Research Article

Solving Fokker-Planck Equations on Cantor Sets Using Local Fractional Decomposition Method

Shao-Hong Yan,^{1,2} Xiao-Hong Chen,¹ Gong-Nan Xie,³ Carlo Cattani,⁴ and Xiao-Jun Yang⁵

¹ College of Geophysics and Information Engineering, China University of Petroleum, Beijing 102249, China

² College of Science, Hebei United University, Tangshan 063000, China

³ School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an, Shaanxi 710048, China

⁴ Department of Mathematics, University of Salerno, Via Ponte don Melillo, Fisciano, 84084 Salerno, Italy

⁵ Department of Mathematics and Mechanics, China University of Mining and Technology, Xuzhou, Jiangsu 221008, China

Correspondence should be addressed to Shao-Hong Yan; shaohong@heuu.edu.cn

Received 4 February 2014; Accepted 6 March 2014; Published 31 March 2014

Academic Editor: Ali H. Bhrawy

Copyright © 2014 Shao-Hong Yan et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The local fractional decomposition method is applied to approximate the solutions for Fokker-Planck equations on Cantor sets with local fractional derivative. The obtained results give the present method that is very effective and simple for solving the differential equations on Cantor set.

1. Introduction

The Fokker-Planck equation [1–16] plays an important role in describing the system dynamics. For example, the Langevin approach for microscopic dynamics [1], the dynamics of energy cascade in turbulence [2], the population dynamics [3], the chaotic universe dynamics [4], the fatigue crack growth dynamics [5], the fission dynamics [6], the dynamics of distributions of heavy quarks [7] and financial returns [8], the spin relaxation dynamics [9], the electron dynamics of plasmas and semiconductors [10], the critical dynamics [11], and the fiber dynamics [12] had been investigated by using the Fokker-Planck equation (see [13, 14] and the cited references therein) and its 3 solution was presented by the different methods. There are some methods for solving the differential equations, such as Adomian decomposition, Homotopy perturbation, Variational iteration, Metropolis Monte Carlo, multiscale finite element and finite difference methods (see, e.g., [15–20]), and others [21–25].

In recent years, fractional calculus was applied to model many fractal dynamical systems [26–30]. The fractional Fokker-Planck equation [26–38], as one of dynamical equations, has interested many researchers. Its solution was also investigated in [39–41]. However, the above fractional

Fokker-Planck equation did not describe the nondifferentiable behaviors of dynamical systems because of the limit of the fractional operators. In order to overcome the above problems, the local fractional calculus was developed and applied to the fractal phenomenon in science and engineering [42–53]. Local fractional Fokker-Planck equation [45], which was an analog of a diffusion equation with local fractional derivative, was suggested as follows:

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = \frac{\Gamma(1+\alpha)}{4} \chi_C(t) \frac{\partial^2}{\partial x^2} u(x, t), \quad (1)$$

where the local fractional operator was the Kolwankar-Gangal local fractional differential operator. In [46], the Fokker-Planck equation on a Cantor set with local fractional derivative was presented as follows:

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x, t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x, t), \quad (2)$$

where the local fractional partial differential operator of order α ($0 < \alpha \leq 1$) was defined as [42–44]

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x_0, t) = \frac{\Delta^\alpha(u(x_0, t) - u(x_0, t_0))}{(t - t_0)^\alpha} \quad (3)$$

with

$$\Delta^\alpha (u(x_0, t) - u(x_0, t_0)) \cong \Gamma(1 + \alpha) [u(x_0, t) - u(x_0, t_0)]. \quad (4)$$

Analytical and approximate solutions for local fractional differential equations were presented by different researchers (see [43, 46–53] and the cited references therein). Applications of local fractional decomposition method were presented (see [51–53] and the cited references therein). Our main purpose of the paper is to apply the local fractional decomposition method to solve the Fokker-Planck equations on a Cantor set.

