Research Article

Dynamics and Motivations of Media Marketing: The Role of Globalization and Empowerment

Cinzia Colapinto¹ and Eleonora Benecchi²

¹ Department of Management, Ca’ Foscari University of Venice, San Giobbe, Cannaregio, 873-30121 Venice, Italy
² Institute of Media and Journalism, Università della Svizzera Italiana, Via Buffi 13, 6900 Lugano, Switzerland

Correspondence should be addressed to Cinzia Colapinto; cinzia.colapinto@unive.it

Received 27 February 2014; Accepted 9 May 2014; Published 3 June 2014

Academic Editor: Simone Marsiglio

Copyright © 2014 C. Colapinto and E. Benecchi. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Audience fragmentation, convergence, and technology development have changed consumption patterns and brand communication. Media companies should consider TV series and movies as brands to be exploiting at national/international level. Using a dynamic diffusion model, we analyze the evolution of popularity in a specific country under different hypotheses. Then we propose an optimal decision model which describes the decision maker’s point of view in a two-sided market.

1. Introduction

Many changes (such as media convergence, transition to digital format, and audience fragmentation) are affecting media companies’ business model and have given to media content a primary role. Indeed, media environment has changed and in order to better satisfy audience’s new tastes and consumption paths, most broadcasters have been transforming into multichannel and multiservice companies, whose supply is characterized by interactivity [1]. Companies, actors, or sportsmen rely more and more on the power of new media and of a viral spread, which is heightened by online communities and the cross-pollination of content on other social media sites [2–4]. The real value of social media consists in rebuilding the buzz and reinforcing the attitude that distinguishes the brand from its rivals. Media companies need to include consumers in their value chain (a real value constellation) recalling Toffler’s [5] concept of prosumer: this term identified consumer involvement in product design; nowadays consumers are involved in value creation. Content is now being produced and consumed in innovative ways; consequently, internet and new media have become both a viewing site and a site of fan engagement. Technological advances lead towards new media content models, mainly based on participatory media [6]. Such a new strategy deeply relies on consumer behaviours. Thus, focusing on the concept of brand, we believe TV series are an interesting case study.

Television programs are a public good, and they are sold through many different channels over different points in time to maximize profits (windowing). As people are doing their best to avoid commercials, companies need to aggregate and to encourage customers to come back for more; as a result, Internet and new and social media are becoming very important marketing tools. The synergistic marketing communication [7] implies the use of different channels (often through synergistic ownership strategies) and approaches to persuasion: companies use multiple promotional outlets to reach desirable audiences. In particular, media marketing has moved toward the Internet discovering the importance of viral marketing, word of mouth, and online communities in all the phases of the life cycle of a product.

In a complex evolving system, TV programs’ and marketing investment choices by media managers are getting more difficult and complex; they need to pursue data-driven decisions. Most analysis tolls consider TV programs ratings and we propose a decisional dynamic model which utilizes a dynamic model explaining the path of program popularity over time.
The paper is organized as follows. Section 2 introduces the concept of connected marketing and recalls the fundamental literature. Section 3 is dedicated to the diffusion model based on the literature of biological field and the related examples: in this way we intend to shed a light on the responses of TV series fans and their behaviour. We point out media managers’ choices as well as viewers’ actions. In Section 4 using real data we test our model and provide the long run behaviour of an imported TV series. Section 5 introduces a model to sustain the decision maker (DM) in the context of management operational policy. Section 6 concludes.


It is increasingly clear that Marketing Departments for TV Shows behave according to the rule that the more endorsements that trickle through blogs, forums, Facebook, and Twitter, the better it is. Whether online activity, especially social media activity, could make or break a TV show is still unclear, however it is a fact that networks are increasingly trying to monitor Internet discussions to gain insight into whether a show will move from hype to hit. Recent deals between the major American networks and research organizations such as Networked Insight are proofs of that.

