Research Article

Generalized Metric Spaces Do Not Have the Compatible Topology

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We study generalized metric spaces, which were introduced by Branciari (2000). In particular, generalized metric spaces do not necessarily have the compatible topology. Also we prove a generalization of the Banach contraction principle in complete generalized metric spaces.

1. Introduction

In 2000, Branciari in [1] introduced a very interesting concept whose name is “ ν -generalized metric space.”

Definition 1 (see Branciari [1]). Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$, and let $\nu \in \mathbb{N}$. Then (X, d) is said to be a ν -generalized metric space if the following hold:

- (N1) $d(x, y) = 0$ if and only if $x = y$ for any $x, y \in X$;
- (N2) $d(x, y) = d(y, x)$ for any $x, y \in X$;
- (N3) $d(x, y) \leq d(x, u_1) + d(u_1, u_2) + \cdots + d(u_\nu, y)$ for any $x, u_1, u_2, \dots, u_\nu, y \in X$ such that $x, u_1, u_2, \dots, u_\nu, y$ are all different.

Example 2. Every metric space (X, d) is a 1-generalized metric space.

A 2-generalized metric space is also said to be a generalized metric space.

Definition 3 (see Branciari [1]). Let X be a set and let d be a function from $X \times X$ into $[0, \infty)$. Then (X, d) is said to be a generalized metric space if the following hold:

- (G1) $d(x, y) = 0$ if and only if $x = y$ for any $x, y \in X$.
- (G2) $d(x, y) = d(y, x)$ for any $x, y \in X$.

- (G3) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$ for any $x, u, v, y \in X$ such that x, u, v, y are all different.

The concept of “generalized metric space” is very similar to that of “metric space.” However, it is very difficult to treat this concept because X does not necessarily have the topology which is compatible with d ; see Example 7. So this concept is very interesting to researchers. See also [2, 3].

Motivated by the above, in this paper, we study generalized metric spaces. In particular, generalized metric spaces do not necessarily have the compatible topology. Also we prove a generalization of the Banach contraction principle in complete generalized metric spaces.

2. ν -Generalized Metric Space

Throughout this paper we denote by $\mathbb{N}$ the set of all positive integers.

In this section, we study ν -generalized metric space. In particular, we give examples in order to understand this concept deeply.

Lemma 4. Let (X, ρ) be a bounded metric space and let M be a real number satisfying

$$\sup \{\rho(x, y) : x, y \in X\} \leq M. \quad (1)$$

Let A and B be two subsets of X with $X = A \cup B$ and $A \cap B = \emptyset$. Define a function d from $X \times X$ into $[0, \infty)$ by

$$\begin{aligned} d(x, x) &= 0 \\ d(x, y) &= d(y, x) = \rho(x, y) \quad \text{if } x \in A, y \in B \\ d(x, y) &= M \quad \text{otherwise.} \end{aligned} \quad (2)$$

Then (X, d) is a generalized metric space.

Proof. (N1) and (N2) are obvious. Let us prove (N3). Let $x, y, u, v \in X$ be all different. Put

$$t = d(x, u) + d(u, v) + d(v, y). \quad (3)$$

In the case where $t \geq M$, (N3) holds because $d(x, y) \leq M$. In the other case, where $t < M$, without loss of generality, we may assume $x \in A$. Then we have $v \in A$ and $u, y \in B$ from the definition of d . Hence,

$$\begin{aligned} d(x, y) &= \rho(x, y) \leq \rho(x, u) + \rho(u, v) + \rho(v, y) \\ &= d(x, u) + d(u, v) + d(v, y). \end{aligned} \quad (4)$$

Thus (N3) holds. $\square$

Definition 5. Let (X, d) be a ν -generalized metric space. Then a net $\{x_\alpha\}$ is said to converge to x if and only if $\lim_\alpha d(x, x_\alpha) = 0$.

Definition 6. Let X be a topological space with topology τ . Let d be a function from $X \times X$ into $[0, \infty)$ satisfying (N1)–(N3) with some $\nu \in \mathbb{N}$. Then τ is compatible with d if and only if the following are equivalent for any net $\{x_\alpha\}$ in X and $x \in X$:

- (a) $\lim_\alpha d(x, x_\alpha) = 0$.
- (b) $\{x_\alpha\}$ converges to x in τ .

The following is a very important example.

