Research Article

Rogue Wave for the (3+1)-Dimensional Yu-Toda-Sasa-Fukuyama Equation

Hanlin Chen,¹ Zhenhui Xu,² and Zhengde Dai³

¹ Joint Laboratory for Extreme Conditions Matter Properties, Southwest University of Science and Technology, Mianyang 621010, China

² Applied Technology College, Southwest University of Science and Technology, Mianyang 621010, China

³ School of Mathematics and Physics, Yunnan University, Kunming 650091, China

Correspondence should be addressed to Zhenhui Xu; xuzhenhui9@163.com

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A new method, homoclinic (heteroclinic) breather limit method (HBLM), for seeking rogue wave solution to nonlinear evolution equation (NEE) is proposed. (3+1)-dimensional Yu-Toda-Sasa-Fukuyama (YTSE) equation is used as an example to illustrate the effectiveness of the suggested method. A new family of two-wave solution, rational breather wave solution, is obtained by extended homoclinic test method, and it is just a rogue wave solution. This result shows rogue wave can come from extreme behavior of breather solitary wave for (3+1)-dimensional nonlinear wave fields.

1. Introduction

It is well known that solitary wave solutions of nonlinear evolution equations play an important role in nonlinear science fields, especially in nonlinear physical science, since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications [1]. In recent years, rogue waves, as a special type of nonlinear waves and also known as freak waves, monster waves, killer waves, extreme waves, and abnormal waves [2], have triggered much interest in various physical branches. Rogue wave is a kind of waves that seems abnormal which is first observed in the deep ocean. It always has two to three times amplitude higher than its surrounding waves and generally forms in a short time for which people think that it comes from nowhere. Rogue waves have been the subject of intensive research in oceanography [3, 4], optical fibres [5–7], superfluids [8], Bose-Einstein condensates, financial markets, and other related fields [9–13]. The first-order rational solution of the self-focusing nonlinear Schrödinger equation (NLS) was first found by Peregrine to describe the rogue waves phenomenon [14]. Recently, by using the Darboux dressing technique or Hirota's bilinear method, rogue waves solutions in complex system were obtained such as nonlinear

Schrödinger equation, Hirota equation, Sasa-Satsuma equation, Davey-Stewartson equation, coupled Gross-Pitaevskii equation, coupled NLS Maxwell-Bloch equation, and coupled Schrödinger-Boussinesq equation [15–26]. In this work, we propose a homoclinic (heteroclinic) breather limit method for seeking rogue wave solution to real NEE. We consider a general nonlinear partial differential equation in the form

$$P(u, u_t, u_x, u_y, \dots) = 0, \quad (1)$$

where P is a polynomial in its arguments, $u : R_x \times R_y \times R_t \rightarrow R$. To determine $u(t, x, y)$ explicitly, we take the following four steps.

Step 1. By Painlevé's analysis, a transformation

$$u = T(f) \quad (2)$$

is made for some new and unknown function f .

Step 2. By using the transformation in Step 1, original equation can be converted into Hirota's bilinear form

$$G(D_t, D_y; f) = 0, \quad (3)$$

where the D -operator [27] is defined by

$$\begin{aligned} D_t^m D_y^n f(t, y) \cdot g(t, y) \\ = \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'} \right)^n [f(t, y) g(t', y')] \Big|_{t'=t, y'=y}. \end{aligned} \quad (4)$$

Step 3. Solve the above equation to get homoclinic (heteroclinic) breather wave solution by using extended homoclinic test approach (EHTA) [28].

Step 4. Letting the period of periodic wave go to infinite in homoclinic (heteroclinic) breather wave solution, we can obtain a rational homoclinic (heteroclinic) wave and this wave is just a rogue wave.

As a example we consider (3+1)-D Yu-Toda-Sasa-Fukuyama equation which is an extension of Bogoyavlenskii-Schiff (BS) equation in higher dimension [29]. It is well known that BS equation is the reduction of the self-dual Yang-Mills equation; it is an integrable system and has an infinite number of conservation laws and N -soliton solutions [30].