Marketing is crucial for media companies to survive and prosper in the current media ecosystem, characterized on one hand by an increasing competition for scarce audience and convergence and on the other by the empowerment of consumers. Media companies are special because they can be utilized as communication tools for self-branding [8]. In particular, considering TV series. It is evident that fans are being exposed to a greater number of products than ever before. That is why developing direct relationship with fans is vital for media companies in the wake of the growing audience for TV producers (fan as super-consumers) and enlightenment by studies portraying fandom as a selected but strategic audience for TV producers (fan as creative subjects) and therefore focusing on the incorporation of fan practices by the TV industry [10–12]; on the other side the power of fandom is recognized in its growing influence on creative and productive choices made by TV shows’ creators (fans as creative subjects) and therefore the focus is on changing relationship between producers and consumers [13–15].

Considering fans either super-consumers or creative subjects, recent field studies demonstrated that media conglomerates have become more aware of how fan creations can add to and affect media product development, marketing, and distribution. Therefore they seek to harness fan activities for low-cost and effective advertisements [14, 16]. Nevertheless, it is also true that the monetization of fan creations is not an easy aim to accomplish: fans show an anticommercial nature and embrace a gift economy model [17], often refusing to be incorporated in the commercial production process [11].

All techniques are exploited in the digital ecosystem. Feedback opportunities provided by Web 2.0 technologies represent both new chances for audience engagement and new challenges for the television industry [13, 18]. In stead of replacing television, the Web seems to have embraced it and, with regard to TV series, the Internet seems to have turned television into a shared event for the most passionate viewers, who have become plugged-in, mainly sharing the show with the other viewers [19–21]. Indeed, through online communities passionate viewers find and share a privileged knowledge which is derived both from the viewing experience and the extratextual elements of the TV program [21]. Fans try to influence TV series production, promotion, and distribution in two main ways: (a) actively petitioning a TV network to keep a show on air and (b) participating online in official and unofficial promotional activities in order to keep communication about the show alive.

As we said, it is interesting to go beyond the opposition between producer and consumer, focusing on the role of fan communities in promoting TV series within the framework of the gift economy [14, 22–24]. The media product is a gift which the receiver can reciprocate through attention, feedback, fandom, or even purchasing advertised products. In particular, digital environments embrace the reply instead of reciprocation: it means giving a feedback to the author or passing along to others. Media fandom is a battleground where the territory between online gift economies and commodity culture will be negotiated.

In this paper we focus on American TV series, which are built in order to be “distributed” to an international audience [15] and use transmedia storytelling [25] as a promotional strategy to engage a wide and dispersed audience. Indeed, many TV series have been distributed with success in European countries, giving birth to strong fan communities [26, 27].

There exists a correlation between TV programs’ ratings and choices (even the choice of doing nothing) by the DM: ratings act as a proxy of the product quality. However, the DM cannot completely control the program popularity because of the consumer empowerment. Media marketing managers usually pursue all possible actions to back a TV program and limit the risk of failure. However, this is a costly behaviour and understanding the impacts of specific actions is crucial to reduce investments and increase returns. To conclude, it is evident that popularity is affected and sustained not only by media managers’ decisions, as viewers are empowered. Indeed, in Section 3 the model will consider both agents.
3. Popularity of TV Series: A Dynamic Model

Connected marketing is used to underpin the popularity of a brand or a product. The success is affected by the capability of generating engagement with customers and the engagement is relevant if we consider media content.

From now on, let \( P(t) \) be a time dependent variable which describes the degree of popularity of a given product at the time \( t \). We now formulate a model (similar models have been used in different contexts such as economic growth, population dynamics, and technology diffusion,) which describes the behaviour of \( P \) over time and allows us to draw long run analysis. We consider a local analysis, in other words, the diffusion path occurring in a specific country. We assume that popularity is driven by the following differential equation:

\[
\frac{dP(t)}{dt} = (\eta - \delta)P(t) + \tau, \tag{1}
\]

where

(i) \( \delta \) is the natural depreciation rate (\( 0 < \delta < 1 \)) of popularity. Every product or service has a life cycle and when the market is mature, it enters the decline stage and may die. This term is also linked to the cultural discount \([28]\) that is a particular program is rooted in one culture and thus is attractive in that environment, whilst it shall have diminished appeal elsewhere. The higher the cultural discount is, the higher the depreciation rate is;

(ii) \( \eta \) describes the actions made by the decision maker to support the campaign and by the other customers to spread information concerning the product;

(iii) \( \tau \) is an exogenous factor, which takes into account the influence of the popularity of the product abroad. A negative value means that the product failed in a foreign market and had a bad reputation which can affect the campaign in the country under analysis. A positive value represents the success of the product or service abroad which can be translated in positive network externalities in the country under analysis. Even if it is a local model, we have to consider the effects derived from information flows and the track records of the product elsewhere.