Example 7. Let

$$X = \{(0, 0)\} \cup ((0, 1] \times [0, 1]). \quad (5)$$

Define a function d from $X \times X$ into $[0, \infty)$ by

$$\begin{aligned} d(x, x) &= 0 \\ d((0, 0), (s, 0)) &= d((s, 0), (0, 0)) = s, \quad \text{if } s \in (0, 1] \\ d((s, 0), (p, q)) &= d((p, q), (s, 0)) = |s - p| + q, \quad \text{if } s, p, q \in (0, 1] \\ d(x, y) &= 3, \quad \text{otherwise.} \end{aligned} \quad (6)$$

Then the following hold:

- (i) (X, d) is not a metric space;
- (ii) (X, d) is a generalized metric space;

- (iii) X does not have a topology which is compatible with d .

Proof. Since

$$\begin{aligned} d((0, 0), (1, 0)) + d((1, 0), (1, 1)) \\ = 1 + 1 = 2 < 3 = d((0, 0), (1, 1)), \end{aligned} \quad (7)$$

(X, d) is not a metric space. Define a metric ρ on X by

$$\rho((s, t), (p, q)) = |s - p| + |t - q|, \quad (8)$$

for $(s, t), (p, q) \in X$. Put

$$A = \{(0, 0)\} \cup ((0, 1] \times (0, 1]), \quad B = (0, 1] \times \{0\}. \quad (9)$$

Then d is equal to the d defined by Lemma 4 with $M = 3$. Therefore, (X, d) is a generalized metric space. In order to show (iii), we will show that the following does not hold.

If a net $\{x_\alpha\}_{\alpha \in D}$ converges to x and for every $\alpha \in D$ a net $\{x_{(\alpha, \beta)}\}_{\beta \in E_\alpha}$ converges to x_α , then $\{x_{(\alpha, \gamma)}\}_{(\alpha, \gamma) \in D \times \prod \{E_\alpha : \alpha \in D\}}$ has a subnet converging to x ; see [4, page 77].

We have that $\{(1/\ell, 0)\}_\ell$ converges to $(0, 0)$ and $\{(1/\ell, 1/m)\}_m$ converges to $(1/\ell, 0)$ for every $\ell \in \mathbb{N}$. However, since $d((0, 0), (1/\ell, 1/m)) = 3$ for $(\ell, m) \in \mathbb{N}^2$, a net $\{(1/\ell, 1/\gamma(\ell))\}_{(\ell, \gamma)}$ does not converge to $(0, 0)$. Therefore there does not exist a topology which is compatible with d . $\square$

Remark 8. For $(\alpha, \gamma) \in D \times \prod \{E_\alpha : \alpha \in D\}$, $x_{(\alpha, \gamma)} = x_{(\alpha, \gamma(\alpha))}$. For $(\alpha_1, \gamma_1), (\alpha_2, \gamma_2) \in D \times \prod \{E_\alpha : \alpha \in D\}$, $(\alpha_1, \gamma_1) \leq (\alpha_2, \gamma_2)$ if and only if $\alpha_1 \leq \alpha_2$ and $\gamma_1(\alpha) \leq \gamma_2(\alpha)$ for any $\alpha \in D$.

Remark 9. Indeed, let τ be the topology induced by a subbase:

$$\{S(x, r) : x \in X, r > 0\}, \quad (10)$$

where $S(x, r) = \{y \in X : d(x, y) < r\}$. Since

$$\begin{aligned} S((0, 0), 2) \cap S((1, 0), 2) \\ = ([0, 1] \times \{0\}) \cap (\{(0, 0), (1, 0)\} \cup ((0, 1] \times (0, 1])) \\ = \{(0, 0), (1, 0)\}, \end{aligned} \quad (11)$$

we have

$$S((0, 0), 2) \cap S((1, 0), 1) = \{(1, 0)\}. \quad (12)$$

Hence $\{(1, 0)\}$ is an open neighborhood of $(1, 0)$. So a sequence $\{(1, 1/n)\}$ does not converge to $(1, 0)$ in τ . Since $\lim_n d((1, 0), (1, 1/n)) = 0$, τ is not compatible with d .

We can easily make an example of a ν -generalized metric space which is not a μ -generalized metric space for $\mu < \nu$.