The (3+1)-dimensional Yu-Toda-Sasa-Fukuyama equation is

$$\begin{aligned} (-4u_t + \Phi(u)u_z)_x + 3u_{yy} &= 0, \\ \Phi(u) &= \partial_x^2 + 4u + 2u_x \partial_x^{-1}. \end{aligned} \quad (5)$$

It is called Yu-Toda-Sasa-Fukuyama (YTSF) equation. YTSF equation is not integrable system [29–31]; it is firstly presented by Yu et al. using the strong symmetry [30, 32]. The nontravelling wave solution was found using auto-Backlund transformation and the generalized projective Riccati equation method [32–34]. Moreover, some soliton-like solutions and periodic solutions for potential YTSF equation were obtained by Hiriota's bilinear method, the tanh-coth method, exp-function method, homoclinic test approach, and extended homoclinic test approach [33–37], respectively. Recently, some analytic solutions for the (3+1)-dimensional potential Yu-Toda-Sasa-Fukuyama equation [38] and the (2+1)-dimensional Ablowitz-Kaup-Newell-Segur equation [39] are obtained by Darvishi using the modified extended homoclinic test approach, some exact solutions of the nonlinear ZK-MEW, and the potential YTSF equations by Zayed and Arnous using the modified simple equation method [40]. Besides these, further result on soliton and its feature for (5) were not studied up to now.

This work focuses on rational breather wave and then rogue wave solutions. Applying HBLM to (3+1)-D YTSF equation we firstly get breather solitary solution and then obtain rational breather solution by letting periodic wave go to infinite in breather solitary solution. Finally, we show that this rational breather wave is just a rogue wave. This is the new physical phenomenon found out up to now.

2. Rational Homoclinic Wave (Rogue Wave)

Let $\xi = x + cz$ in (5); for simplicity we take constant $c > 0$ ($c < 0$ is similar), notice that $\partial_x = \partial_\xi$, $\partial_z = c\partial_\xi$ and so $c = \partial_z \partial_x^{-1}$, then (5) can be converted into the following form:

$$-4u_{\xi t} + cu_\xi^4 + 3c(u^2)_{\xi\xi} + 3u_{yy} = 0. \quad (6)$$

Setting $\eta = \xi - bt = x + cz - bt$ in (6) gives

$$3u_{yy} + 4bu_{\eta\eta} + 3c(u^2)_{\eta\eta} + cu_{\eta\eta} = 0. \quad (7)$$

Setting $\zeta = iy$ in (7) gives

$$3u_{\zeta\zeta} - 4bu_{\eta\eta} - 3c(u^2)_{\eta\eta} - cu_{\eta\eta} = 0. \quad (8)$$

It is easy to see that (7) has an equilibrium solution u_0 which is an arbitrary constant.

We suppose that

$$u = u_0 + 2(\ln f)_{\eta\eta}, \quad (9)$$

where $f(\eta, \zeta)$ is unknown real function. Substituting (9) into (8) we obtain the following bilinear form:

$$(3D_\zeta^2 - (4b + 6cu_0)D_\eta^2 - cD_\eta^4 - A)f \cdot f = 0, \quad (10)$$

where A is an integration constant, $D_\eta^4 f \cdot f = 2(ff_{4\eta} - 4f_\eta f_{3\eta} + 3f_{2\eta}^2)$, $D_\eta^2 f \cdot f = 2(f_{\eta\eta}f - f_\eta^2)$. With regard to (9), using the homoclinic test technique we can seek the solution in the form

$$f = e^{-p_1(\eta - \alpha\zeta)} + \delta_1 \cos(p(\eta + \beta\zeta)) + \delta_2 e^{p_1(\eta - \alpha\zeta)}, \quad (11)$$

where $p_1, p, \delta_1, \delta_2$ are real constants to be determined and α, β are constants to be determined.