It is possible to show that model (1) admits a solution of the form \( (\eta \neq \delta) \)

\[
P(t) = \left( P(0) + \frac{\tau}{\eta - \delta} \right) e^{-\delta \eta t} - \frac{\tau}{\eta - \delta}. \tag{2}
\]

This kind of dynamic model formulated in terms of differential equation has been widely used in the economic and social science literature to describe the evolution of variables such as innovation, population, and human capital (see for instance \([29–32]\)). If \( \eta = \delta \) the above model (1) becomes trivial and it admits either a constant solution or a linear behavior. (\( \tau = 0 \) or \( \tau \neq 0, \) resp).

4. Model Discussion

In the following discussion we present for each possible solution a set of cases that could represent a concrete occurrence of the solution itself. The examples we are providing are only explanatory and limited to the field of American TV series imported in Italy. We decided to focus on American TV series for different reasons. First, foreign TV series and especially American ones have historically dominated the Italian television market. This gave us a sample which was internally coherent but provided also a wide set of cases to test our model with. Since we are taking into consideration not only marketing actions undertaken by the local networks in order to support specific TV series, but also cultural practices and fan activities that helped even unsupported TV series to gain strength in the local market, the fact that a remarkable number of American TV series had successfully conquered the Italian public, despite the inherent cultural differences between those shows and the local audience strategic \([33]\), was also an influential factor in choosing our sample.

This was the typical case of American TV series bought through an output deal, where the Italian broadcasters agreed to buy everything a specific major produces for an established period of time. This kind of blind agreement, obviously, leads to the purchase of TV series that are not strategic or even adapted to the broadcasting network entitled to air them. In many cases Italian TV channels used those “residual” products to fill up the night schedule or the many “void” spaces of a full day schedule. This meant that the popularity and ratings of the TV series abroad were not influential in choosing the time slot in which the products were aired, nor the broadcasting network felt the need to promote or push the product inside the local market.

One classic example of the case in which there is no influence from abroad and no actions are undertaken. The solution to (1) is given by \( P(t) = P(0)e^{-\delta t} \). As a result \( P \) goes to zero in the long run (Figure 1): the popularity tends naturally to fade away. If the decision maker does not intervene, the attention of consumers will focus on something else.

A negative value means that the product failed in a foreign market and had a bad reputation which can affect the campaign in the country under analysis. A positive value represents the success of the product or service abroad which can be translated in positive network externalities in the country under analysis. Even if it is a local model, we have to consider the effects derived from information flows and the track records of the product elsewhere.
The first five years of *The Bold and the Beautiful* (1987) in US were not a good start for the CBS soap; it was in fact the network’s lowest-rated soap. In this case though, the aim of the producers was to create an international success and not a national one. In fact since its first airing the soap has become the most-watched soap in the world with an audience of an estimated 26.2 million viewers [37]. In Italy the investment made on the soap, simply known as *Beautiful*, was huge in terms of promotion and distribution since the beginning of its airing in 1990. The soap was scheduled in a crucial daytime timeslot (13.45) on Raidue and later on even in prime time with a rhythm of 3 episodes per night. In 1994 the soap was bought by Mediaset but the daytime slot remained the same even when the series started its airing on Canale5. This was a crucial point in the promotional campaign which stated that “the lunch time is always beautiful” stating the happy marriage between Beautiful and the 13.40 time slot. The faithfulness of the Italian audience toward the soap was built over time and based on the strategic scheduling, the collaboration between the American producers and Mediaset group which lead to the setting of many storylines in Italy (f. i. Venezia 1997, Como 2000, and Portofino 2007), the investments in the promotional area that lead to localized promos with the leading actors and frequent visits from the cast in Italian talk shows and entertainment programs (last of which was the participation of Ronn Moss, the leading male actor, in the *Dancing with the stars* Italian edition) and the investments in the distribution area, with special edition DVD sold in kiosks, specialized volumes published by Mediaset Group and a rear of the daytime episodes in prime time on Mediaset digital platforms (La5 channel). The ratings for the American soap reflected its popularity in Italy with episodes reaching the record of 7 million viewers and a 37% share and more than 400 episodes with 6 million of viewers each (Auditel data, 34). Interesting enough the international success in this case reverberated on the original country of production in two ways: for once the soap was kept under production because of the selling power registered abroad; secondly, the popularity of the show abroad seemed to affect the national audiences and by the mid-to-late 1990s the numbers of the soap began to climb and in the 2000s constantly held the second position in the Nielsen weekly ratings for daytime dramas.
(b) If $\eta < \delta$ the product is not able to gain attention and in the long run people will forget it.