In this paper, Section 2 gives the recent results for local fractional integral operator. The local fractional decomposition method is analyzed in Section 3. The approximate solutions are presented in Section 4. Finally, conclusions are given in Section 5.

2. The Local Fractional Integral Operator

In this section, we introduce the local fractional integral operator and its recent results.

Definition 1 (see [42, 46–53]). Let the function $f(x) \in C_\alpha(a, b)$, if it is valid for

$$|f(x) - f(x_0)| < \varepsilon^\alpha, \quad (5)$$

where $|x - x_0| < \delta$, for $\varepsilon > 0$ and $\varepsilon \in R$.

Definition 2 (see [42, 43, 46–53]). Let $f(x) \in C_\alpha[a, b]$. The local fractional integral of $f(x)$ of order α in the interval $[a, b]$ is defined as

$$\begin{aligned} {}_a I_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1 + \alpha)} \int_a^b f(t) (dt)^\alpha \\ &= \frac{1}{\Gamma(1 + \alpha)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)^\alpha, \end{aligned} \quad (6)$$

where the partitions of the interval $[a, b]$ are denoted by (t_j, t_{j+1}) , $j = 0, \dots, N-1$, $t_0 = a$ and $t_N = b$ with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_0, \Delta t_1, \Delta t_2, \dots\}$.

Some properties of local fractional integrals operator used in the paper are listed as follows [42, 51]:

$$\begin{aligned} {}_a I_x^{(\alpha)} f(x) g^{(\alpha)}(x) &= [f(x) g(x)] \Big|_a^x - {}_a I_x^{(\alpha)} f^{(\alpha)}(x) g(x), \\ {}_a I_b^{(\alpha)} {}_c I_d^{(\alpha)} \psi(x, y) &= {}_c I_d^{(\alpha)} {}_a I_b^{(\alpha)} \psi(x, y), \\ {}_a I_x^{(\alpha)} {}_a I_t^{(\alpha)} f(t) &= {}_a I_x^{(\alpha)} \left[\frac{(x-t)^\alpha}{\Gamma(1 + \alpha)} f(t) \right]. \end{aligned} \quad (7)$$

The formulas of local fractional integrals operator used in the paper are listed as follows [42]:

$$\begin{aligned} {}_0 I_x^{(\alpha)} E_\alpha(x^\alpha) &= E_\alpha(x^\alpha) - 1, \\ {}_0 I_x^{(\alpha)} \frac{x^{n\alpha}}{\Gamma(1 + n\alpha)} &= \frac{x^{(n+1)\alpha}}{\Gamma(1 + (n+1)\alpha)}. \end{aligned} \quad (8)$$

Definition 3 (see [42, 43, 46–53]). Let $f(x) \in C_\alpha[a, b]$. The local fractional derivative of $f(x)$ of order α in the interval $[a, b]$ is defined as

$$\frac{d^\alpha f(x_0)}{dx^\alpha} = \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha}, \quad (9)$$

where

$$\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1 + \alpha) [f(x) - f(x_0)]. \quad (10)$$

The formulas of local fractional differential operator used in the paper are listed as follows [42]:

$$\begin{aligned} \frac{d^\alpha}{dx^\alpha} E_\alpha(x^\alpha) &= E_\alpha(x^\alpha), \\ \frac{d^\alpha}{dx^\alpha} E_\alpha(-x^\alpha) &= -E_\alpha(x^\alpha), \\ \frac{d^\alpha}{dx^\alpha} \frac{x^{3\alpha}}{\Gamma(1 + 3\alpha)} &= \frac{x^{2\alpha}}{\Gamma(1 + 2\alpha)}. \end{aligned} \quad (11)$$

3. Analysis of the Method

In this section, the local fractional decomposition method for a class of differential equations defined on Cantor is given.

