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Research Article

Robust Monotonically Convergent Iterative Learning Control for Discrete-Time Systems via Generalized KYP Lemma

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This paper addresses the problem of P-type iterative learning control for a class of multiple-input multiple-output linear discrete-time systems, whose aim is to develop robust monotonically convergent control law design over a finite frequency range. It is shown that the 2 D iterative learning control processes can be taken as 1 D state space model regardless of relative degree. With the generalized Kalman-Yakubovich-Popov lemma applied, it is feasible to describe the monotonically convergent conditions with the help of linear matrix inequality technique and to develop formulas for the control gain matrices design. An extension to robust control law design against systems with structured and polytopic-type uncertainties is also considered. Two numerical examples are provided to validate the feasibility and effectiveness of the proposed method.

1. Introduction

The well-known iterative learning control (ILC) algorithm can effectively improve the transient responses and tracking performance for systems that execute the same task over a finite duration repetitively, the key idea of which is to iteratively reduce the tracking error by refining the control input signal based on the information from previous trials [1]. As demonstrated in survey papers [2–6], ILC has attracted considerable research attention in many areas during the past few decades. Extensive applications of ILC have been used for many practical problems coming from, for example, batch processes [7–9], point-to-point control [10, 11], and positioning control [12–15].

In fact, among all types of ILC research issues, both theoretical and practical, robustness and monotonic convergence have been studied as two major topics. Many uncertain factors such as model uncertainties, variable initial conditions, stochastic noises, and packet dropout need to be taken into consideration with regard to robust ILC design. For example, a kind of so-called adaptive ILC has been developed for local Lipschitz continuous (LLC) uncertain nonlinear systems with unknown parameters, and composite energy function (CEF) is usually constructed to facilitate the convergence analysis [2]. Considering the inherent two-dimensional (2 D) structure of every ILC process, 2 D system theory has been developed to design ILC based on linear repetitive processes [16–18], Roesser model [19, 20], and Fornasini-Marchsini model [21, 22]. Moreover, the robust ILC has been particularly extended to networked control system [23, 24] and switched systems [25, 26].

To achieve good learning transients, the monotonic convergence is particularly important in ILC design problems. For example, first-order and second-order P-type ILC schemes are used for continuous linear time-invariant (LTI) systems, where the monotonic convergence of tracking error is guaranteed in the sense of Lebesgue-p norm [27]. It is also noticed that the so-called super-vector formulation for discrete-time ILC has been prevalent for monotonic convergence analysis under different appropriate norm topology. In [28], the monotonic convergence analysis for interval ILC systems is presented for discrete-time systems. A gradient-based optimal ILC scheme is proposed for ensuring robust monotonic convergence [29]. A new semisliding window ILC algorithm is developed for discrete-time LTI systems [30]. Recently, by integrating the technique of linear matrix inequality (LMI), the well-established H_∞ norm has been used for deriving monotonical convergence conditions that

can be described as LMIs and formulas for the control law design, and the tracking error can be ensured to converge monotonically in the sense of $\mathscr{L}_2$ norm [31–34].

However, the aforementioned monotonically convergent ILC works treat control law design over the complete frequency range which is not practical in many cases. In particular, the reference signal and design specification are often given for a certain frequency range of relevance. This viewpoint motivates the present study. In this paper, an integrated ILC framework is developed for multiple-input multiple-output (MIMO), LTI discrete systems with a relative degree, and the frequency design can be specified over a finite range. This benefits from the well-established generalized Kalman-Yakubovich-Popov (KYP) lemma that can tie together a frequency domain inequality (FDI) over finite frequency range and an LMI. It is shown that monotonic convergence conditions can be described in terms of LMIs, as well as formulas obtained for the control law design. Furthermore, this approach is extended to handling the robust issues for the systems with norm-bounded and polytopic-type uncertainties.

Briefly, the paper is organized as follows. Section 2 introduces several useful LMIs. The problem formulation is supplied in Section 3. The convergence performance of the proposed scheme is analyzed in Section 4. Section 5 provides two illustrative examples. Finally, some concluding remarks are given in Section 6.

Throughout this paper, the following notations are employed. Z^+ denotes the set of nonnegative integers. For a matrix X, its transpose, complex conjugate transpose, and orthogonal complement are denoted by X^T, X^*, and $X^\perp$, respectively. I and 0 are the identity matrix and the zero matrix with appropriate dimensions, respectively. $X > 0$ and $X < 0$ denote positive definiteness and negative definiteness, respectively. The symbol $(\star)$ represents the transposed elements in a symmetric matrix and $\rho(\cdot)$ denotes the spectral radius of its matrix argument. For matrices X and Y, $X \otimes Y$ denotes the Kronecker product. q is a forward shift operator along the discrete-time axis; that is, $qx(k) = x(k+1)$.

2. Preliminary Knowledge

Before presenting the main results, the following well-known results are briefly introduced in this section.

Lemma 1 (Schur complement, [35]). *Given a symmetric matrix $S = \left[\begin{smallmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{smallmatrix}\right]$, S_{11} and S_{22} are square, and then the following inequalities are equivalent:*

(1) $S < 0$

(2) $S_{11} < 0$ *and* $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$

(3) $S_{22} < 0$ *and* $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2 (see [36]). *Assume X, Y, and $Z = Z^T$ are real matrices with appropriated dimensions. Then for any matrix Σ satisfying $\Sigma^T \Sigma \le I$, the following inequality:*

$$Z + X\Sigma Y + Y^T \Sigma^T X^T < 0 \tag{1}$$

Table 1

	Low frequency	Middle frequency	High frequency
Θ	$\lvert\theta\rvert \le \theta_l$	$\theta_1 \le \theta \le \theta_2$	$\lvert\theta\rvert \ge \theta_h$
Ψ	$\begin{bmatrix} 0 & 1 \\ 1 & -2\cos(\theta_l) \end{bmatrix}$	$\begin{bmatrix} 0 & e^{j\theta_c} \\ e^{-j\theta_c} & -2\cos(\theta_w) \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 2\cos(\theta_h) \end{bmatrix}$

where $\theta_c = (\theta_1 + \theta_2)/2$, $\theta_w = (\theta_2 - \theta_1)/2$.

holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$Z + \varepsilon X X^T + \varepsilon^{-1} Y^T Y < 0. \tag{2}$$

Lemma 3 (see [37]). *Assume X, Y, and $Z = Z^T$ are real matrices with appropriated dimensions. There exists a matrix W such that the following inequality:*

$$Z + X^T W Y + Y^T W^T X < 0. \tag{3}$$

holds if and only if the following two inequalities with respect to W are satisfied:

$$X^{\perp^T} Z X^\perp < 0, \qquad Y^{\perp^T} Z Y^\perp < 0 \tag{4}$$

Lemma 4 (generalized KYP lemma, [38]). *For a discrete LTI system with transfer function $G(z)$ and frequency response matrix $G(e^{j\theta}) = \mathbb{C}(e^{j\theta} I - \mathbb{A})^{-1}\mathbb{B} + \mathbb{D}$, the following statements are equivalent:*