Substituting (10) into (9) we can get an algebraic equation of $e^{p_1(\eta - \alpha\zeta)}$. Then equating the coefficients of all powers of $e^{jp(\xi - \alpha\zeta)}$ ($j = -1, 0, 1$) to zero, we get

$$\begin{aligned} &2\delta_1 c p p_1^3 + (4b + 6cu_0)\delta_1 p p_1 + 3\delta_1 \alpha \beta p p_1 - 2\delta_1 c p^3 p_1 \\ &= 0, \\ &(4b + 6cu_0)\delta_1 \delta_2 p p_1 + 2\delta_1 \delta_2 p p_1^3 \\ &+ 3\delta_1 \delta_2 \alpha \beta p p_1 - 2\delta_1 \delta_2 c p^3 p_1 = 0, \\ &12\delta_2 \alpha^2 p_1^2 - 4\delta_1^2 c p^4 + (4b + 6u_0)\delta_1^2 p^2 - 3\delta_1^2 \beta^2 p^2 \\ &- 4(4b + 6cu_0)\delta_2 p_1^2 - 16\delta_2 c p_1^4 = 0, \\ &6\delta_1 c p^2 p_1^2 - (4b + 6u_0)\delta_1 p_1^2 + (4b + 6u_0)\delta_1 p^2 \\ &- \delta_1 c p_4 - \delta_1 c p_1^4 + 3\delta_1 \alpha^2 p_1^2 - 3\delta_1 \beta^2 p^2 = 0, \\ &-\delta_1 \delta_2 c p_1^4 + 6\delta_1 \delta_2 c p^2 p_1^2 - (4b + 6cu_0)\delta_1 \delta_2 p_1^2 - 3\delta_1 \delta_2 \beta^2 p^2 \\ &- \delta_1 \delta_2 c p^4 + (4b + 6cu_0)\delta_1 \delta_2 p^2 + 3\delta_1 \delta_2 \alpha^2 p_1^2 = 0. \end{aligned} \quad (12)$$

Take $p_1 = p$; then (12) can be reduced into the following:

$$\begin{aligned} (4b + 6cu_0) \delta_1 p^2 + 3\delta_1 \alpha \beta p^2 &= 0, \\ (4b + 6cu_0) \delta_1 \delta_2 p^2 + 3\delta_1 \delta_2 \alpha \beta p^2 &= 0, \\ 4\delta_1 c p^4 + 3\delta_1 \alpha^2 p^2 - 3\delta_1 \beta^2 p^2 &= 0, \\ 4\delta_1 \delta_2 c p^4 - 3\delta_1 \delta_2 \beta^2 p^2 \delta_1 + 3\delta_1 \delta_2 \alpha^2 p^2 &= 0, \\ 12\delta_2 \alpha^2 p^2 - 4\delta_1^2 c p^4 + (4b + 6cu_0) \delta_1^2 p^2 - 3\delta_1 \beta^2 p^2 \\ - 4(4b + 6cu_0) \delta_2 p^2 - 16\delta_2 c p^4 &= 0. \end{aligned} \quad (13)$$

Solving (13) yields

$$\begin{aligned} \delta_1 &= \pm 2 \frac{\sqrt{(6\alpha^2 - 2(2b - 3u_0) - 3\beta^2) \delta_2}}{\sqrt{6\beta^2 - 2(2b - 3u_0) - 3\alpha^2}}, \\ \alpha \beta &= -\frac{1}{3} (4b + 6cu_0), \quad p^2 = \frac{3}{4c} (\beta^2 - \alpha^2), \end{aligned} \quad (14)$$

where b, c, δ_2 are some free real constants and α, β are some free constants. Setting $c = -1$ in (14) gives

$$\begin{aligned} \delta_1 &= \pm 2 \frac{\sqrt{(6\alpha^2 - 2(2b - 3u_0) - 3\beta^2) \delta_2}}{\sqrt{6\beta^2 - 2(2b - 3u_0) - 3\alpha^2}}, \\ \alpha \beta &= -\frac{2}{3} (2b - 3u_0), \quad p^2 = \frac{3}{4} (\alpha^2 - \beta^2). \end{aligned} \quad (15)$$