We now write (2) in the following form:

$$L_t^{(\alpha)} u(x, t) = -L_x^{(\alpha)} u(x, t) + L_{xx}^{(2\alpha)} u(x, t), \quad (12)$$

where $L_{xx}^{(2\alpha)} = \partial^{2\alpha} / \partial x^{2\alpha}$ is a 2α th local fractional differential operator with respect to x , $L_t^{(\alpha)} = \partial^\alpha / \partial t^\alpha$ is a α th local fractional differential operator with respect to t , and $L_x^{(\alpha)} = \partial^\alpha / \partial x^\alpha$ is α th local fractional differential operator with respect to x .

The initial condition reads as follows:

$$u(x, 0) = f(x) \quad (0 \leq x \leq l). \quad (13)$$

We now define the α th-fold local fractional integral operator in the form

$$L_t^{(-\alpha)} m(t) = {}_0 I_t^{(\alpha)} m(s). \quad (14)$$

In view of (13), we structure

$$L_t^{(-\alpha)} L_t^{(\alpha)} u(x, t) = L_t^{(-\alpha)} \{-L_x^{(\alpha)} u(x, t) + L_{xx}^{(2\alpha)} u(x, t)\}. \quad (15)$$

Hence, from (15), we have

$$u(x, t) = r(x) - L_t^{(-\alpha)} L_x^{(\alpha)} u(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u(x, t), \quad (16)$$

where the nondifferentiable term $r(x)$ is obtained from the initial condition.

Making use of (16), for $n \geq 0$, we give the recurrence relationship in the following form:

$$u_{n+1}(x, t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x, t), \quad (17)$$

FIGURE 1: Graph of $u(x, t)$ with the parameter $\alpha = \ln 2 / \ln 3$.

FIGURE 2: Graph of $u(x, t)$ with the parameter $\alpha = \ln 2 / \ln 3$.

subject to the initial value

$$u_0(x, t) = r(x). \quad (18)$$

Finally, the approximation form of the solution reads as

$$u(x, t) = \lim_{n \rightarrow \infty} \phi_n(x, t) = \lim_{n \rightarrow \infty} \sum_{i=0}^{\infty} u_i(x, t). \quad (19)$$

4. The Approximate Solutions

In this section, we investigate the approximate solutions for Fokker-Planck equations on Cantor sets with local fractional derivative by using the local fractional decomposition method.

Example 1. Let us consider the following Fokker-Planck equation on a Cantor set with local fractional derivative in the form

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x, t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x, t), \quad (20)$$

subject to the initial value

$$u(x, 0) = \frac{x^{2\alpha}}{\Gamma(1 + 2\alpha)}. \quad (21)$$

In view of (17), we have the recurrence formulas in the form

$$u_{n+1}(x, t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x, t), \quad (22)$$

$$u_0(x, t) = u(x, 0) = \frac{x^{2\alpha}}{\Gamma(1 + 2\alpha)}. \quad (23)$$

From (23), we obtain the following approximate formulas:

$$\begin{aligned} u_1(x, t) &= -\frac{x^\alpha}{\Gamma(1 + \alpha)} \frac{t^\alpha}{\Gamma(1 + \alpha)} + \frac{t^\alpha}{\Gamma(1 + \alpha)}, \\ u_2(x, t) &= \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)}, \\ u_3(x, t) &= 0, \\ &\vdots \\ u_n(x, t) &= 0. \end{aligned} \quad (24)$$

Hence, the nondifferentiable solution for (20) with the initial value (21) is

$$\begin{aligned} u(x, t) &= \frac{x^{2\alpha}}{\Gamma(1 + 2\alpha)} + \frac{t^{2\alpha}}{\Gamma(1 + 2\alpha)} \\ &\quad - \frac{x^\alpha}{\Gamma(1 + \alpha)} \frac{t^\alpha}{\Gamma(1 + \alpha)} + \frac{t^\alpha}{\Gamma(1 + \alpha)}, \end{aligned} \quad (25)$$

and its graph is shown in Figure 1.