(1) the frequency domain inequality

$$\begin{bmatrix} \left(e^{j\theta} I - \mathbb{A}\right)^{-1}\mathbb{B} \\ I \end{bmatrix}^* \begin{bmatrix} \mathbb{C} & \mathbb{D} \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} \mathbb{C} & \mathbb{D} \\ 0 & I \end{bmatrix} \begin{bmatrix} \left(e^{j\theta} I - \mathbb{A}\right)^{-1}\mathbb{B} \\ I \end{bmatrix} < 0 \quad \forall \theta \in \Theta \tag{5}$$

or

$$\begin{bmatrix} G(e^{j\theta}) \\ I \end{bmatrix}^* \Pi \begin{bmatrix} G\left(e^{j\theta}\right) \\ I \end{bmatrix} < 0 \quad \forall \theta \in \Theta \tag{6}$$

holds for all $e^{j\theta} \in \Lambda(\Phi, \Psi)$, where Π is a given real symmetric matrix and

$$\Lambda(\Phi, \Psi) = \left\{ e^{j\theta} \in C : \begin{bmatrix} e^{j\theta} \\ 1 \end{bmatrix}^* \Phi \begin{bmatrix} e^{j\theta} \\ 1 \end{bmatrix} = 0, \begin{bmatrix} e^{j\theta} \\ 1 \end{bmatrix}^* \Psi \begin{bmatrix} e^{j\theta} \\ 1 \end{bmatrix} \ge 0, \ \forall \theta \in \Theta \right\}, \tag{7}$$

where $\Phi = \left[\begin{smallmatrix} -1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ and Θ denotes the frequency ranges specified by Ψ as shown in Table 1.

(2) There exists Hermitian matrices P, Q such that $Q > 0$ and

$$\begin{bmatrix} \mathbb{A} & \mathbb{B} \\ I & 0 \end{bmatrix}^* (\Phi \otimes P + \Psi \otimes Q) \begin{bmatrix} \mathbb{A} & \mathbb{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} \mathbb{C} & \mathbb{D} \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} \mathbb{C} & \mathbb{D} \\ 0 & I \end{bmatrix} < 0. \tag{8}$$

3. Problem Formulation

3.1. System Description. Consider the following MIMO discrete-time LTI system over $[0, T]$:

$$\begin{aligned} x_j(k+1) &= Ax_j(k) + Bu_j(k), \\ y_j(k) &= Cx_j(k) + Du_j(k), \\ x_j(0) &= x_0, \quad k \in \mathcal{N} = \{0, 1, \ldots, N\}, \end{aligned} \tag{9}$$

where j is the iteration number, which denotes the jth repetitive operation of the system. The task interval $[0, T]$ is finite and discretized in a set $\mathcal{N}$ that consists of sampled instances $0, 1, \ldots, N$. $x_j(k) \in R^n$ is the state vector, $u_j(k) \in R^m$ is the control input vector, and $y_j(k) \in R^p$ is the output vector. $A, B, C,$ and D are constant matrices of appropriate dimensions. x_0 is the initial condition for each iteration. The relative degree r ($r \geq 0$) of system (9) can be defined by [34]

(1) $r = 0$ if $D \neq 0$;

(2) $r \geq 1$ if it holds that

(a) $CA^{r-1}B \neq 0$;

(b) $D = 0$ and $CA^iB = 0$ for all $i < r - 1$.

Let $y_d(k)$ denote the reference vector, and then the tracking error on iteration j is

$$e_j(k) = y_d(k) - y_j(k). \tag{10}$$

The control target is to design appropriate control signal $u_j(k)$ and present some LMI conditions such that the system output can converge monotonically to the reference trajectory $y_d(k)$ over a finite frequency range when the iteration number j tends to infinity, even if there exist system uncertainties.

In order to complete the above control task, the following assumptions are imposed on system (9).

Assumption 5. The initial resetting condition is satisfied; that is, $e_j(0) = 0$, $\forall j \in Z^+$. Without loss of generality, it is considered that $x_j(0) = 0$.

Remark 6. Obviously, the transfer function matrix from $u_j(k)$ to $y_j(k)$ can be expressed as

$$y_j(k) = G_p(q)\, u_j(k), \tag{11}$$

where

$$G_p(q) = C(qI - A)^{-1}B + D =: \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{12}$$

is usually obtained by discretizing the original continuous-time domain model using a sampling mechanism that consists of a sampler with the sampling interval T_s and a zero-order hold.

Remark 7. The relative degree r is exactly the steps of delay in the output $y_j(k)$ in order to have the control input $u_j(k)$ appearing. Note that the relative degree of one, that is, $D = 0$ and $CB \neq 0$, is usually considered in the literature for discrete-time ILC [5].

3.2. Design of ILC. In this section, the ILC law is introduced as follows:

$$u_{j+1}(k) = u_j(k) + L(q)\, e_j(k), \tag{13}$$

where $L(q)$ denotes an $m \times p$ polynomial gain operator to be designed.

Subtract $e_j(k)$ from $e_{j+1}(k)$ and then use (11) and (13) to obtain

$$\begin{aligned} e_{j+1}(k) - e_j(k) &= y_j(k) - y_{j+1}(k) \\ &= G_p(q)\left[u_j(k) - u_{j+1}(k)\right] \\ &= -G_p(q)\, L(q)\, e_j(k) \end{aligned} \tag{14}$$

which leads to

$$e_{j+1}(k) = G_e(q)\, e_j(k), \tag{15}$$

where $G_e(q) = I - G_p(q)L(q)$.

4. Convergence Analysis

4.1. Super-Vector Approach. Using the lifting approach, system (9) and ILC law (13) can be described respectively as

$$\begin{aligned} \mathbf{Y}_j &= \mathbf{G_p}\mathbf{U}_j, \\ \mathbf{U}_{j+1} &= \mathbf{U}_j + \mathbf{L}\mathbf{E}_j, \end{aligned} \tag{16}$$

where $\mathbf{U}_j$, $\mathbf{Y}_j$, and $\mathbf{Y}_d$, $\mathbf{E}_j = \mathbf{Y}_d - \mathbf{Y}_j$, are the supervectors which are lifted to contain N sampled points, and $\mathbf{G_p}$ and $\mathbf{L}$ are two lower triangular block Toeplitz matrices. The elements of $\mathbf{G_p}$ are the system Markov parameters (or the pulse response coefficients).