Women Murders Club, a legal drama which ran on ABC from October 12, 2007, to May 13, 2008, was affected by the 2007/2008 Writers Guild of America and by the subsequent erratic programming: after the first 10 episodes were aired, people had to wait 4 months to see the last 3 episodes, the show was also put in the Friday night prime time slot which is considered a “death slot,” due to the concept that many shows scheduled on or moved to Friday nights would not last long before cancellation due to low ratings. From a reception point of view not only the ratings were low (first season ranked 57th with a medium of 8.964 million viewers), but also the critical response was tepid with Goodman [38], the San Francisco Chronicle TV critic, publishing a highly critical review assigning the paper’s lowest possible rating and the New York Times TV critic Stanley describing the show as “all right, but not good enough” [39]. In fact, Women Murders Club was cancelled on May 12, 2008. Mediaset had purchased the drama series before the strike, judging just from the pilot, and could not of course forecast the damages the strike would have had on the development and even the ratings of many American TV shows. Women Murders Club suddenly became an unwanted product because of the fast cancellation and the lack of popularity in the original country. Mediaset ran the TV series first on the digital channel Joy, part of its Premium Gallery, and later, because of the general lack of attention, decides to use it to fill up the prime time slot of Canale 5 during the summer of 2010. The choice of programming a new drama series outside the guarantee period (11 January–30 June) is unusual but wise in this case: the tepid reception of the TV series first run both in US and in Italy, the lack of a previous successful campaign and of previous good results to build from and the small number of episodes available, convinced the network that the series could not be considered strategic, despite the fact that it was consistent to the channel identity and target. Women Murders Club is then offered to the summer audience as a gift, with a promotion built around its unusual scheduling (“Canale 5 offers you something new, even if it is Summer”) and its resemblance to more popular dramas (“A mix between CSI, Sex and The City, and Grey’s Anatomy”). The series is then used up as fast as possible: in the first week 4 episodes are aired in two nights (Tuesday and Thursday), and from the second week the series is aired with a rhythm of 2 episodes per night on Thursday. After a good start (14.72% share and winner of the prime time) the ratings drop down week after week and the TV series is constantly surpassed by old episodes of the scientific program Quark, aired by RAI in the same timeslot.

(C) $\tau \neq 0$ Under these hypotheses the solution is $P(t) = (P(0) + \tau/(\eta-\delta))e^{(\tau-\delta)t} - \tau/(\eta-\delta)$. Different scenarios can occur:

(a) If $\eta > \delta$ then $P$ will increase providing a successful campaign, whatever is the influence from abroad; great investments in creating a fandom would overturn the performance and decrease the effects of bad reputation.

(b) If $\eta < \delta$ the popularity will tend to $-\tau/(\eta-\delta)$. In this case the reputation and popularity abroad really drive the internal performance.

(b1) If $\tau$ is positive, then the long run results of the campaign will end up in a success case, whose intensity is proportional to $\tau$.