Choosing $u_0 \neq 2b/3$ and $\delta_2 > 0$, we get from (15)

$$\begin{aligned} |\beta| &> |\alpha|, \quad \alpha = -\frac{2(2b - 3u_0)}{3\beta}, \\ \alpha^2 &> \frac{2(2b - 3u_0)}{3\beta}, \quad \left(u_0 < \frac{2b}{3} \text{ or } > \frac{2b}{3}\right) \\ \text{or } \alpha^2 &< \frac{2b}{3} \cdot \left(u_0 > \frac{2b}{3}\right). \end{aligned} \quad (16)$$

Substituting (15)-(16) into (11), we have

$$\begin{aligned} f_1(\eta, \zeta) &= 2\sqrt{\delta_2} \cosh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \\ &\quad + h_1 \cos(p(\eta + \beta\zeta)), \\ f_2(\eta, \zeta) &= 2\sqrt{\delta_2} \cosh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \\ &\quad - h_1 \cos(p(\eta + \beta\zeta)), \end{aligned} \quad (17)$$

where $h_1 = 2\sqrt{(6\alpha^2 - 2(2b - 3u_0) - 3\beta^2)\delta_2} / \sqrt{6\beta^2 - 2(2b - 3u_0) - 3\alpha^2}$, $p = \pm(\sqrt{3(\alpha^2 - \beta^2)}/2)$, and α, β are some free constants. Substituting (17) into (9) yields the solutions of (8) as follows, respectively:

$$\begin{aligned} u_1(\eta, \zeta) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\ &\quad \left. \left. + 2m_1 \sinh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \right. \right. \\ &\quad \left. \left. \times \sin(p(\eta + \beta\zeta))\right) \right. \\ &\quad \left. \times \left(\left(\cosh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \right. \right. \right. \\ &\quad \left. \left. \left. + m_1 \cos(p(\eta + \beta\zeta))\right)^2\right)^{-1}\right), \\ u_2(\eta, \zeta) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\ &\quad \left. \left. - 2m_1 \sinh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \right. \right. \\ &\quad \left. \left. \times \sin(p(\eta + \beta\zeta))\right) \right. \\ &\quad \left. \times \left(\left(\cosh\left(p\left(\eta + \frac{2(2b - 3u_0)}{3\beta}\zeta\right) + \frac{1}{2}\ln(\delta_2)\right) \right. \right. \right. \\ &\quad \left. \left. \left. - m_1 \cos(p(\eta + \beta\zeta))\right)^2\right)^{-1}\right), \end{aligned} \quad (18)$$

where $m_0 = 9(\beta^2 - \alpha^2)/(6\beta^2 - 2(2b - 3u_0) - 3\alpha^2)$, $m_1 = \sqrt{6\alpha^2 - 2(2b - 3u_0) - 3\beta^2} / \sqrt{6\beta^2 - 2(2b - 3u_0) - 3\alpha^2} < 1$, and $p = \pm(\sqrt{3(\alpha^2 - \beta^2)}/2)$.

Taking $\zeta = iy$, $\alpha = ai$, and $\beta = \omega i$ into (18) yields the solutions of (7) as follows, respectively:

$$\begin{aligned}
 u_1(\eta, y) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\
 &\quad \left. \left. + 2m_1 \sinh \left(p \left(\eta + \frac{2(2b-3u_0)}{3\omega} y \right) \right. \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \times \sin(p(\eta - \omega y)) \right) \\
 &\quad \times \left(\left(\cosh \left(p \left(\eta + \frac{2(2b-3u_0)}{3\omega} y \right) \right) + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \left. + m_1 \cos(p(\eta - \omega y)) \right)^2 \right)^{-1} \Bigg), \\
 u_2(\eta, y) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\
 &\quad \left. \left. - 2m_1 \sinh \left(p \left(\eta + \frac{2(2b-3u_0)}{3\omega} y \right) \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \times \sin(p(\eta - \omega y)) \right) \\
 &\quad \times \left(\left(\cosh \left(p \left(\eta + \frac{2(2b-3u_0)}{3\omega} y \right) \right) + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \left. - m_1 \cos(p(\eta - \omega y)) \right)^2 \right)^{-1} \Bigg),
 \end{aligned} \tag{19}$$