Example 10. Put $X = \mathbb{N}$ and let $\nu \in \mathbb{N}$ satisfy $\nu \geq 2$. Define a function d from $X \times X$ into $[0, \infty)$ by

$$\begin{aligned} d(x, x) &= 0, \\ d(1, s) &= d(s, 1) = \nu + 1, \quad \text{if } s \in \mathbb{N} \setminus \{1, 2\}, \\ d(x, y) &= 1, \quad \text{otherwise.} \end{aligned} \quad (13)$$

Then the following hold:

- (i) (X, d) is not a μ -generalized metric space for $\mu \in \mathbb{N}$ with $\mu < \nu$;
- (ii) (X, d) is a μ -generalized metric space for $\mu \in \mathbb{N}$ with $\mu \geq \nu$.

Proof. (N1) and (N2) obviously hold. Let $\mu \in \mathbb{N}$ satisfy $\mu < \nu$. Since

$$\sum_{j=1}^{\mu+1} d(j, j+1) = \mu + 1 < \nu + 1 = d(1, \mu + 2), \quad (14)$$

(N3) does not hold. So (X, d) is not a μ -generalized metric space. Let $\mu \in \mathbb{N}$ satisfy $\mu \geq \nu$. Let $x, u_1, u_2, \dots, u_\mu, y \in X$ be all different. Then we have

$$d(x, y) \leq \nu + 1 \leq \mu + 1 \leq d(x, u_1) + d(u_1, u_2) + \dots + d(u_\mu, y). \quad (15)$$

Thus (N3) holds. Hence (X, d) is a μ -generalized metric space. $\square$

We give some definitions. The reason of these definitions is that (X, d) does not necessarily have the topology which is compatible with d . So (X, d) does not necessarily have the uniformity which is compatible with d .

Definition 11. Let (X, d) be a ν -generalized metric space.

- (a) A sequence $\{x_j\}$ is said to be *Cauchy* if and only if $\lim_j \sup_{m>j} d(x_j, x_m) = 0$.
- (b) X is said to be *complete* if and only if every Cauchy sequence converges to some point in X .
- (c) X is said to be *Hausdorff* if and only if $\lim_j d(x, x_j) = \lim_j d(y, x_j) = 0$ implies $x = y$.

Lemma 12. Let (X, d) be a ν -generalized metric space and let $x, u_1, \dots, u_\nu, y \in X$ such that $x, u_1, \dots, u_\nu$ are all different and $u_1, \dots, u_\nu, y$ are all different. Then

$$d(x, y) \leq d(x, u_1) + d(u_1, u_2) + \dots + d(u_\nu, y) \quad (16)$$

holds.

Proof. In the case where $x = y$, the conclusion obviously holds from (N1). In the other case, where $x \neq y$, the conclusion obviously holds from (N3). $\square$

3. The CJM Fixed Point Theorem

In this section, we generalize the CJM fixed point theorem; see Ćirić [5], Jachymski [6], and Matkowski [7, 8].

Theorem 13. Let (X, d) be a complete ν -generalized metric space and let T be a CJM contraction on X ; that is, the following hold:

- (i) for every $\varepsilon > 0$, there exists $\delta > 0$ such that $d(x, y) < \varepsilon + \delta$ implies $d(Tx, Ty) \leq \varepsilon$ for any $x, y \in X$;
- (ii) $x \neq y$ implies $d(Tx, Ty) < d(x, y)$ for any $x, y \in X$.

Then T has a unique fixed point z of T . Moreover, $\lim_j d(T^j x, z) = 0$ for any $x \in X$.

Proof. We first note that T is nonexpansive by (ii); that is

$$d(Tx, Ty) \leq d(x, y) \quad (17)$$

for any $x, y \in X$. Fix $u \in X$ and define a sequence $\{u_j\}$ in X by $u_j = T^j u$ for $j \in \mathbb{N}$. We next show that $\{u_j\}$ converges to a fixed point of T , dividing the following three cases:

- (a) there exists $n \in \mathbb{N}$ such that $u_{n+1} = u_n$;
- (b) $u_{j+1} \neq u_j$ for all $j \in \mathbb{N}$ and there exist $m, n \in \mathbb{N}$ such that $m + 2 \leq n$ and $u_m = u_n$;
- (c) $u_1, u_2, \dots$ are all different.