Equation (16) gives

$$\mathbf{E}_{j+1} = \mathbf{G_e}\mathbf{E}_j, \tag{17}$$

where $\mathbf{G_e} = I - \mathbf{G_p}\mathbf{L}$. For more details of the developments on (16) and (17), refer to [5].

From (17), the monotonic convergence condition can be simply defined in an appropriate norm topology

$$\left\|\mathbf{E}_{j+1}\right\| \leq \left\|\mathbf{G_e}\right\| \left\|\mathbf{E}_j\right\|. \tag{18}$$

Remark 8. Clearly, when the state-space model matrices A, B, C, and D have structured and polytopic-type uncertainties, it is difficult to derive learning gain matrix from condition (18).

4.2. Frequency Domain Approach. The following proposition will be helpful for developing frequency-domain monotonic convergence condition.

Proposition 9 (see [31]). *Assume $G_e(q) = I - G_p(q)L(q)$ is stable and causal, if*

$$\left\|G_e(e^{j\theta})\right\|_\infty < 1 \tag{19}$$

and then $\|\mathbf{G_e}\|_2 < 1$; that is, the monotonic convergence of tracking error $\mathbf{E}_j$ can be accomplished in $\mathcal{L}_2$ norm.

With Proposition 9, there exist matrices $\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$, and $\mathbb{D}$ such that $G_e(q)$ can be expressed by

$$\begin{aligned} G_e(q) &= \mathbb{C}(qI-\mathbb{A})^{-1}\mathbb{B}+\mathbb{D} \\ &=: \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B} \\ \hline \mathbb{C} & \mathbb{D} \end{array}\right]. \end{aligned} \tag{20}$$

It is seen that the condition (19) can be resolved by combining a robust H_∞ control theory and the LMI technique. However, condition (19) requires control law design over the entire frequency and is a very strict condition. By utilizing the generalized KYP Lemma, this paper develops monotonically convergent ILC design restricted within a finite frequency range. Accordingly, (19) is replaced by following condition:

$$\left\|G_e(e^{j\theta})\right\|_\infty < 1 \quad \forall\theta\in\Theta, \tag{21}$$

where Θ denotes a finite frequency range.

Moreover, condition (21) is replaced with

$$\left\|G_e(e^{j\theta})\right\|_\infty < \gamma \quad \forall\theta\in\Theta,\ \gamma\in(0,1]. \tag{22}$$

However, in this case, inequality (22) is no longer a standard H_∞ problem. To this end, we denote $\gamma=\epsilon_1\epsilon_2^{-1}$ for scalars $\epsilon_1>0$ and $\epsilon_2>0$. Then if

$$\epsilon_1\le\epsilon_2 \tag{23}$$

holds, $\gamma\in(0,1]$ follows immediately, and at the same time (22) equivalently becomes

$$\left\|\overline{G}_e(e^{j\theta})\right\|_\infty < \epsilon_1 \quad \forall\theta\in\Theta, \tag{24}$$

where $\overline{G}_e(e^{j\theta})=\epsilon_2 G_e(e^{j\theta})$.

Thus, condition (24) can be viewed as an H_∞ problem that is subject to a linear constraint condition (23).

Now with controlled system (15), let us further consider how to solve the condition (24) under the generalized KYP Lemma framework. Consider the frequency response matrix $\overline{G}_e(e^{j\theta})$ and choose the matrix Π of Lemma 4 as

$$\Pi=\begin{bmatrix}\epsilon_1^{-1}I & 0\\ 0 & -\epsilon_1 I\end{bmatrix}. \tag{25}$$

We can get

$$\overline{G}_e\left(e^{j\theta}\right)^*\overline{G}_e\left(e^{j\theta}\right) < \epsilon_1^2 I \quad \forall\theta\in\Theta. \tag{26}$$

Obviously, inequality (26) is equivalent to condition (24).

4.2.1. Zero Relative Degree ($r=0$). Consider first that system (9) has a zero relative degree, resulting in

$$D\ne 0. \tag{27}$$

Accordingly, the ILC law (13) is applied with the following gain operator:

$$L(q)\equiv L, \tag{28}$$

where L is an $m\times p$ matrix to be determined.

Moreover, it is easy to see that $G_e(q)$ and $\overline{G}_e(q)$ satisfy (20) because it can be modeled by

$$G_e(q)=\left[\begin{array}{c|c} A & -BL \\ \hline C & I-DL \end{array}\right], \qquad \overline{G}_e(q)=\left[\begin{array}{c|c} A & -\epsilon_2 BL \\ \hline C & \epsilon_2(I-DL) \end{array}\right]. \tag{29}$$

Now with Lemma 4 and (29), the following theorem can be presented.

Theorem 10. *Consider the ILC system* (9) *and* (13) *satisfying* $r=0$ *and Assumption 5, and the gain operator matrix* $L(q)$ *is defined by* (28). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $|\theta|\le\theta_l$ *when* $j\to\infty$, *if there exist scalars* $\epsilon_1>0$, $\epsilon_2>0$ *and matrices* $\widehat{P}>0$, $\widehat{Q}>0$, R, X *satisfying* (23) *and the following LMI:*

$$\begin{bmatrix} -\widehat{P} & \star & \star & \star \\ \widehat{Q}-R & \widehat{P}-2\cos(\theta_l)\widehat{Q}+R^TA^T+AR & \star & \star \\ 0 & X^TB^T & -\epsilon_1 I & \star \\ 0 & CR & \epsilon_2 I+DX & -\epsilon_1 I \end{bmatrix} < 0 \tag{30}$$

If the LMIs of (23) and (30) are feasible, then the gain matrix L is given by

$$L=-\epsilon_2^{-1}X. \tag{31}$$

Proof. Applying Lemma 4 gives that condition (26) holds if there exist symmetric matrices P and Q such that $Q>0$ and

$$\begin{aligned} &\begin{bmatrix}\mathbb{A} & \mathbb{B}\\ I & 0\end{bmatrix}^T(\Phi\otimes P+\Psi\otimes Q)\begin{bmatrix}\mathbb{A} & \mathbb{B}\\ I & 0\end{bmatrix} \\ &\quad+\begin{bmatrix}\mathbb{C} & \mathbb{D}\\ 0 & I\end{bmatrix}^T\Pi\begin{bmatrix}\mathbb{C} & \mathbb{D}\\ 0 & I\end{bmatrix}<0, \end{aligned} \tag{32}$$

where $\mathbb{A}=A$, $\mathbb{B}=-\epsilon_2 BL$, $\mathbb{C}=C$, $\mathbb{D}=\epsilon_2(I-DL)$, $\Pi=\begin{bmatrix}\epsilon_1^{-1}I & 0\\ 0 & -\epsilon_1 I\end{bmatrix}$, and Ψ is the only matrix whose block entries depend on the chosen frequency range. Without loss of generality, the low frequency range is considered; that is, $|\theta|\le\theta_l$, which gives that

$$\Phi\otimes P+\Psi\otimes Q=\begin{bmatrix}-P & Q\\ Q & P-2\cos(\theta_l)Q\end{bmatrix}. \tag{33}$$