In this case the popularity of the TV series in the local market is connected with the popularity of the TV series abroad. CSI Miami (2002), the first spin-off of the forensic drama CSI, arrived in Italy at the end of 2003, after the huge success registered by the original CSI on the channel Italia 1 and the even more huge success of the show in the worldwide market. It was not necessary to build up a big promotional campaign for the TV series because the international success of the CSI franchise was speaking for itself. Later on, CSI Miami was even named the most popular program in the world because it featured in more countries’ top 10 rankings for 2005 than any other [40] and this data was used in the Italian promos for the show. Building from the international success of the franchise and of the TV series itself CSI Miami soon became one of the most viewed programs of Wednesday night and the most viewed series between the CSI franchises in Italy. The confidence in the Italian success of CSI Miami is so strong that in 2006 the series was even used to counterbalance the flop of the third season of The OC, replacing the teen drama on Sunday night and airing twice a week for months.
This was the case of *Defying Gravity*, a multinationally produced space travel drama, which first aired on ABC in August 2009. The low ratings convinced the American network to suspend the series, even if only 8 episodes out of the 13 produced had been aired. When the debacle of the series abroad was clear, the Italian satellite channel Sky had already purchased the full first season and decided to air the product anyway on Fox Life starting 12 June 2010 in the late prime time-slot (21:55). The promotion and distribution of the show could not, in this case, profit from the international success and had instead to fight against the bad popularity of the TV series, which was even more evident because of the high profile promotion made in the original country. *Defying Gravity* was in fact supposed to have 6 seasons, 6 years was in fact the length of the interplanetary voyage which planned to visit most of the planets of the Solar System, and was marketed as "*Grey's anatomy* set in space" establishing a direct link with the most successful medical drama of ABC and playing with the fact that the two series had the same producing team. This kind of promotion made the series' flop even more problematic for the international broadcasters. Sky reused the ABC reference to *Grey's Anatomy*, a huge success in the Italian market too, but tried to obscure any reference to sci-fi, selling the show as a soap and adding to the main title the subtitle "the galaxy of hearts." The choice of scheduling *Defying Gravity* on Fox Life—dedicated to family drama and medicals—and the late night summer slot-time, though, were clear indications that the product was not considered strategic anymore because of the bad reputation it had gained abroad.

If $\eta = 0$ then the popularity will tend to $\tau/\delta$. This is the case in Italy of the *Law and Order* franchise (1990), the success ($\tau > 0$) of which is obviously disconnected from the amount of local investments: the dedication of the broadcasting networks in the promotion and distribution area for the single series, in fact, has always tended to zero in Italy. This is shown by the small number of promos realized for each series all through their airing, by the scheduling of all the series in the late night or even in the night-time slots and by the fact that they have been running both on RAI and Mediaset channels, with no intention of connecting the single series to the specific channels. *Law and Order SVU*, for instance, has been used by Rete4 to fill up void spaces in the late night schedule (between October and November 2007, 10 out of the 22 episodes of the first season were aired, in no particular order), before becoming the regular fill for the late night time slot of the channel. *Law and Order* timeslot on Raidue often changed from daytime to late night and back again, depending on the nature of the single episode, while the 17th season (last one of the original series) has been moved to the prime time of Raitre. Despite all the changes and lack of promotion the constant popularity of the TV series abroad and the strong niche following in Italy has made of the *Law and Order* franchise a reliable show, with special regards to the ratings which remain constant.

4.1. *An Empirical Evidence*. As mention above, in Italy, Supernatural’s performance has been affected by the lack of (or scarce) promotion and erratic scheduling (in particular shifting from springtime to summer, a season characterized by lower audience data).

Figure 2 shows the Auditel data [41] of the first four series of Supernatural broadcast on Rai2 channel in the period 2007–2010: in detail the first series was broadcast during February–July 2007, the second June–September 2008, the third July–September 2009, and the fourth July–August 2010. The x axis shows the sequence of episodes, whilst the y axis lists the corresponding audience data.

Figure 2 suggests that there are fluctuations of the data around the black trend line, whose intensity is decreasing over time. However, Supernatural’s long-run behavior can be well approximated by the function $P(t) = 10^6e^{-0.013t}$ that shows an exponential decay of popularity. In particular this empirical evidence shows that $\eta - \delta = -0.013$, that is, $\eta < \delta$, implying that either $\eta = 0$ (lack of promotion) or nonzero $\eta < \delta$ (scare promotion).