In the first case, u_n is a fixed point of T . By (N1), $\{u_j\}$ converges to u_n . In the second case, from (ii), we have $\{d(u_j, u_{j+1})\}$ is strictly decreasing. So, since $u_{m+1} = u_{n+1}$, we have

$$d(u_m, u_{m+1}) = d(u_n, u_{n+1}) < d(u_m, u_{m+1}). \quad (18)$$

This is a contradiction. Thus, the second case cannot be possible. In the third case, from (ii), we have $\{d(u_j, u_{j+k})\}$ is strictly decreasing for any $k \in \mathbb{N}$. So $\{d(u_j, u_{j+k})\}$ converges to some $\varepsilon_1 \geq 0$. Then we note that $d(u_j, u_{j+k}) > \varepsilon_1$ for every $j \in \mathbb{N}$. Arguing by contradiction, we assume $\varepsilon_1 > 0$. From (i), there exists $\delta_1 > 0$ such that

$$d(x, y) < \varepsilon_1 + \delta_1 \text{ implies } d(Tx, Ty) \leq \varepsilon_1. \quad (19)$$

From the definition of ε_1 , there exists $n \in \mathbb{N}$ such that $d(u_n, u_{n+k}) < \varepsilon_1 + \delta_1$. Then we have $d(u_{n+1}, u_{n+k+1}) \leq \varepsilon_1$. This is a contradiction. Therefore we obtain $\varepsilon_1 = 0$. That is, $\lim_j d(u_j, u_{j+k}) = 0$ holds for any $k \in \mathbb{N}$. Thus

$$\lim_{j \rightarrow \infty} \max \{d(u_j, u_{j+k}) : k = 1, 2, \dots, \nu + 1\} = 0 \quad (20)$$

holds. Fix $\varepsilon_2 > 0$. Then, by (i), there exists $\delta_2 \in (0, \varepsilon_2)$ such that

$$d(x, y) < \varepsilon_2 + 2\nu\delta_2 \text{ implies } d(Tx, Ty) \leq \varepsilon_2. \quad (21)$$

Let $\ell \in \mathbb{N}$ such that

$$\max \{d(u_j, u_{j+k}) : k = 1, 2, \dots, \nu + 1\} < \delta_2, \quad (22)$$

for all $j \in \mathbb{N}$ with $j \geq \ell$. We will show

$$d(u_\ell, u_{\ell+m}) < \varepsilon_2 + \nu\delta_2, \quad (23)$$

for $m \in \mathbb{N}$ by induction. For $m = 1, 2, \dots, \nu + 1$, we have

$$d(u_\ell, u_{\ell+m}) < \delta_2 < \varepsilon_2 + \nu\delta_2, \quad (24)$$

and, thus, (23) holds. We assume (23) holds for some $m \in \mathbb{N}$ with $m > \nu$. We have, by (N3),

$$\begin{aligned} & d(u_{\ell+\nu}, u_{\ell+m}) \\ & \leq \sum_{j=1}^{\nu} d(u_{\ell+j}, u_{\ell+j-1}) + d(u_\ell, u_{\ell+m}) \\ & < \nu\delta_2 + \varepsilon_2 + \nu\delta_2 = \varepsilon_2 + 2\nu\delta_2. \end{aligned} \quad (25)$$

Hence $d(u_{\ell+\nu+1}, u_{\ell+m+1}) \leq \varepsilon_2$. We put

$$\alpha = \begin{cases} d(u_\ell, u_{\ell+\nu+1}) & \text{if } \nu = 1 \\ d(u_\ell, u_{\ell+1}) + d(u_{\ell+1}, u_{\ell+\nu+1}) & \text{if } \nu = 2 \\ \sum_{j=\ell}^{\ell+\nu-2} d(u_j, u_{j+1}) + d(u_{\ell+\nu-1}, u_{\ell+\nu+1}) & \text{if } \nu > 2. \end{cases} \quad (26)$$

We note $\alpha < \nu\delta_2$. By (N3), we have

$$d(u_\ell, u_{\ell+m+1}) \leq \alpha + d(u_{\ell+\nu+1}, u_{\ell+m+1}) < \nu\delta_2 + \varepsilon_2. \quad (27)$$

Thus, (23) holds for $m := m + 1$. So, by induction, (23) holds for every $m \in \mathbb{N}$. Therefore we have shown

$$\lim_{\ell \rightarrow \infty} \sup_{\ell < m} d(u_\ell, u_m) \leq \varepsilon_2 + \nu\delta_2 < (\nu + 1)\varepsilon_2. \quad (28)$$