Example 2. We consider the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x, t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x, t), \quad (26)$$

together with initial condition

$$u(x, 0) = E_\alpha(x^\alpha). \quad (27)$$

From (17), we get the recurrence formulas in the form

$$u_{n+1}(x, t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x, t), \quad (28)$$

$$u_0(x, t) = u(x, 0) = E_\alpha(x^\alpha). \quad (29)$$

FIGURE 3: Graph of $u(x, t)$ with the parameter $\alpha = \ln 2 / \ln 3$. FIGURE 4: Graph of $u(x, t)$ with the parameter $\alpha = \ln 2 / \ln 3$.

Making use of (29), we reach the following formulas:

$$\begin{aligned} u_1(x, t) &= 0, \\ u_2(x, t) &= 0, \\ &\vdots \\ u_n(x, t) &= 0. \end{aligned} \quad (30)$$

Therefore, the nondifferentiable solution of (26) with the initial value (27) reads as follows:

$$u(x, t) = E_\alpha(x^\alpha) \quad (31)$$

with plot shown in Figure 2.

Example 3. We present the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x, t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x, t) \quad (32)$$

and suggest the initial condition

$$u(x, 0) = E_\alpha(-x^\alpha). \quad (33)$$

In view of (17), the recurrence formulas can be written as

$$u_{n+1}(x, t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x, t), \quad (34)$$

$$u_0(x, 0) = u(x, 0) = E_\alpha(-x^\alpha). \quad (35)$$

From (35), we get the following approximate equalities:

$$\begin{aligned} u_1(x, 0) &= \frac{2t^\alpha}{\Gamma(1+\alpha)} E_\alpha(-x^\alpha), \\ u_2(x, 0) &= \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)} E_\alpha(-x^\alpha), \\ u_3(x, 0) &= \frac{8t^{3\alpha}}{\Gamma(1+3\alpha)} E_\alpha(-x^\alpha) \\ &\vdots \\ u_n(x, t) &= \frac{2^n t^{n\alpha}}{\Gamma(1+n\alpha)} E_\alpha(-x^\alpha). \end{aligned} \quad (36)$$

Therefore, the nondifferentiable solution of (32) with the initial value (33) reads as follows:

$$u(x, t) = E_\alpha(2t^\alpha) E_\alpha(-x^\alpha), \quad (37)$$

together with plot shown in Figure 3.

Example 4. We suggest the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x, t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x, t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x, t), \quad (38)$$

and the initial condition is

$$u(x, 0) = -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \quad (39)$$

From (17), the recurrence formulas read as follows:

$$u_{n+1}(x, t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x, t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x, t), \quad (40)$$

$$u_0(x, t) = u(x, 0) = -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \quad (41)$$

Hereby, from (41), we have the following formulas:

$$\begin{aligned}
 u_1(x, t) &= \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{x^\alpha}{\Gamma(1+\alpha)}, \\
 u_2(x, t) &= -\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{2t^{2\alpha}}{\Gamma(1+2\alpha)}, \\
 u_3(x, t) &= \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \\
 u_3(x, t) &= 0, \\
 &\vdots \\
 u_n(x, t) &= 0.
 \end{aligned} \tag{42}$$

So the nondifferentiable solution of (32) with the initial value (33) is

$$\begin{aligned}
 u(x, t) &= \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{2t^{2\alpha}}{\Gamma(1+2\alpha)} \\
 &\quad - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} - \frac{t^\alpha}{\Gamma(1+\alpha)} \frac{x^\alpha}{\Gamma(1+\alpha)} \\
 &\quad - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \frac{x^\alpha}{\Gamma(1+\alpha)},
 \end{aligned} \tag{43}$$

together with plot illustrated in Figure 4.

5. Conclusions

In this work, we had used the local fractional decomposition method to solve the Fokker-Planck equations on Cantor sets which were described by the local fractional differential operator. The nondifferentiable solutions were obtained. The obtained results show that the present method is a very effective and powerful mathematical tool for finding the nondifferentiable solutions for the local fractional differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by the National Scientific and Technological Support Projects (no. 2012BAE09B00), the National Natural Science Foundation of China (no. 51274270), and the National Natural Science Foundation of Hebei Province (no. E2013209215).

