Erratum

Erratum to “Compact Operators for Almost Conservative and Strongly Conservative Matrices”

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We redefine the space f and state the results of [1] in this light.

Let $\mathcal{B}$ be a semigroup of positive regular matrices $B = (b_{nk})$.

A bounded sequence $x = (x_k)$ is said to be $\mathcal{B}$ -almost convergent to the value l if and only if $t_{pn}(x) \rightarrow l$, as $p \rightarrow \infty$ uniformly in n , where

$$t_{pn}(x) = \frac{1}{p+1} \sum_{m=0}^p B_{m+n}(x); \quad (p, n \in \mathbb{N}), \quad (1)$$

and $B_n(x) = \sum_{k=1}^{\infty} b_{nk} x_k$ which is B -transform of a sequence x (see Mursaleen [2]). The number l is called the generalized limit of x , and we write $l = f\text{-}\lim x$. We write

$$f = \left\{ x \in \ell_{\infty} : \lim_{p \rightarrow \infty} t_{pn}(x) = L \text{ uniformly in } n \right\}. \quad (2)$$

Using the idea of $\mathcal{B}$ -almost convergence, we define the following.

An infinite matrix $A = (a_{nk})_{n,k=1}^{\infty}$ is said to be $\mathcal{B}$ -almost conservative if $Ax \in f$ for all $x \in c$, and we denote it by $A \in (c, f)$. An infinite matrix $A = (a_{nk})_{n,k=1}^{\infty}$ is said to be $\mathcal{B}$ -strongly conservative if $Ax \in c$ for all $x \in f$, and we denote it by $A \in (f, c)$.

Now, we restate Theorem 11 and Theorem 15 of [1] as follows, respectively.

Theorem 11. Let $A = (a_{nk})$ be a $\mathcal{B}$ -almost conservative matrix. Then, one has

$$0 \leq \|L_A\|_{\chi} \leq \limsup_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\tilde{a}_{nk}| \right), \quad (3)$$

$$L_A \text{ is compact if } \lim_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\tilde{a}_{nk}| \right) = 0,$$

where $\tilde{a}_{nk} = \sum_{j=1}^{\infty} a_{nj} b_{jk}$.

Proof. It follows on the same lines as of Theorem 11 [1] by only replacing a_{nk} by $\tilde{a}_{nk}$. $\square$

Theorem 15. Let B be a normal positive regular matrix. Let $A = (a_{nk})$ be an infinite matrix. Then, one has the following.

(i) If $A \in (f, c_0)$, then

$$\|L_A\|_{\chi} = \limsup_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\hat{a}_{nk}| \right). \quad (4)$$

(ii) If $A \in (f, c)$, then

$$\begin{aligned} & \frac{1}{2} \cdot \limsup_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\hat{a}_{nk} - \alpha_k| \right) \\ & \leq \|L_A\|_{\chi} \leq \limsup_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\hat{a}_{nk} - \alpha_k| \right), \end{aligned} \quad (5)$$

where $\alpha_k = \lim_{n \rightarrow \infty} \hat{a}_{nk}$ for all $k \in \mathbb{N}$.

(iii) If $A \in (f, \ell_\infty)$, then

$$0 \leq \|L_A\|_\chi \leq \limsup_{n \rightarrow \infty} \left(\sum_{k=1}^{\infty} |\hat{a}_{nk}| \right), \quad (6)$$

where $\hat{A} = (\hat{a}_{nk})$ is the composition of the matrices A and B^{-1} ; that is, $\hat{a}_{nk} = \sum_{j=1}^{\infty} a_{nj} b_{jk}^{-1}$.

Proof. It follows on the same lines as Theorem 15 of [1] by only replacing a_{nk} by $\hat{a}_{nk}$. $\square$

Remark 1 (see [2]). If $\mathcal{B}$ consists of the iterates of the operator T defined on ℓ_∞ by $Tx = (x_{\sigma(n)})$, where σ is an injection of the set of positive integers into itself having no finite orbits, then $\mathcal{B}$ -invariant mean is reduced to the σ -mean and $\mathcal{B}$ -almost convergence is reduced to σ -convergence. In this case, our results are reduced to the results of [3].

References

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