Since $\varepsilon_2 > 0$ is arbitrary, we obtain that $\{u_j\}$ is Cauchy. Since X is complete, $\{u_j\}$ converges to some point $z \in X$. We have by Lemma 12 and the nonexpansiveness of T

$$\begin{aligned} d(z, Tz) &\leq \left(d(z, u_{m+1}) + \sum_{j=1}^{\nu-1} d(u_{m+j}, u_{m+j+1}) + d(u_{m+\nu}, Tz) \right) \\ &\leq \left(d(z, u_{m+1}) + \sum_{j=1}^{\nu-1} d(u_{m+j}, u_{m+j+1}) + d(u_{m+\nu-1}, z) \right), \end{aligned} \quad (29)$$

for sufficiently large $m \in \mathbb{N}$. As m tends to ∞ , we obtain $d(z, Tz) = 0$. Thus, z is a fixed point of T . The uniqueness of the fixed point is obviously followed by (ii). $\square$

Remark 14. In [9], there is another fixed point theorem which is independent of Theorem 13.

By Theorem 13, we obtain a generalization of the Banach contraction principle [10, 11].

Corollary 15 (see Branciari [1]). *Let (X, d) be a complete ν -generalized metric space and let T be a contraction on X ; that is, there exists $r \in [0, 1)$ such that*

$$d(Tx, Ty) \leq rd(x, y), \quad (30)$$

for any $x, y \in X$. Then T has a unique fixed point z of T . Moreover, $\lim_j d(T^j x, z) = 0$ for any $x \in X$.

Remark 16. The authors in [12] stated the proof in [1] is incorrect and gave a proof under the assumption that (X, d) is Hausdorff and $\nu = 2$. See also [13].

In order to show that Theorem 13 is a generalization of Theorem 3.1 in [14], we prove the following. See also [15]. The idea on the proof of the following proposition appears in [16, 17].

Proposition 17. *Let (X, d) be a ν -generalized metric space and let T be a mapping on X . Assume that there exist functions φ, ψ from $[0, \infty)$ into $[0, \infty)$ such that the following hold:*

- (i) $\psi(d(Tx, Ty)) \leq \psi(d(x, y)) - \varphi(d(x, y))$ for any $x, y \in X$;
- (ii) ψ is nondecreasing;
- (iii) $\inf \varphi([s, t]) > 0$ for any $s, t \in (0, \infty)$ with $s < t$.

Then T is a CJM contraction.

Proof. Since $\varphi(t) > 0$ for any $t \in (0, \infty)$, (ii) of the definition of CJM contraction obviously holds. We will show (i) of the definition of CJM contraction. Fix $\varepsilon > 0$. From (iii), we can put

$$\eta := \inf \{ \varphi(t) : \varepsilon \leq t \leq \varepsilon + 1 \} > 0. \quad (31)$$

We choose $\delta \in (0, 1)$ such that

$$\psi(\varepsilon + \delta) < \lim_{t \rightarrow \varepsilon+0} \psi(t) + \eta. \quad (32)$$

Let $x, y \in X$ satisfy $d(x, y) < \varepsilon + \delta$. In the case where $d(x, y) = 0$, we have $d(Tx, Ty) = 0$ because $x = y$. In the case where $0 < d(x, y) \leq \varepsilon$, we have

$$\begin{aligned} \psi(d(Tx, Ty)) &\leq \psi(d(x, y)) - \varphi(d(x, y)) < \psi(d(x, y)) \leq \psi(\varepsilon), \end{aligned} \quad (33)$$

which implies $d(Tx, Ty) < \varepsilon$. In the other case, where $d(x, y) > \varepsilon$, we have

$$\begin{aligned} \psi(d(Tx, Ty)) &\leq \psi(d(x, y)) - \varphi(d(x, y)) \leq \psi(\varepsilon + \delta) - \eta \\ &< \lim_{t \rightarrow \varepsilon+0} \psi(t) + \eta - \eta = \lim_{t \rightarrow \varepsilon+0} \psi(t), \end{aligned} \quad (34)$$

which implies $d(Tx, Ty) \leq \varepsilon$. Hence we have $d(Tx, Ty) \leq \varepsilon$ in all cases. Therefore T is a CJM contraction. $\square$

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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