Then, (32) becomes

$$\begin{aligned} &\begin{bmatrix}-\mathbb{A}^TP\mathbb{A}+Q\mathbb{A}+\mathbb{A}^TQ+P-2\cos(\theta_l)Q & -\mathbb{A}^TP\mathbb{B}+Q\mathbb{B}\\ -\mathbb{B}^TP\mathbb{A}+\mathbb{B}^TQ & -\mathbb{B}^TP\mathbb{B}\end{bmatrix} \\ &\quad+\begin{bmatrix}\epsilon_1^{-1}\mathbb{C}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{C}^T\mathbb{D}\\ \epsilon_1^{-1}\mathbb{D}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{D}^T\mathbb{D}-\epsilon_1 I\end{bmatrix}<0. \end{aligned} \tag{34}$$

The condition of (34) cannot, however, be directly applied to control law design since it involves product terms $\mathbb{A}^TP\mathbb{B}$ and $Q\mathbb{B}$.

To separate the matrices P and Q from the process model matrices, rewrite (34) as

$$\begin{bmatrix} \mathbb{A}^T & I & 0 \\ \mathbb{B}^T & 0 & I \end{bmatrix} \begin{bmatrix} -P & Q & 0 \\ Q & P - 2\cos(\theta_l) Q + \epsilon_1^{-1}\mathbb{C}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{C}^T\mathbb{D} \\ 0 & \epsilon_1^{-1}\mathbb{D}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{D}^T\mathbb{D} - \epsilon_1 I \end{bmatrix} \times \begin{bmatrix} \mathbb{A} & \mathbb{B} \\ I & 0 \\ 0 & I \end{bmatrix} < 0. \tag{35}$$

To apply the result of Lemma 3, we can set $Z = \begin{bmatrix} -P & Q & 0 \\ Q & P-2\cos(\theta_l)Q+\epsilon_1^{-1}\mathbb{C}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{C}^T\mathbb{D} \\ 0 & \epsilon_1^{-1}\mathbb{D}^T\mathbb{C} & \epsilon_1^{-1}\mathbb{D}^T\mathbb{D}-\epsilon_1 I \end{bmatrix}$, $X^\perp = \begin{bmatrix} \mathbb{A} & \mathbb{B} \\ I & 0 \\ 0 & I \end{bmatrix}$, $Y^\perp = \begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}$, $Y = \begin{bmatrix} 0 & I & 0 \end{bmatrix}$, $X = \begin{bmatrix} -I & \mathbb{A} & \mathbb{B} \end{bmatrix}$.

Then we can have

$$Y^{\perp^T} Z Y^\perp = \begin{bmatrix} -P & 0 \\ 0 & \epsilon_1^{-1}\mathbb{D}^T\mathbb{D} - \epsilon_1 I \end{bmatrix} < 0. \tag{36}$$

The above inequality holds if and only if the diagonal blocks satisfy

$$-P < 0, \qquad \epsilon_1^{-1}\mathbb{D}^T\mathbb{D} - \epsilon_1 I < 0. \tag{37}$$

Hence $P > 0$ is required. Moreover, $\epsilon_1^{-1}\mathbb{D}^T\mathbb{D} - \epsilon_1 I < 0$ is equivalent to $\rho[(I - DL)^T(I - DL)] < \epsilon_1\epsilon_2^{-1} = \gamma$, which is naturally set up according to asymptotic convergence of the ILC system.

Next, application of Lemma 3 gives that (35) and (36) are feasible if there exists a matrix W satisfying

$$\begin{bmatrix} -P & Q - W & 0 \\ Q - W^T & P - 2\cos(\theta_l) Q + \epsilon_1^{-1}C^TC + \mathbb{A}^TW + W^T\mathbb{A} & \epsilon_1^{-1}C^T\mathbb{D} + W^T\mathbb{B} \\ 0 & \epsilon_1^{-1}\mathbb{D}^T\mathbb{C} + \mathbb{B}^TW & \epsilon_1^{-1}\mathbb{D}^T\mathbb{D} - \epsilon_1 I \end{bmatrix} < 0. \tag{38}$$

Applying the Schur's complement to (38) and inserting $\mathbb{A} = A$, $\mathbb{B} = -\epsilon_2 BL$, $\mathbb{C} = C$, and $\mathbb{D} = \epsilon_2(I - DL)$ give that

$$\begin{bmatrix} -P & Q - W & 0 & 0 \\ Q - W^T & P - 2\cos(\theta_l) Q + A^TW + W^TA & -W^T\epsilon_2 BL & C^T \\ 0 & -\epsilon_2 L^TB^TW & -\epsilon_1 I & \epsilon_2(I - DL)^T \\ 0 & C & \epsilon_2(I - DL) & -\epsilon_1 I \end{bmatrix} < 0, \tag{39}$$

pre- and postmultiplying this last inequality by $\mathrm{diag}\{R^T, R^T, I, I\}$ and $\mathrm{diag}\{R, R, I, I\}$ to obtain

$$\begin{bmatrix} -R^TPR & R^T(Q - W)R & 0 & 0 \\ R^T(Q - W^T)R & R^T(P - 2\cos(\theta_l) Q + A^TW + W^TA)R & -R^TW^T\epsilon_2 BL & R^TC^T \\ 0 & -\epsilon_2 L^TB^TRW & -\epsilon_1 I & \epsilon_2(I - DL)^T \\ 0 & CR & \epsilon_2(I - DL) & -\epsilon_1 I \end{bmatrix} < 0. \tag{40}$$

Finally, introduce the change of variables:

$$X = -\epsilon_2 L, \qquad W = R^{-1}, \qquad \widehat{P} = R^TPR, \qquad \widehat{Q} = R^TQR \tag{41}$$

giving immediately that (40) is equivalent to the LMI of (30) and the proof is complete. □

Next it will be shown that Theorem 10 can be further developed to address system (9) with structured uncertainty matrices of the form:

$$G_p(q) = \left[\begin{array}{c|c} A + \Delta A & B + \Delta B \\ \hline C + \Delta C & D + \Delta D \end{array}\right], \tag{42}$$

where ΔA, ΔB, ΔC, and ΔD represent admissible uncertainties which are assumed to satisfy

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \Sigma \begin{bmatrix} F_1 & F_2 \end{bmatrix}, \tag{43}$$

where H_1, H_2, F_1, and F_2 are known matrices of appropriate dimensions, and Σ is an unknown matrix satisfying

$$\Sigma^T\Sigma \le I. \tag{44}$$

In this case, the following robust result can be presented.