The parametric estimation has been done using the technique presented in [42, 43] and based on the solution of an inverse problem for a differential equation.

5. *An Optimal Decision Model: The Decision Maker View Point*

In a media company the decision maker faces the problem of maximizing profits subject to some constraints. Media companies must cover their costs (license fees, production costs of programs, ...) and create returns just as any other business [44]. The above formulated model is purely dynamic and that does not provide any optimal decision rule. The purpose of this section is indeed to formulate a dynamic programming model that can fully describe the complexity of a decision making process [45, 46]. We assume we have a closed market in which there is no influence of the popularity of the product abroad and then $\tau$ is assumed to be
zero. With respect to the purely dynamic model previously described in which the term \( \eta \) was supposed to be constant, in this context we assume that \( \eta(P(t)) = \Omega P(t) - \pi(t) \) where \( \Omega P(t) \) represents the revenues (depending on the level of popularity) that a TV company receives from the sale of ad spaces to advertisers (\( \Omega \) can be interpreted as the price of ad space) while \( \pi(t) \) are the profits to be maximized by the DM.

The difference \( \Omega P(t) - \pi(t) \) is the investment in a marketing campaign. The DM tries to maximize his/her intertemporal utility subject to a dynamic constraint that describes the evolution of popularity and it can be formulated as follows:

\[
\text{Max} \quad \int_0^{+\infty} U(\pi(t)) e^{-\rho t} dt
\]

s.t.
\[
\frac{dP(t)}{dt} = (-\delta + \Omega P(t) - \pi(t)) P(t)
\]

(3)
\[
P(0) = P_0,
\]

where \( \rho \) is the discount rate.

The variable \( P(t) \) is the state variable while \( \pi(t) \) is the control. For simplicity we assume a logarithmic utility \( U \), that is, \( U(\pi(t)) = \ln(\pi(t)) \), even if other utility expressions could be considered analogously. The following result states the existence of a nontrivial equilibrium for the above dynamic model.

**Proposition 1.** Suppose \( \rho > \delta \). Then the above model shows the existence of a nonzero equilibrium \((P^*, \pi^*)\) where

\[
P^* = \frac{\rho}{\Omega}, \quad \pi^* = \rho - \delta.
\]

**Proof.** It is provided in the Appendix. \( \square \)

In other words, if the technical condition \( \rho > \delta \) is satisfied, this dynamic model shows the existence of a nontrivial equilibrium \((P^*, \pi^*)\). The equilibrium of popularity \( P^* \) depends on the price \( \Omega \) and the higher will be the cost of a media product for a customer the lower will be the level of \( P^* \). However, as the analytical discussion in the Appendix shows, this equilibrium is unstable; this practically means that only under strict conditions the popularity will not vanish in the long run.

6. Conclusions

Internet provides tools that can alter the whole experience of watching television, giving access to the process, not just the results. Through online communities, in fact, passionate viewers find and share a privileged knowledge which is derived both from the viewing experience and the extratextual elements of the show [21]. According to recent studies, fans are making a new kind of television scene that transcends place and shakes up long-standing balances of power between fans and the television makers. In general the most recent literature published in the field of fan studies suggests that the line between consumption and production is blurring into a new paradigm, with fans growing up to be creators and creators trying to connect and engage with fan communities when not labeling themselves as fans [13, 21, 47].

The most successful TV series analyzed in this study have, in fact, been created, promoted, and distributed not only taking into account the existence of an active and internet-based fan audience but also relying on this same audience in order to make the series an international success. Compared with the ones that did not profit from the presence of a fan audience or did not use connected marketing strategies, these TV series have indeed succeeded in generating deep, perpetual audience engagement.

From a previous desk research and exploratory analysis [48], it emerged in fact that (a) there is a willingness to consume texts offered by producers in different formats outside of the narrative space of the show itself and (b) part of the reason for this extratextual success lies in the initial construction and marketing of the show itself.

Building from these results, the model we propose here tries to identify and explain the main factors on which the popularity of American TV series in Italy is based. All the examples confirm in fact the relevance of collateral actions, specifically web orientated, to underpin the success of television’s content as well as the influence of foreign popularity (globalization), which can be considered the explaining variable involved in the success of a media product.