References

- [1] T. D. Frank, "A Langevin approach for the microscopic dynamics of nonlinear Fokker-Planck equations," *Physica A: Statistical Mechanics and its Applications*, vol. 301, no. 1-4, pp. 52-62, 2001.
- [2] A. Naert, R. Friedrich, and J. Peinke, "Fokker-Planck equation for the energy cascade in turbulence," *Physical Review E - Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics*, vol. 56, no. 6, pp. 6719-6722, 1997.
- [3] M. Saad and T. Goudon, "On a Fokker-Planck equation arising in population dynamics," *Revista Matemática Complutense*, vol. 11, no. 2, pp. 353-372, 1998.
- [4] D. H. Coule and K. O. Olynyk, "Chaotic universe dynamics using a Fokker-Planck equation," *Modern Physics Letters A*, vol. 2, no. 11, pp. 811-817, 1987.
- [5] A. Tsurui and H. Ishikawa, "Application of the Fokker-Planck equation to a stochastic fatigue crack growth model," *Structural Safety*, vol. 4, no. 1, pp. 15-29, 1986.
- [6] J. R. Nix, A. J. Sierk, H. Hofmann, F. Scheuter, and D. Vautherin, "Stationary Fokker-Planck equation applied to fission dynamics," *Nuclear Physics, Section A*, vol. 424, no. 2, pp. 239-261, 1984.
- [7] D. B. Walton and J. Rafelski, "Equilibrium distribution of heavy quarks in Fokker-Planck dynamics," *Physical Review Letters*, vol. 84, no. 1, pp. 31-34, 2000.
- [8] D. Sornette, "Fokker-Planck equation of distributions of financial returns and power laws," *Physica A: Statistical Mechanics and its Applications*, vol. 290, no. 1-2, pp. 211-217, 2001.
- [9] A. Yoshimori and J. Korrington, "A Fokker-Planck equation for spin relaxation," *Physical Review*, vol. 128, no. 3, pp. 1054-1060, 1962.
- [10] V. I. Kolobov, "Fokker-Planck modeling of electron kinetics in plasmas and semiconductors," *Computational Materials Science*, vol. 28, no. 2, pp. 302-320, 2003.
- [11] C. P. Enz, "Fokker-Planck description of classical systems with application to critical dynamics," *Physica A: Statistical Mechanics and its Applications*, vol. 89, no. 1, pp. 1-36, 1977.
- [12] A. Klar, P. Reuterswärd, and M. Seaïd, "A semi-lagrangian method for a fokker-planck equation describing fiber dynamics," *Journal of Scientific Computing*, vol. 38, no. 3, pp. 349-367, 2009.
- [13] L. P. Kadanoff, *Statistical Physics: Statics, Dynamics and Renormalization*, World Scientific, 2000.
- [14] T. D. Frank and A. Daffertshofer, "Multivariate nonlinear Fokker-Planck equations and generalized thermostatics," *Physica A: Statistical Mechanics and its Applications*, vol. 292, no. 1-4, pp. 392-410, 2001.
- [15] M. Tatari, M. Dehghan, and M. Razzaghi, "Application of the Adomian decomposition method for the Fokker-Planck equation," *Mathematical and Computer Modelling*, vol. 45, no. 5-6, pp. 639-650, 2007.
- [16] J. Biazar, K. Hosseini, and P. Gholamin, "Homotopy perturbation method Fokker-Planck equation," *International Mathematical Forum*, vol. 3, no. 19, pp. 945-954, 2008.
- [17] J. Biazar, P. Gholamin, and K. Hosseini, "Variational iteration method for solving Fokker-Planck equation," *Journal of the Franklin Institute*, vol. 347, no. 7, pp. 1137-1147, 2010.
- [18] K. Kikuchi, M. Yoshida, T. Maekawa, and H. Watanabe, "Metropolis Monte Carlo method as a numerical technique to solve the Fokker-Planck equation," *Chemical Physics Letters*, vol. 185, no. 3-4, pp. 335-338, 1991.
- [19] A. Masud and L. A. Bergman, "Application of multi-scale finite element methods to the solution of the Fokker-Planck equation," *Computer Methods in Applied Mechanics and Engineering*, vol. 194, no. 12-16, pp. 1513-1526, 2005.
- [20] P. Kumar and S. Narayanan, "Solution of Fokker-Planck equation by finite element and finite difference methods for nonlinear systems," *Sadhana*, vol. 31, no. 4, pp. 445-461, 2006.