Corollary 11. *Consider the ILC system* (9) *and* (13) *satisfying* $r = 0$ *and Assumption 5. Assume that the plant has uncertain*

matrices described by (43) *and* (44), *and the gain operator matrix* $L(q)$ *is defined by* (28). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $\theta \le |\theta_l|$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$ *and matrices* $\widehat{P} > 0$, $\widehat{Q} > 0$, *and* R, X *satisfying* (23) *and the following LMI:*

$$\begin{bmatrix} -\widehat{P} & \star & \star & \star & \star & \star \\ \widehat{Q} - R & \widehat{P} - 2\cos(\theta_l)\widehat{Q} + R^T A^T + AR & \star & \star & \star & \star \\ 0 & X^T B^T & -\epsilon_1 I & \star & \star & \star \\ 0 & CR & \epsilon_2 I + DX & -\epsilon_1 I & \star & \star \\ 0 & \epsilon_3 H_1^T & 0 & \epsilon_3 H_2^T & -\epsilon_3 I & \star \\ 0 & F_1 R & F_2 X & 0 & 0 & -\epsilon_3 I \end{bmatrix} < 0. \tag{45}$$

If the LMIs of (23) and (45) are feasible, then the gain matrix L is given by (31).

Proof. With Theorem 10 applied, this proof can be expressed as the requirement that

$$\Upsilon + \mathscr{H}\Sigma\mathscr{F} + \mathscr{H}^T\Sigma\mathscr{F}^T < 0, \tag{46}$$

where

$$\Upsilon = \begin{bmatrix} -\widehat{P} & \star & \star & \star \\ \widehat{Q} - R & \widehat{P} - 2\cos(\theta_l)\widehat{Q} + R^T A^T + AR & \star & \star \\ 0 & X^T B^T & -\epsilon_1 I & \star \\ 0 & CR & \epsilon_2 I + DX & -\epsilon_1 I \end{bmatrix},$$
$$\mathscr{H} = \begin{bmatrix} 0 \\ H_1 \\ 0 \\ H_2 \end{bmatrix},$$
$$\mathscr{F} = \begin{bmatrix} 0 & F_1 R & F_2 X & 0 \end{bmatrix}. \tag{47}$$

With Lemma 2 applied, one has that (46) holds for all Σ satisfying $\Sigma^T\Sigma \le I$ if and only if there exists a scalar $\epsilon_3 > 0$ such that

$$\Upsilon + \epsilon_3 \mathscr{H}\mathscr{H}^T + \epsilon_3^{-1}\mathscr{F}^T\mathscr{F} < 0. \tag{48}$$

It is noted that the above inequality can be rewritten to obtain

$$\Upsilon + \begin{bmatrix} \epsilon_3^{1/2}\mathscr{H} & \epsilon_3^{-1/2}\mathscr{F}^T \end{bmatrix} \begin{bmatrix} \epsilon_3^{1/2}\mathscr{H}^T \\ \epsilon_3^{-1/2}\mathscr{F} \end{bmatrix} < 0 \tag{49}$$

Using the Schur complement Lemma, one knows that (49) is equivalent to

$$\begin{bmatrix} \Upsilon & \star \\ \begin{bmatrix} \epsilon_3^{1/2}\mathscr{H}^T \\ \epsilon_3^{-1/2}\mathscr{F} \end{bmatrix} & -I \end{bmatrix} < 0. \tag{50}$$

Now pre- and post-multiplying (50) by $\mathrm{diag}\{I, I, I, I, \epsilon_3^{1/2}I, \epsilon_3^{1/2}I\}$ result in (45). This proof is completed. □

Furthermore, it will be demonstrated that Theorem 10 can be extended to address the case, where the matrices of system (9) are known to lie within a convex bounded uncertain domain Ω:

$$\Omega = \left\{ (A, B, C, D) \mid (A, B, C, D) = \sum_{i=1}^{N} \tau_i (A_i, B_i, C_i, D_i); \quad \tau_i \ge 0, \ \sum_{i=1}^{N} \tau_i = 1 \right\}. \tag{51}$$

The following result enables robust ILC design when the model matrices of system (9) belong to a polytope-type uncertain domain Ω.

Corollary 12. *Consider the ILC system* (9) *and* (13) *satisfying* $r = 0$ *and Assumption 5. Assume that the plant matrices have polytope uncertainty described by* (51), *and the gain operator matrix* $L(q)$ *is defined by* (28). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $\theta \le |\theta_l|$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$ *and matrices* $\widehat{P}_i > 0$, $\widehat{Q}_i > 0$, R, X *satisfying* (23) *and the following LMIs:*

$$\begin{bmatrix} -\widehat{P}_i & \star & \star & \star \\ \widehat{Q}_i - R & \widehat{P}_i - 2\cos(\theta_l)\widehat{Q}_i + R^T A_i^T + A_i R & \star & \star \\ 0 & X^T B_i^T & -\epsilon_1 I & \star \\ 0 & C_i R & \epsilon_2 I + D_i X & -\epsilon_1 I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, N. \tag{52}$$

If the LMIs of (23) and (52) are feasible, then the gain matrix L is given by (31).

Proof. For all systems of the type of (9) falling within Ω, let $\widehat{P} = \sum_{i=1}^{N} \tau_i \widehat{P}_i$, $\widehat{Q} = \sum_{i=1}^{N} \tau_i \widehat{Q}_i$. Then the LMI of (30) can be

achieved from the set of LMIs in (52). The remaining of this proof is omitted since it can follow the same lines of the proof of Theorem 10. □

4.2.2. Higher-Order Relative Degree ($r \geq 1$). Consider now that system (9) has a higher-order relative degree of $r \geq 1$, leading to systems of the form (9) with matrices satisfying

$$D = 0, \quad CA^{r-1}B \neq 0, \quad CA^{i}B = 0 \quad (i = 0, \ldots, r-2). \tag{53}$$

In order to compensate for the influence of r, the ILC law (13) is considered with an anticipatory gain operator of the form:

$$L(q) = L_r q^r, \tag{54}$$

where L_r is an $m \times p$ matrix.

