma^2 I
\end{bmatrix}
\]

\[
\kappa [A_{11i} A_{12i} B_{12i}] < 0.
\]
Using Schur complement, it can be obtained
\[
\begin{bmatrix}
-\mathbf{P} + \mathbf{Q} & 0 & 0 & \mathbf{A}_1^T & \mathbf{Z}_i \\
* & -\mathbf{Q} & 0 & \mathbf{A}_2^T & \mathbf{Z}_i \\
* & * & -\mathbf{I} & \mathbf{B}_{12}^T & \mathbf{Z}_i \\
* & * & * & -\mathbf{K}^{-1} & \mathbf{Z}_i \\
\end{bmatrix}
< 0. \quad (35)
\]

Let \( \mathbf{X}_i = \mathbf{P}_i^{-1}, \mathbf{R} = \mathbf{Q}_i^{-1}, \mathbf{Y}_i = \mathbf{K}_i \mathbf{X}_i, \) and \( \mathbf{Z}_i = \mathbf{H}_i \mathbf{X}_i. \) Pre-and postmultiplying (35) by \( \text{diag} \{ \mathbf{X}_i, \mathbf{R}, \mathbf{I}, \mathbf{I} \} \) and then using Schur complement, then inequality (32) is obtained. Implying Theorem 6, we can conclude that the corresponding closed-loop system is finite-time \( L_2-L_\infty \) control. This completes the proof. \( \square \)

4. Numerical Example

To illustrate the proposed results, a numerical example is considered for finite-time \( L_2-L_\infty \) control. The system is described by (1) and assumed to have two modes; \( \Omega = \{1, 2\}. \) The mode switching is governed by a Markov chain that has the following transition probability matrix:
\[
\mathbf{P} = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}. \quad (36)
\]
The system matrices are as follows:
\[
\mathbf{A}_{111} = \mathbf{A}_{211} = \begin{bmatrix} 0.3 & 0.102 \\ -0.663 & 0.3 \end{bmatrix},
\]
\[
\mathbf{A}_{112} = \mathbf{A}_{212} = \begin{bmatrix} 0.8 & 0.0539 \\ -0.8655 & 0.8 \end{bmatrix},
\]
\[
\mathbf{A}_{121} = \mathbf{A}_{221} = \begin{bmatrix} 0.5 & 0.06 \\ -0.843 & 0.5 \end{bmatrix},
\]
\[
\mathbf{A}_{122} = \mathbf{A}_{222} = \begin{bmatrix} 0.9 & 0.0766 \\ -0.7661 & 0.9 \end{bmatrix},
\]
\[
\mathbf{B}_{111} = \mathbf{B}_{211} = \begin{bmatrix} 0.0005 \\ 0.0539 \end{bmatrix},
\]
\[
\mathbf{B}_{212} = \mathbf{B}_{112} = \begin{bmatrix} 0.005 \\ 0.1078 \end{bmatrix},
\]
\[
\mathbf{B}_{121} = \mathbf{B}_{221} = \begin{bmatrix} 0.0045 \\ 0.0539 \end{bmatrix},
\]
\[
\mathbf{B}_{122} = \mathbf{B}_{222} = \begin{bmatrix} 0.0045 \\ 0.1078 \end{bmatrix},
\]
\[
\mathbf{C}_{111} = \mathbf{C}_{211} = \mathbf{C}_{112} = \mathbf{C}_{212} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix},
\]
\[
\mathbf{C}_{121} = \mathbf{C}_{221} = \mathbf{C}_{122} = \mathbf{C}_{222} = \begin{bmatrix} 0.3 & 0 \end{bmatrix},
\]
\[
\mathbf{D}_{111} = \mathbf{D}_{211} = \mathbf{D}_{112} = \mathbf{D}_{212} = 0.3.
\]

Assume \( L_2-L_\infty \) performance of level \( \gamma = 0.3; \) by applying Theorem 7, we can explicitly compute the optimally achievable closed-loop \( L_2-L_\infty \) performance \( \gamma \) from Theorem 7 as
\[
\gamma = 0.2056. \]

5. Conclusion

The problem of finite-time \( L_2-L_\infty \) control for MJS has been studied. By using the Lyapunov functional approach, a sufficient condition is derived such that the closed-loop MJS are stochastic FTB and satisfy the given level. The controller can be obtained by using the exiting LMI optimization techniques. Finally, numerical and simulation results demonstrate the effectiveness of the results of the paper.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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