- [21] A. H. Bhrawy and A. S. Alofi, "The operational matrix of fractional integration for shifted Chebyshev polynomials," *Applied Mathematics Letters*, vol. 26, pp. 25–31, 2013.
- [22] A. H. Bhrawy and M. Alshomrani, "A shifted Legendre spectral method for fractional-order multi-point boundary value problems," *Advances in Difference Equations*, vol. 2012, pp. 1–19, 2012.
- [23] A. H. Bhrawy and D. Baleanu, "A spectral Legendre-Gauss-Lobatto collocation method for a space-fractional advection diffusion equations with variable coefficients," *Reports in Mathematical Physics*, vol. 72, no. 2, pp. 219–233, 2013.
- [24] D. Baleanu, A. H. Bhrawy, and T. M. Taha, "Two efficient generalized Laguerre spectral algorithms for fractional initial value problems," *Abstract and Applied Analysis*, vol. 2013, Article ID 546502, 10 pages, 2013.
- [25] A. H. Bhrawy and M. A. Alghamdi, "A shifted Jacobi-Gauss-Lobatto collocation method for solving nonlinear fractional Langevin equation involving two fractional orders in different intervals," *Boundary Value Problems*, vol. 2012, no. 1, pp. 1–13, 2012.
- [26] J. Klafter, S. C. Lim, and R. Metzler, *Fractional Dynamics: Recent Advances*, World Scientific, 2012.
- [27] D. Baleanu, J. A. T. Machado, and A. C. Luo, *Fractional Dynamics and Control*, Springer, 2012.
- [28] V. E. Tarasov, *Fractional Dynamics*, Springer, 2010.
- [29] G. M. Zaslavsky, *Hamiltonian Chaos and Fractional Dynamics*, Oxford University Press, Oxford, UK, 2005.
- [30] E. Goldfain, "Fractional dynamics, Cantorian space-time and the gauge hierarchy problem," *Chaos, Solitons and Fractals*, vol. 22, no. 3, pp. 513–520, 2004.
- [31] R. Metzler and T. F. Nonnenmacher, "Space- and time-fractional diffusion and wave equations, fractional Fokker-Planck equations, and physical motivation," *Chemical Physics*, vol. 284, no. 1-2, pp. 67–90, 2002.
- [32] R. Metzler, E. Barkai, and J. Klafter, "Deriving fractional fokker-planck equations from a generalised master equation," *Europhysics Letters*, vol. 46, no. 4, pp. 431–436, 1999.
- [33] S. A. El-Wakil and M. A. Zahran, "Fractional Fokker-Planck Equation," *Chaos, Solitons and Fractals*, vol. 11, no. 5, pp. 791–798, 2000.
- [34] A. V. Chechkin, J. Klafter, and I. M. Sokolov, "Fractional Fokker-Planck equation for ultraslow kinetics," *Europhysics Letters*, vol. 63, no. 3, pp. 326–332, 2003.
- [35] A. A. Stanislavsky, "Subordinated Brownian motion and its fractional Fokker-Planck equation," *Physica Scripta*, vol. 67, no. 4, pp. 265–268, 2003.
- [36] K. Kim and Y. S. Kong, "Anomalous behaviors in fractional Fokker-Planck equation," *Journal of the Korean Physical Society*, vol. 40, no. 6, pp. 979–982, 2002.
- [37] I. M. Sokolov, "Thermodynamics and fractional Fokker-Planck equations," *Physical Review E, Statistical, Nonlinear, and Soft Matter Physics*, vol. 63, no. 5, pp. 561111–561118, 2001.
- [38] V. E. Tarasov, "Fokker-Planck equation for fractional systems," *International Journal of Modern Physics B*, vol. 21, no. 6, pp. 955–967, 2007.