Moreover, $G_e(q)$ and $\overline{G}_e(q)$ can still satisfy (20) as shown in the following:

$$\begin{aligned} G_e(q) &= \begin{bmatrix} A & -A^r BL_r \\ C & I - CA^{r-1}BL_r \end{bmatrix}, \\ \overline{G}_e(q) &= \begin{bmatrix} A & -\epsilon_2 A^r BL_r \\ C & \epsilon_2 \left(I - CA^{r-1}BL_r\right) \end{bmatrix} \end{aligned} \tag{55}$$

or

$$\begin{aligned} G_e(q) &= \begin{bmatrix} A & -BL_r \\ CA^r & I - CA^{r-1}BL_r \end{bmatrix}, \\ \overline{G}_e(q) &= \begin{bmatrix} A & -\epsilon_2 BL_r \\ CA^r & \epsilon_2 \left(I - CA^{r-1}BL_r\right) \end{bmatrix}. \end{aligned} \tag{56}$$

Remark 13. Using the fact that $q(qI - A)^{-1} = I + (qI - A)^{-1}A$ or $q(qI - A)^{-1} = I + A(qI - A)^{-1}$ repetitively yields that $q^r(qI - A)^{-1}$ can be expressed as

$$q^r(qI - A)^{-1} = \sum_{l=0}^{r-1} q^{r-1-l}A^l + (qI - A)^{-1}A^r \tag{57}$$

or

$$q^r(qI - A)^{-1} = \sum_{l=0}^{r-1} q^{r-1-l}A^l + A^r(qI - A)^{-1}. \tag{58}$$

Then $G_e(q)$ can be derived as

$$\begin{aligned} G_e(q) &= I - G_p(q)L(q) \\ &= I - Cq^r(qI - A)^{-1}BL_r \\ &= I - C\sum_{l=0}^{r-1} q^{r-1-l}A^l BL_r - C(qI - A)^{-1}A^r BL_r \\ &= -C(qI - A)^{-1}A^r BL_r + I - CA^{r-1}BL_r \end{aligned} \tag{59}$$

or

$$\begin{aligned} G_e(q) &= I - G_p(q)L(q) \\ &= I - C(qI - A)^{-1}q^r BL_r \\ &= I - C\sum_{l=0}^{r-1} q^{r-1-l}A^l BL_r - C(qI - A)^{-1}A^r BL_r \\ &= -CA^r(qI - A)^{-1}BL_r + I - CA^{r-1}BL_r. \end{aligned} \tag{60}$$

Theorem 14. *Consider the ILC system* (9), (13), *and* (55) *satisfying* $r \geq 1$ *and Assumption 5, and the gain operator matrix* $L(q)$ *is defined by* (54). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $|\theta| \leq \theta_l$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$, *and matrices* $\widehat{P} > 0$, $\widehat{Q} > 0$, R, *and* X_r *satisfying* (23) *and the following LMI:*

$$\begin{bmatrix} -\widehat{P} & \star & \star & \star \\ \widehat{Q} - R & \widehat{P} - 2\cos(\theta_l)\widehat{Q} + R^T A^T + AR & \star & \star \\ 0 & X_r^T B^T A^{rT} & -\epsilon_1 I & \star \\ 0 & CR & \epsilon_2 I + CA^{r-1}BX_r & -\epsilon_1 I \end{bmatrix} < 0. \tag{61}$$

If the LMIs of (23) and (61) are feasible, then the gain matrix L_r is given by

$$L_r = -\epsilon_2^{-1}X. \tag{62}$$

Proof. This proof is omitted since it follows identical steps to that of Theorem 10. □

Even if a higher-order relative degree exists, LMIs of (23) and (61) can still be obtained to achieve the monotonic convergence. Since LMI (61) contains product of matrices of the plant $G_p(q)$, Theorem 14 cannot be extended like that done in Corollaries 11 and 12. However, it is noted that, for the case $r = 1$, the results of Theorem 14 are feasible to deal with the norm-bounded and polytopic-type uncertainties.

Next let us consider only the case where the uncertain model is of the form:

$$G_p(q) = \left[\begin{array}{c|c} A + \Delta A & B \\ \hline C + \Delta C & 0 \end{array}\right]. \tag{63}$$

The matrices ΔA and ΔC represent admissible uncertainties that are assumed to satisfy

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \Sigma F_1, \tag{64}$$

where H_1, H_2, and F_1 are known constant matrices, and Σ is an unknown matrix satisfying $\Sigma^T \Sigma \leq I$. Now the effects of norm-bounded uncertainty can be addressed via the following result.

Corollary 15. *Consider the ILC system* (9), (13), *and* (55) *satisfying* $r = 1$ *and Assumption 5. Assume that the plant has uncertain matrices described by* (63) *and* (64), *and the gain operator matrix* $L(q)$ *is defined by* (54). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $\theta \leq |\theta_l|$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$, $\epsilon_3 > 0$ *and matrices* $\widehat{P} > 0$, $\widehat{Q} > 0$, R, X_r *satisfying* (23) *and the following LMI:*

$$\begin{bmatrix} -\widehat{P} & \star & \star & \star & \star & \star \\ \widehat{Q} - R & \widehat{P} - 2\cos(\theta_l)\widehat{Q} + R^T A^T + AR & \star & \star & \star & \star \\ 0 & X_r^T B^T A^T & -\epsilon_1 I & \star & \star & \star \\ 0 & CR & \epsilon_2 I + CBX & -\epsilon_1 I & \star & \star \\ 0 & \epsilon_3 H_1^T & 0 & \epsilon_3 H_2^T & -\epsilon_3 I & \star \\ 0 & F_1 R & F_1 B X_r & 0 & 0 & -\epsilon_3 I \end{bmatrix} < 0. \tag{65}$$

If the LMIs of (23) and (65) are feasible, then the gain matrix L_r is given by (62).

Proof. This proof is omitted since it follows in an identical manner to that of Corollary 11. □

Furthermore, the matrices A and C are known to lie within an uncertainty polytope Ω_1 where

$$\Omega_1 = \left\{ (A, C) \mid (A, C) = \sum_{i=1}^{N} \tau_i (A_i, C_i);\ \tau_i \geq 0,\ \sum_{i=1}^{N} \tau_i = 1 \right\}. \tag{66}$$

The following result is able to address the problems of polytope-type uncertainty.

Corollary 16. *Consider the ILC system* (9), (13), *and* (55) *satisfying* $r = 1$ *and Assumption 5. Assume that the plant matrices have polytope uncertainty described by* (66), *and the gain operator matrix* $L(q)$ *is defined by* (54). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $\theta \leq |\theta_l|$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$ *and matrices* $\widehat{P}_i > 0$, $\widehat{Q}_i > 0$, R, X *satisfying* (23) *and the following LMIs:*

$$\begin{bmatrix} -\widehat{P}_i & \star & \star & \star \\ \widehat{Q}_i - R & \widehat{P}_i - 2\cos(\theta_l)\widehat{Q}_i + R^T A_i^T + A_i R & \star & \star \\ 0 & X_r^T B^T {A_i}^T & -\epsilon_1 I & \star \\ 0 & C_i R & \epsilon_2 I + C_i B X_r & -\epsilon_1 I \end{bmatrix} < 0. \tag{67}$$

If the LMIs of (23) and (67) are feasible, then the gain matrix L_r is given by (62).

