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*Research Article*

# Two New Types of Fixed Point Theorems in Complete Metric Spaces

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We introduce two new types of fixed point theorems in the collection of multivalued and single-valued mappings in complete metric spaces.

## 1. Introduction

Let $T$ be a mapping on a complete (or compact) metric space $(X, d)$. We do not assume richer structure such as convex metric spaces and Banach spaces. There are thousands of theorems which assure the existence of a fixed point of $T$. We can categorize these theorems into the following four types.

(T1) Leader type [1]: $T$ has a unique fixed point and $\{T^n x\}$ converges to the fixed point for all $x \in X$. Such a mapping is called a *Picard operator* in [2].

(T2) Unnamed type: $T$ has a unique fixed point and $\{T^n x\}$ does not necessarily converge to the fixed point.

(T3) Subrahmanyam type [3]: $T$ may have more than one fixed point and $\{T^n x\}$ converges to a fixed point for all $x \in X$. Such a mapping is called a *weakly Picard operator* in [3, 4].

(T4) Caristi type [5, 6]: $T$ may have more than one fixed point and $\{T^n x\}$ does not necessarily converge to a fixed point.

We know that most of the theorems such as Banach's [7], Ćirić's [8], Kannan's [9], Kirk's [10], Matkowski's [11], Meir and Keeler's [12], and Suzuki's [13, 14] belong to $(T1)$. Also, very recently, Suzuki [15] characterized $(T1)$. Subrahmanyam's theorem [3] belongs to $(T3)$, and Caristi's theorem [5, 6] and its generalizations [15–17] belong to $(T4)$. On the other hand, as far as the authors do know, there are no theorems belonging to $(T2)$; see Kirk's survey [18]. Also, recently many interesting fixed point theorems are proved in the framework of ordered metric spaces; see [18–35] and others.

In this paper, motivated by the above, we introduce two new types of fixed point theorems in the collection of multivalued and single-valued mappings and will prove them, which belong to $(T3)$.

Let $(X, d)$ be a metric space, and let $P_{\mathrm{cl,bd}}(X)$ denote the class of all nonempty, closed, and bounded subsets of $X$. Let $T : X \to P_{\mathrm{cl,bd}}(X)$ be a multivalued mapping on $X$. A point $x \in X$ is called a fixed point of $T$ if $x \in Tx$. Set $\mathrm{Fix}(T) = \{x \in X : x \in Tx\}$.

A famous theorem on multivalued mappings is due to Nadler [36], which extended the Banach contraction principle to multivalued mappings. Many authors have studied the existence and uniqueness of strict fixed points for multivalued mappings in metric spaces; see, for example, [37–44] and references therein.

Let $H$ be the Hausdorff metric on $P_{\mathrm{cl,bd}}(X)$ induced by $d$; that is,

$$H(A, B) := \max\left\{\sup_{x\in B} d(x, A), \sup_{x\in A} d(x, B)\right\}, \quad A, B \in P_{\mathrm{cl,bd}}(X). \tag{1}$$

Denote $\delta(x, A) = \sup\{d(x, y) : y \in A\}$ and $D(x, A) = \inf\{d(x, y) : y \in A\}$, where $A \in P_{\mathrm{cl,bd}}(X)$.

## 2. Main Results

The following is the first our main results.

**Theorem 1.** *Let $(X, d)$ be a complete metric space and let $T$ be a mapping from $X$ into itself. Suppose that $T$ satisfies the following condition:*

$$d(Tx, Ty) \le \left(\frac{d(x, Ty) + d(y, Tx)}{d(x, Tx) + d(y, Ty) + 1}\right) d(x, y), \quad (2)$$

*for all $x, y \in X$. Then*

(a) *$T$ has at least one fixed point $\dot{x} \in X$;*

(b) *$\{T^n x\}$ converges to a fixed point, for all $x \in X$;*

(c) *if $\dot{x}, \dot{y}$ are two distinct fixed points of $T$, then $d(\dot{x}, \dot{y}) \ge 1/2$.*

*Proof.* Let $x_0 \in X$ be arbitrary and choose a sequence $\{x_n\}$ such that $x_{n+1} = Tx_n$. We have

$$\begin{aligned} d(x_{n+1}, x_n) &= d(Tx_n, Tx_{n-1}) \\ &\le \left(\frac{d(x_n, x_n) + d(x_{n-1}, x_{n+1})}{d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + 1}\right) d(x_n, x_{n-1}) \\ &= \left(\frac{d(x_{n-1}, x_{n+1})}{d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + 1}\right) d(x_n, x_{n-1}) \\ &\le \left(\frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + 1}\right) d(x_n, x_{n-1}). \end{aligned} \quad (3)$$