where a, ω are some free real constants, $m_0 = 9(\omega^2 - a^2)/(6\omega^2 + 2(2b - 3u_0) - 3a^2)$, $m_1 = \sqrt{6a^2 + 2(2b - 3u_0) - 3\omega^2}/\sqrt{6\omega^2 + 2(2b - 3u_0) - 3a^2} < 1$, and $p = \pm(\sqrt{3(\omega^2 - a^2)}/2)$.

The solution $u_1(\eta, y)$ (resp., $u_2(\eta, y)$) shows a new family of two-wave, breather solitary wave, which is a solitary wave and meanwhile is a periodic wave whose amplitude periodically oscillates with the evolution of time. It shows elastic interaction between a left-propagation (backward-direction) periodic wave with speed b and homoclinic wave of different direction with speed $2(2b - 3u_0)/3\omega$.

Taking $\eta = \xi - bt = x - z - bt$ into (19) gives and yields the breather-type soliton solutions of the (3+1)-D YTSF equation as follows, respectively (see Figures 1 and 2):

$$\begin{aligned}
 u_1(t, x, y, z) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\
 &\quad \left. \left. + 2m_1 \sinh \left(p \left(x + \frac{2(2b-3u_0)}{3\omega} y - z - bt \right) \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \times \sin(p(x - \omega y - z - bt)) \right) \\
 &\quad \times \left(\left(\cosh \left(p \left(x + \frac{2(2b-3u_0)}{3\omega} y - z - bt \right) \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \left. + m_1 \cos(p(x - \omega y - z - bt)) \right)^2 \right)^{-1} \Bigg), \\
 u_2(t, x, y, z) &= u_0 + \left(2p^2 \left(m_0 \right. \right. \\
 &\quad \left. \left. - 2m_1 \sinh \left(p \left(x + \frac{2(2b-3u_0)}{3\omega} y - z - bt \right) \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \times \sin(p(\eta - \omega y - z - bt)) \right) \\
 &\quad \times \left(\left(\cosh \left(p \left(x + \frac{2(2b-3u_0)}{3\omega} y - z - bt \right) \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \ln(\delta_2) \right) \right. \\
 &\quad \left. \left. - m_1 \cos(p(x - \omega y - z - bt)) \right)^2 \right)^{-1} \Bigg),
 \end{aligned} \tag{20}$$

where a, ω are some free real constants, $m_0 = 9(\omega^2 - a^2)/(6\omega^2 + 2(2b - 3u_0) - 3a^2)$, $m_1 = \sqrt{6a^2 + 2(2b - 3u_0) - 3\omega^2}/\sqrt{6\omega^2 + 2(2b - 3u_0) - 3a^2} < 1$, and $p = \pm(\sqrt{3(\omega^2 - a^2)}/2)$.

FIGURE 1: The figure of $u_1(t, x, y, z)$ as $u_0 = -2$ and $b = 3$.

FIGURE 2: The figure of $u_2(t, x, y, z)$ as $u_0 = -2$, $b = 3$, $\omega = 3.5$, $p = 0.6092400758$, $x = -1$, and $z = -2$.