Proof. This proof is omitted since it follows identical steps to that of Corollary 12. □

Remark 17. Considering the dual transfer function $G_e(q)$ that is described by (56), the following result can be obtained in an identical manner to that of Theorem 18.

Theorem 18. *Consider the ILC system* (9), (13), *and* (56) *satisfying* $r \geq 1$ *and Assumption 5, and the gain operator matrix* $L(q)$ *is defined by* (54). *Then,* $\|\mathbf{E}_j\|_2$ *converges monotonically to zero over the low frequency range* $|\theta| \leq \theta_l$ *when* $j \to \infty$, *if there exist scalars* $\epsilon_1 > 0$, $\epsilon_2 > 0$, *and matrices* $\widehat{P} > 0$, $\widehat{Q} > 0$, R, *and* X_r *satisfying* (23) *and the following LMI:*

$$\begin{bmatrix} -\widehat{P} & \star & \star & \star \\ \widehat{Q} - R & \widehat{P} - 2\cos(\theta_l)\widehat{Q} + R^T A^T + AR & \star & \star \\ 0 & X_r^T B^T & -\epsilon_1 I & \star \\ 0 & CA^r R & \epsilon_2 I + CA^{r-1} B X_r & -\epsilon_1 I \end{bmatrix} < 0. \tag{68}$$

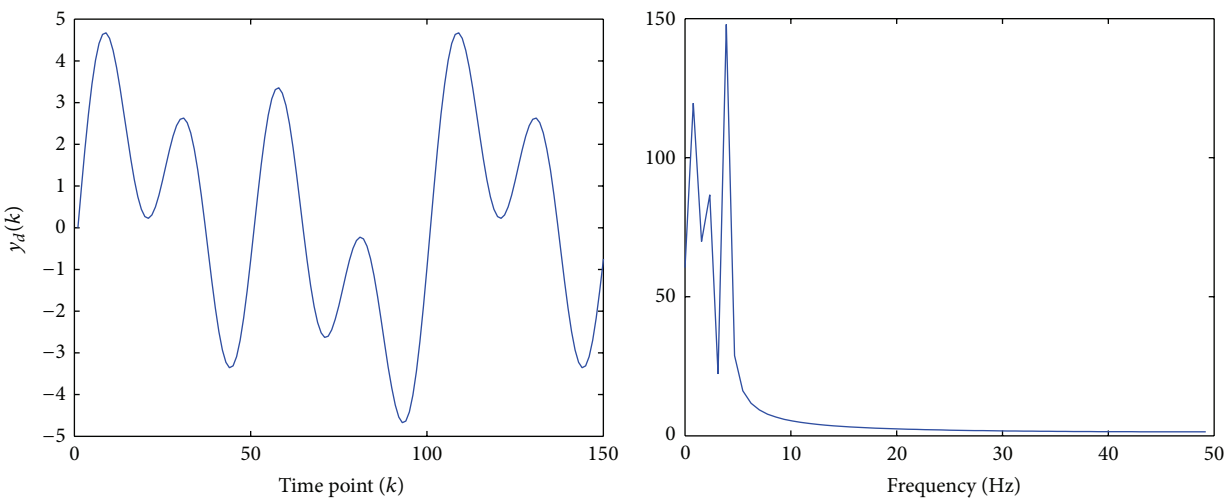

FIGURE 1: (a) The reference trajectory and (b) the frequency spectrum for the reference trajectory.

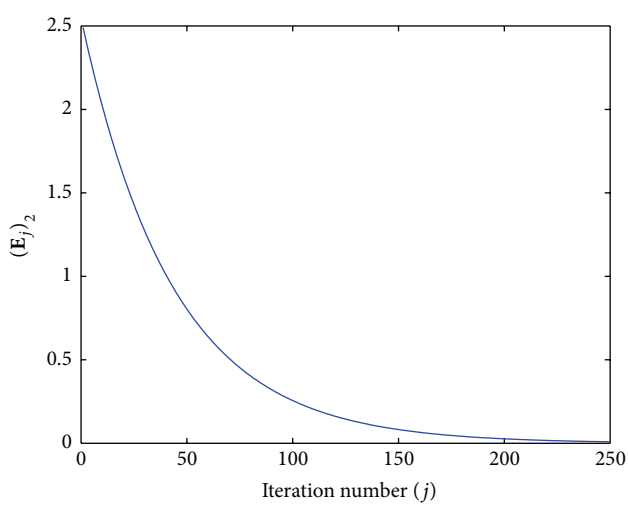

FIGURE 2: Example 1, $\mathscr{L}_2$ norm of tracking error with respect to the iteration number.

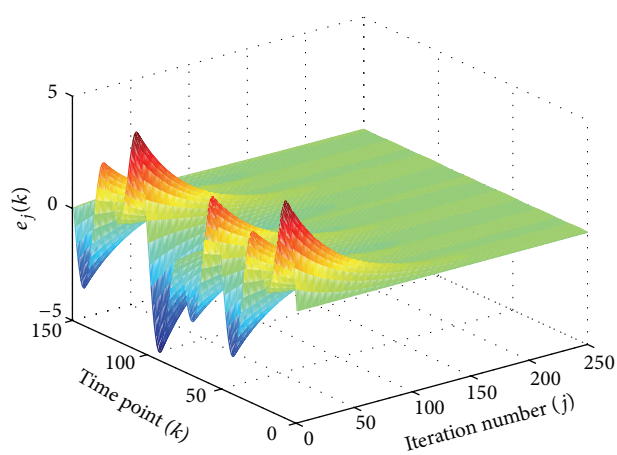

FIGURE 3: Example 1, process of the tracking error along both the iteration axis and the time axis.

If the LMIs of (23) and (68) are feasible, then the gain matrix L_r is given by (62).

Based on Theorem 18, the robust results with $r = 1$ are omitted since it follows identical steps to those of Corollary 15 and Corollary 16.

5. Simulation Examples

Example 1. Consider the following SISO system given by [28]:

$$A = \begin{bmatrix} 0.72 & 0 & 0 \\ 1.0 & -1.04 & -0.81 \\ 0 & 0.81 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \tag{69}$$
$$C = \begin{bmatrix} 1 & -0.98 & -1.09 \end{bmatrix}, \qquad D = 0.$$

Obviously, the considered system has a relative degree of one. Each iteration duration T is 1.5 s and the sampling frequency is set to 100 Hz. The reference trajectory is shown in Figure 1(a) and associated frequency spectrum in Figure 1(b). Inspecting the amplitudes in the frequency spectrum, it is shown that significant harmonics in the range from 0 to 10 Hz, which can be taken as the low frequency range. And hence θ_l is chosen as $\theta_l = 0.6284$.