Given

$$\beta_n = \frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + 1}, \quad (4)$$

we have

$$\begin{aligned} d(x_{n+1}, x_n) &\le \beta_n d(x_n, x_{n-1}) \\ &\le (\beta_n \beta_{n-1}) d(x_{n-1}, x_{n-2}) \\ &\vdots \\ &\le (\beta_n \beta_{n-1} \cdots \beta_1) d(x_1, x_0). \end{aligned} \quad (5)$$

Observe that $(\beta_n)$ is nonincreasing, with positive terms. So $\beta_1 \cdots \beta_n \le \beta_1^n$ and $\beta_1^n \to 0$. It follows that

$$\lim_{n\to\infty} (\beta_1 \beta_2 \cdots \beta_n) = 0. \quad (6)$$

Thus, it is verified that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0. \quad (7)$$

Now for all $m, n \in \mathbb{N}$ we have

$$\begin{aligned} d(x_m, x_n) &\le d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) \\ &\quad + \cdots + d(x_{m-1}, x_m) \\ &\le [(\beta_n \beta_{n-1} \cdots \beta_1) + (\beta_{n+1} \beta_n \cdots \beta_1) \\ &\quad + \cdots + (\beta_{m-1} \beta_{m-2} \cdots \beta_1)] d(x_1, x_0) \\ &= \sum_{k=n}^{m-1} (\beta_k \beta_{k-1} \cdots \beta_1) d(x_1, x_0). \end{aligned} \quad (8)$$

Suppose that $a_k = (\beta_k \beta_{k-1} \cdots \beta_1)$. Since

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 0 \quad (9)$$

$\sum_{k=1}^{\infty} a_k < \infty$. It means that

$$\sum_{k=n}^{m-1} (\beta_k \beta_{k-1} \cdots \beta_1) \longrightarrow 0, \quad (10)$$

as $m, n \to \infty$. In other words, $\{x_n\}$ is a Cauchy sequence and so converges to $\dot{x} \in X$.

We claim that $\dot{x}$ is a fixed point.

Note that

$$d(x_{n+1}, T\dot{x}) \le \left(\frac{d(x_n, T\dot{x}) + d(\dot{x}, Tx_n)}{d(\dot{x}, T\dot{x}) + d(x_n, Tx_n) + 1}\right) d(x_n, \dot{x}). \quad (11)$$

On taking limit on both sides of (11), we have $d(\dot{x}, T\dot{x}) = 0$. Thus, $T\dot{x} = \dot{x}$.

If there exist two distinct fixed points $\dot{x}, \dot{y} \in X$, then

$$\begin{aligned} d(\dot{x}, \dot{y}) &= d(T\dot{x}, T\dot{y}) \\ &\le [d(\dot{x}, T\dot{y}) + d(T\dot{x}, \dot{y})] d(\dot{x}, \dot{y}) \\ &= 2[d(\dot{x}, \dot{y})]^2. \end{aligned} \quad (12)$$

Therefore, $d(\dot{x}, \dot{y}) \ge 1/2$ and we find the desired results. □

In the following, two examples of such type of mappings, which satisfy (2), are given.

*Example 2.* Let $X = \{0, 1/2, 1\}$ and let $d : X \times X \to [0, \infty)$ be defined by

$$\begin{gathered} d\left(0, \frac{1}{2}\right) = 2, \qquad d\left(1, \frac{1}{2}\right) = \frac{5}{2}, \qquad d(0, 1) = 3, \\ d(0, 0) = d\left(\frac{1}{2}, \frac{1}{2}\right) = d(1, 1) = 0, \\ d(a, b) = d(b, a), \quad \forall\, a, b \in X. \end{gathered} \quad (13)$$