- [39] F. Liu, V. Anh, and I. Turner, "Numerical solution of the space fractional Fokker-Planck equation," *Journal of Computational and Applied Mathematics*, vol. 166, no. 1, pp. 209–219, 2004.
- [40] W. Deng, "Finite element method for the space and time fractional fokker-planck equation," *SIAM Journal on Numerical Analysis*, vol. 47, no. 1, pp. 204–226, 2008.
- [41] Z. Odibat and S. Momani, "Numerical solution of Fokker-Planck equation with space- and time-fractional derivatives," *Physics Letters A: General, Atomic and Solid State Physics*, vol. 369, no. 5-6, pp. 349–358, 2007.
- [42] X.-J. Yang, *Advanced Local Fractional Calculus and Its Applications*, World Science Publisher, New York, NY, USA, 2012.
- [43] X.-J. Yang, *Local Fractional Functional Analysis and Its Applications*, Asian Academic Publisher, Hong Kong, China, 2011.
- [44] X.-J. Yang, H. M. Srivastava, J. H. He, and D. Baleanu, "Cantor-type cylindrical-coordinate method for differential equations with local fractional derivatives," *Physics Letters A*, vol. 377, no. 28–30, pp. 1696–1700, 2013.
- [45] K. M. Kolwankar and A. D. Gangal, "Local fractional Fokker-Planck equation," *Physical Review Letters*, vol. 80, no. 2, pp. 214–217, 1998.
- [46] X.-J. Yang and D. Baleanu, "Local fractional variational iteration method for Fokker-Planck equation on a Cantor set," *Acta Universitaria*, vol. 23, no. 2, pp. 3–8, 2013.
- [47] C. F. Liu, S. S. Kong, and S. J. Yuan, "Reconstructive schemes for variational iteration method within Yang-Laplace transform with application to fractal heat conduction problem," *Thermal Science*, vol. 17, no. 3, pp. 715–721, 2013.
- [48] S. Q. Wang, Y. J. Yang, and H. K. Jassim, "Local fractional function decomposition method for solving inhomogeneous wave equations with local fractional derivative," *Abstract and Applied Analysis*, vol. 2014, Article ID 176395, 7 pages, 2014.
- [49] C. G. Zhao, A. M. Yang, H. Jafari, and A. Haghbin, "The Yang-Laplace transform for solving the IVPs with local fractional derivative," *Abstract and Applied Analysis*, vol. 2014, Article ID 386459, 5 pages, 2014.
- [50] A. M. Yang, Y. Z. Zhang, and Y. Long, "The Yang-Fourier transforms to heat-conduction in a semi-infinite fractal bar," *Thermal Science*, vol. 17, no. 3, pp. 707–713, 2013.
- [51] X.-J. Yang, D. Baleanu, and W. P. Zhong, "Approximate solutions for diffusion equations on Cantor space-time," *Proceedings of the Romanian Academy A*, vol. 14, no. 2, pp. 127–133, 2013.
- [52] X.-J. Yang, D. Baleanu, M. P. Lazarević, and M. S. Cajić, "Fractal boundary value problems for integral and differential equations with local fractional operators," *Thermal Science*, 2013.
- [53] D. Baleanu, J. A. T. Machado, C. Cattani, M. C. Baleanu, and X.-J. Yang, "Local fractional variational iteration and decomposition methods for wave equation on Cantor sets within local fractional operators," *Abstract and Applied Analysis*, vol. 2014, Article ID 535048, 6 pages, 2014.