The paper also introduces a decisional dynamic model able to sustain the DM in his/her decisions. In the Italian scenario, commercial television plays a predominant role and competes to attract viewers’ attention and then sells that attention to advertisers [49]. In other words a broadcasting media platform can only succeed if it has viewers; otherwise, its revenues from advertising would be zero. As it is an advertisement-supported media, from the two-sided markets literature we know that ad prices reflect both the value of reaching a given audience and the marginal effect of the advertisement sale on total audience size. Thus popular TV programs indirectly generate returns for a media company attracting viewers. Our model provides the DM the optimal solution balancing the investment in sustaining program popularity and the maximization of profits.

For future works we aim at modifying this model by introducing a stochastic term able to catch the natural fluctuations arising from audience data.

**Appendix**

In an optimal decision making process, the DM tries to solve the following model:

\[
\text{Max} \quad \int_0^{+\infty} \ln(\pi(t)) e^{-\rho t} dt
\]

s.t.
\[
\frac{dP(t)}{dt} = (-\delta + \Omega P(t) - \pi(t)) P(t)
\]

(5)
\[
P(0) = P_0.
\]
The Hamiltonian associated with model (5) is
\[ H(P(t), \pi(t)) = \ln(\pi(t)) e^{-\rho t} + \lambda(t) \times (-\delta P(t) + (\Omega P(t) - \pi(t)) P(t)) . \] (6)

Optimality conditions for (6) read as
\[ 0 = \frac{\partial H}{\partial \pi} = e^{-\rho t} - \lambda(t) P(t), \]
\[ -\dot{\lambda}(t) = \frac{\partial H}{\partial P} = \lambda(t) (-\delta + 2\Omega P(t) - \pi(t)), \]
\[ \dot{P}(t) = \frac{\partial H}{\partial \lambda} = -\delta P(t) + (\Omega P(t) - \pi(t)) P(t). \] (7)

The system (7) is equivalent to
\[ \pi(t) = \frac{e^{-\rho t}}{\lambda(t) P(t)} \Rightarrow \frac{\dot{\pi}(t)}{\pi(t)} = -\rho - \frac{\dot{\lambda}(t)}{\lambda(t)} - \frac{\dot{P}(t)}{P(t)} , \]
\[ -\dot{\lambda}(t) = \lambda(t)(-\delta + 2\Omega P(t) - \pi(t)) \]
\[ \Rightarrow -\frac{\dot{\lambda}(t)}{\lambda(t)} = -\delta + 2\Omega P(t) - \pi(t) , \]
\[ \dot{P}(t) = -\delta P(t) + (\Omega P(t) - \pi(t)) P(t) \]
\[ \Rightarrow \frac{\dot{P}(t)}{P(t)} = -\delta + \Omega P(t) - \pi(t) . \] (8)

which implies
\[ \frac{\dot{\pi}(t)}{\pi(t)} = -\rho + \Omega P(t) , \]
\[ \frac{\dot{P}(t)}{P(t)} = -\delta + \Omega P(t) - \pi(t) . \] (9)

The only nontrivial equilibrium is
\[ P^\ast = \frac{\rho}{\Omega} , \]
\[ \pi^\ast = \rho - \delta > 0 . \] (10)

The Jacobian of the linearized system at \((P^\ast, \pi^\ast)\) is equal to
\[ J = \begin{bmatrix} 0 & \Omega (\rho - \delta) \\ -\rho & \rho \end{bmatrix} . \] (11)

The determinant of \(J\) is equal to
\[ \text{det}(J) = \rho (\rho - \delta) > 0 , \] (12)

while the trace of \(J\) is equal to
\[ \text{trace}(J) = \rho > 0 , \] (13)

which implies that \((P^\ast, \pi^\ast)\) is an unstable equilibrium. It is possible to show that this system admits three more equilibria, namely \((0, 0)\), \((P^\ast, 0)\) and \((0, \pi^\ast)\). Among them, \((0, 0)\) and \((0, \pi^\ast)\), are locally asymptotically stable.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References


Submit your manuscripts at http://www.hindawi.com