$(X, d)$ is a complete metric space. Let $T : X \to X$ be defined by

$$T(0) = 0, \qquad T\left(\frac{1}{2}\right) = \frac{1}{2}, \qquad T(1) = 0$$
$$\begin{aligned} d(T0, T1) &= d(0,0) = 0, \\ d\left(T0, T\left(\frac{1}{2}\right)\right) &= d\left(0, \frac{1}{2}\right) = 2, \\ d\left(T1, T\left(\frac{1}{2}\right)\right) &= d\left(0, \frac{1}{2}\right) = 2, \end{aligned} \tag{14}$$

and we have

$$\begin{aligned} d\left(T0, T\left(\frac{1}{2}\right)\right) &= d\left(0, \frac{1}{2}\right) = 2 \\ &\le \left(\frac{d(0, T(1/2)) + d(1/2, T(0))}{d(0, T0) + d(1/2, T(1/2)) + 1}\right) \\ &\quad \times d\left(0, \frac{1}{2}\right) = 8 \end{aligned} \tag{15}$$

and also

$$\begin{aligned} d\left(T1, T\left(\frac{1}{2}\right)\right) &= d\left(0, \frac{1}{2}\right) = 2 \\ &\le \left(\frac{d(1, T(1/2)) + d(1/2, T(1))}{d(1, T1) + d(1/2, T(1/2)) + 1}\right) \\ &\quad \times d\left(1, \frac{1}{2}\right) \\ &= \left(\frac{5/2 + 2}{4}\right) \times \frac{5}{2} = \frac{45}{16}. \end{aligned} \tag{16}$$

Therefore, $T$ satisfies all the conditions of Theorem 1. Also, $T$ has two distinct fixed points $\{0, 1/2\}$ and $d(0, 1/2) = 2 \ge 1/2$.

*Example 3.* Let $X = [0, 2 - \sqrt{3}]$ be endowed with Euclidean metric and let $T : X \to X$ be defined by

$$Tx = \begin{cases} 0 & 0 \le x < 2 - \sqrt{3} \\ 2 - \sqrt{3} & x = 2 - \sqrt{3}. \end{cases} \tag{17}$$

Then we claim that $T$ satisfies all the conditions of Theorem 1.

If $x = 2 - \sqrt{3}$ and $0 \le y < 2 - \sqrt{3}$, we have

$$\begin{aligned} &|Tx - Ty| \left(|x - Tx| + |y - Ty| + 1\right) \\ &\quad = \left(2 - \sqrt{3}\right)\left(|y| + 1\right) = \left(2 - \sqrt{3}\right)(y + 1) \\ &\quad \le \left(2 - \sqrt{3} - y\right)^2 - \left(2 - \sqrt{3}\right)\left(2 - \sqrt{3} - y\right) \\ &\quad = \left(|x - Ty| + |y - Tx|\right)|x - y|. \end{aligned} \tag{18}$$

Thus,

$$|Tx - Ty| \le \left(\frac{|x - Ty| + |y - Tx|}{|x - Tx| + |y - Ty| + 1}\right)|x - y|. \tag{19}$$

Similar argument holds for the other conditions.

*Remark 4.* Note that in (2) the ratio

$$\frac{d(x, Ty) + d(y, Tx)}{d(x, Tx) + d(y, Ty) + 1} \tag{20}$$

might be greater or less than 1 and has not introduced an upper bound. Note that if, for every $x, y \in X$, $d(x, y) < 1/2$, then we have

$$\begin{aligned} &d(x, Ty) + d(y, Tx) \\ &\quad \le 2d(x, y) + d(x, Tx) + d(y, Ty) \\ &\quad < d(x, Tx) + d(y, Ty) + 1. \end{aligned} \tag{21}$$

It means that

$$\frac{d(x, Ty) + d(y, Tx)}{d(x, Tx) + d(y, Ty) + 1} < 1, \tag{22}$$

and thus Theorem 1 is a special case of Banach contraction principle. Therefore, when $(X, d)$ is a complete metric space such that, for all $x, y \in X$, $d(x, y) \ge 1/2$, Theorem 1 is valuable because (20) might be greater than 1. Example 2 shows this note precisely.

The following is the second in our main results.