Use (19) and take $\delta_2 = 1$; then $(1/2) \ln(\delta_2) = 0$ in u_2 . So, solution u_2 can be rewritten as follows:

$$\begin{aligned}
 & u_2^{(1)}(\eta, y) \\
 &= u_0 + \left(2p^2 \left(m_0 - 2m_1 \sinh \left(p \left(\eta + \frac{2(2b - 3u_0)}{3\omega} y \right) \right) \right. \right. \\
 &\quad \times \sin(p(\eta - \omega y)) \Bigg) \\
 &\quad \times \left(\left(\cosh \left(p \left(\eta + \frac{2(2b - 3u_0)}{3\omega} y \right) \right) \right. \right. \\
 &\quad \left. \left. - m_1 \cos(p(\eta - \omega y)) \right)^2 \right)^{-1} \Bigg), \tag{21}
 \end{aligned}$$

where $m_0 = 12p^2/(4p^2 + 2(2b - 3u_0) + 3\omega^2)$ and $m_1 = \sqrt{3a^2 + 2(2b - 3u_0) - 4p^2}/\sqrt{4p^2 + 2(2b - 3u_0) + 3\omega^2}$.

Now we consider a limit behavior of $u_2^{(1)}$ as the period $2\pi/p$ of periodic wave $\cos(p(\eta - \omega y))$ goes to infinite; that is, $p \rightarrow 0$. By computing, we obtain the following result:

$$\begin{aligned}
 & U_{\text{rogue wave}} \\
 &= u_0 + \frac{16(6A - (\eta + (2(2b - 3u_0)/3\omega)y)(\eta - \omega y))}{((\eta + (2(2b - 3u_0)/3\omega)y)^2 + (\eta - \omega y)^2 + 8A)^2}, \tag{22}
 \end{aligned}$$

where $A = 1/(3\omega^2 + 2(2b - 3u_0))$; here we have used $m_1 \rightarrow 1$ and $\omega = a$ as $p \rightarrow 0$.

U contains two waves with different velocities and directions. It is easy to verify that $U_{\text{rogue wave}}$ is a rational solution of (7). Moreover, we can show that $U_{\text{rogue wave}}$ also is breather-type solution. In fact, $U \rightarrow 0$ for fixed η as $y \rightarrow \pm \infty$. So, U is not only a rational breather solution but also a rogue wave solution which has two to three times amplitude higher than its surrounding waves and generally forms in a short time. It is an example that the rogue wave can come from breather solitary wave solution for real equation. One can think whether the energy collection and superposition of breather solitary wave in many many periods leads to a rogue wave or not.

Taking $\eta = \xi - bt = x - z - bt$ into (21), we obtain the rogue wave solutions of the (3+1)-D YTSF equation as follows (see Figure 3):

$$\begin{aligned}
 & U_{(\text{ytsf}) \text{ rogue wave}} \\
 &= u_0 + \left(16 \left(6A - \left(x + \frac{2(2b - 3u_0)}{3\omega} y - z - bt \right) \right. \right. \\
 &\quad \times (x - \omega y - z - bt) \Bigg) \\
 &\quad \times \left(\left(\left(x + \frac{2(2b - 3u_0)}{3\omega} y - z - bt \right)^2 \right. \right. \\
 &\quad \left. \left. + (x - \omega y - z - bt)^2 + 8A \right)^2 \right)^{-1} \Bigg), \tag{23}
 \end{aligned}$$

where $A = 1/(3\omega^2 + 2(2b - 3u_0))$; here we have used $m_1 \rightarrow 1$ and $\omega = a$ as $p \rightarrow 0$.

3. Conclusion

In this paper, we propose a new method for seeking rogue wave, homoclinic (heteroclinic) breather limit method (HBLM). Applying this method to the real (3+1)-D YTSF equation, we obtain a family of homoclinic breather solution and rational homoclinic solution. Furthermore, rational homoclinic solution obtained here is just a rogue wave solution, and then we obtain the rogue wave solutions of the (3+1)-D YTSF equation. In future, we intend to study the interaction between breather wave and solitary wave. What

FIGURE 3: The figure of $U_{(ytsf)}$ rogue wave as $a = \omega$, $u_0 = 1$, $b = 2$, $\omega = \sqrt{6}/3$, $x = -1$, and $z = -2$.

is more, can we obtain similar results to another integrable or nonintegrable system with homoclinic or heteroclinic breather wave? How can one use the homoclinic breather wave to obtain rogue wave under contained conditions?

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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