For uncertainties modeled by (64), assume that $H_1 = \mathrm{diag}\{-0.05, 0.1, 0.1\}$, $H_2 = [-0.12, 0.15, 0.15]$, $F_1 = I$, and $\Sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \sigma_3\}$, where σ_1, σ_2, and σ_3 vary randomly between -1 and 1. Then apply Corollary 15 and use the LMI solver "feasp" in the Matlab toolbox to obtain $L = 0.0304$.

The simulation results are shown in Figures 2, 3, and 4. From these three figures, it is clearly demonstrated that the tracking error converges monotonically to zero along the iteration axis.

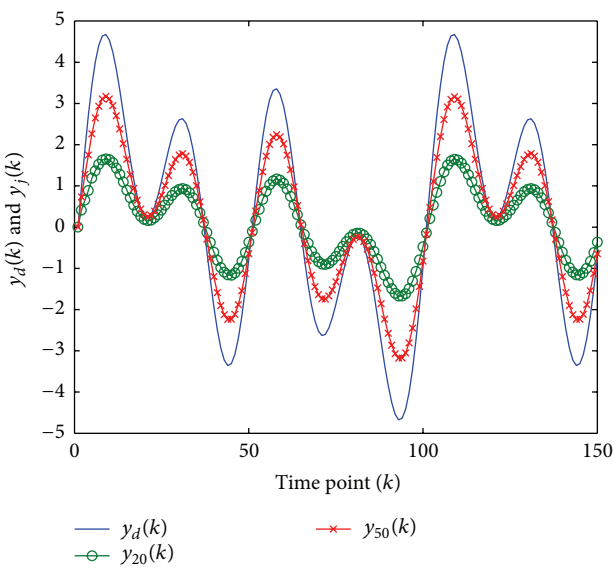

FIGURE 4: Example 1, the reference $y_d(k)$ and output $y_j(k)$ for $j = 20$ and $j = 50$.

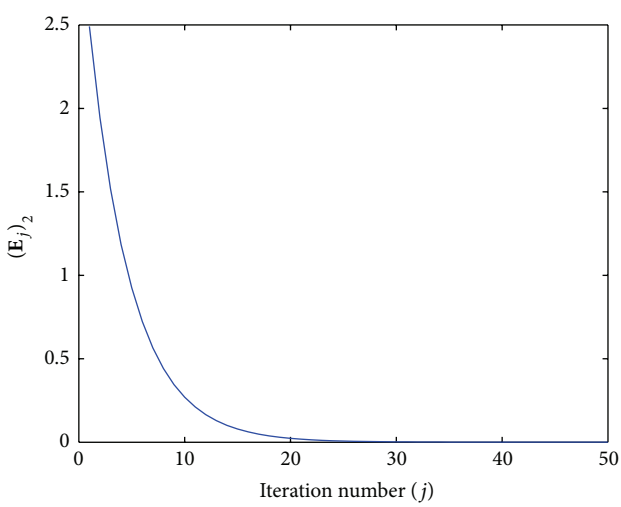

FIGURE 5: Example 2, $\mathscr{L}_2$ norm of tracking error with respect to the iteration number.

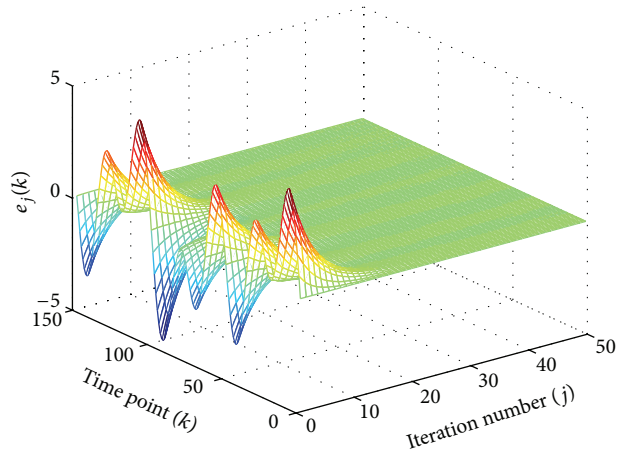

FIGURE 6: Example 2, process of the tracking error along both the iteration axis and the time axis.

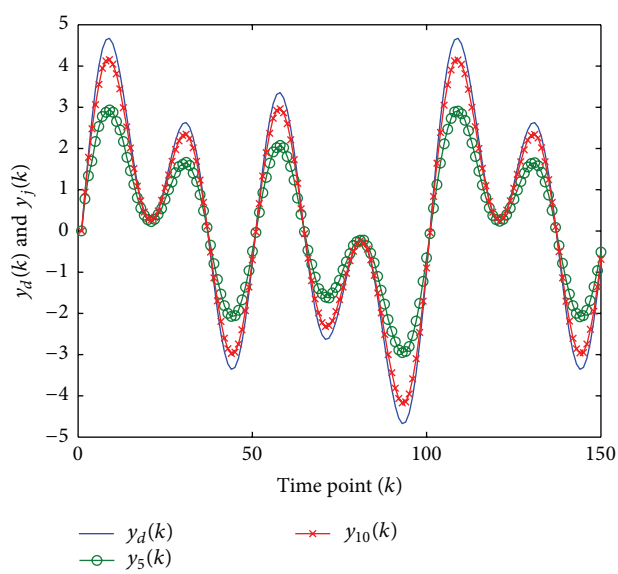

FIGURE 7: Example 2, the reference $y_d(k)$ and output $y_j(k)$ for $j = 5$ and $j = 10$.

Example 2. In this example, the system is considered with matrices given by

$$A = \begin{bmatrix} 0.72 & 0 & 0 \\ 1.0 & -1.04 & -0.81 \\ 0 & 0.81 + a & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \tag{70}$$

$$C = \begin{bmatrix} 1 & -0.98 + c & -1.09 \end{bmatrix}, \qquad D = 0, \tag{71}$$

where a and c are uncertain parameters in the form of $(a, c) = \tau_1(a_1, c_1) + \tau_2(a_2, c_2)$. Here, $(a_1, c_1) = (-0.4, -0.4)$, $(a_2, c_2) = (0.4, 0.4)$, and $\tau_1 + \tau_2 = 1$, and τ_1 and τ_2 are uncertain variables lying in the interval $[0, 1]$.

The simulation condition is performed identically to Example 1. Then apply Corollary 16 to such a system and solve LMIs (23) and (67) to obtain $L_r = 0.3326$. The simulation results are shown in Figures 5, 6, and 7, from which it is seen that the tracking error also decays monotonically to zero along the iteration axis.

6. Conclusion

This paper deals with tracking problem of uncertain MIMO discrete-time systems with a relative degree. Based on the idea of generalized Kalman-Yakubovich-Popov lemma, the proposed ILC scheme achieves robust monotonically convergent control law design over a finite frequency range, and sufficient conditions in terms of LMIs have been developed. The effectiveness of the controller design is validated through two numerical examples.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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