**Theorem 5.** *Let $(X, d)$ be a complete metric space and let $T$ be a multivalued mapping from $X$ into $P_{cl,bd}(X)$. Let $T$ satisfy the following:*

$$H(Tx, Ty) \le \left(\frac{D(x, Ty) + D(y, Tx)}{\delta(x, Tx) + \delta(y, Ty) + 1}\right) d(x, y), \tag{23}$$

*for all $x, y \in X$. Then $T$ has a fixed point $\dot{x} \in X$.*

*Proof.* Let $x_0 \in X$ and $x_1 \in Tx_0$. For each $0 < h_1 < 1$ one can choose $x_2 \in Tx_1$ such that

$$\begin{aligned} d(x_1, x_2) &< H(Tx_0, Tx_1) + \left(1 - \frac{1}{h_1}\right) H(Tx_0, Tx_1) \\ &= \frac{1}{h_1} H(Tx_0, Tx_1). \end{aligned} \tag{24}$$

For each $0 < h_n < 1$ we can choose $x_{n+1} \in Tx_n$ such that

$$\begin{aligned} d(x_n, x_{n+1}) &< H(Tx_{n-1}, Tx_n) + \left(1 - \frac{1}{h_n}\right) H(Tx_0, Tx_1) \\ &= \frac{1}{h_n} H(Tx_0, Tx_1). \end{aligned} \tag{25}$$

Specifically if

$$\begin{aligned} h_n &= \sqrt{\frac{d(x_{n-1} + x_{n+1})}{d(x_{n-1} + x_n) + d(x_n + x_{n+1}) + 1}} \\ &= \sqrt{\beta_n}, \end{aligned} \tag{26}$$

then

$$d(x_n, x_{n+1}) \le \sqrt{\beta_n} d(x_{n-1}, x_n) \le \beta_n d(x_{n-1}, x_n). \quad (27)$$

Therefore,

$$\begin{aligned} d(x_{n+1}, x_n) &\le \beta_n d(x_n, x_{n-1}) \\ &\le (\beta_n \beta_{n-1}) d(x_{n-1}, x_{n-2}) \\ &\vdots \\ &\le (\beta_n \beta_{n-1} \cdots \beta_1) d(x_1, x_0). \end{aligned} \quad (28)$$

It can easily be seen that

$$\lim_{n\to\infty} (\beta_1 \beta_2 \cdots \beta_n) = 0. \quad (29)$$

Thus, it is easily verified that

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0. \quad (30)$$

Now for all $m, n \in \mathbb{N}$ we have

$$\begin{aligned} d(x_m, x_n) &\le d(x_n, x_{n+1}) + d(x_n, x_{n+1}) + \cdots + d(x_{m-1}, x_m) \\ &\le [(\beta_n \beta_{n-1} \cdots \beta_1) + (\beta_{n+1} \beta_n \cdots \beta_1) \\ &\quad + \cdots + (\beta_{m-1} \beta_{m-2} \cdots \beta_1)] d(x_1, x_0) \\ &= \sum_{k=n}^{m-1} (\beta_k \beta_{k-1} \cdots \beta_1) d(x_1, x_0). \end{aligned} \quad (31)$$

Suppose that $a_k = (\beta_k \beta_{k-1} \cdots \beta_1)$. Since

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = 0, \quad (32)$$

$\sum_{k=1}^{\infty} a_k < \infty$. It means that

$$\sum_{k=n}^{m-1} (\beta_k \beta_{k-1} \cdots \beta_1) \longrightarrow 0, \quad (33)$$

as $m, n \to \infty$. In other words, $\{x_n\}$ is a Cauchy sequence and so converges to $\dot{x} \in X$. We claim that $\dot{x}$ is a fixed point. Consider

$$\begin{aligned} D(\dot{x}, T\dot{x}) &\le d(\dot{x}, x_{n+1}) + D(x_{n+1}, T\dot{x}) \\ &\le H(Tx_n, T\dot{x}) + d(\dot{x}, x_{n+1}) \\ &\le \left( \frac{D(\dot{x}, Tx_n) + D(x_n, T\dot{x})}{\delta(\dot{x}, T\dot{x}) + \delta(x_n, Tx_n) + 1} \right) \\ &\quad \times d(x_n, \dot{x}) + d(\dot{x}, x_{n+1}) \\ &\le [D(\dot{x}, x_{n+1}) + D(x_n, T\dot{x})] \\ &\quad \times d(x_n, \dot{x}) + d(\dot{x}, x_{n+1}). \end{aligned} \quad (34)$$

On taking limit on both sides of (31) we have $D(\dot{x}, T\dot{x}) = 0$. It means that $\dot{x} \in T\dot{x}$. □

*Remark 6.* Note that Theorem 5 is a generalization of Theorem 1 because by taking $Fx = \{Tx\}$ and applying Theorem 5 for $F$ we obtain Theorem 1.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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