Research Article

An Improved Internal Model Principle Based Multivariable Nonlinear Control Method with Multiclass Nonharmonic Disturbances and Its Application to Speed Control of a Motor Drive System

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We study the global disturbance rejection problem for a class of general multivariable nonlinear systems with multiclass nonharmonic disturbances. The paper first introduces the importance and state of the art for disturbance rejection problem and describes the control problem in the form of mathematical expressions. It stresses the multiclass disturbances produced by the exosystem satisfying certain characteristic conditions. Then, the nonlinear internal models are designed in accordance with different characteristics of multiclass external disturbances. On the basis of introduction of the control law for disturbance-free system, a multivariable state feedback controller is devised in terms of the designed internal model equations and corresponding assumptions. A Lyapunov function is constructed to theoretically prove the global uniform boundness of all signals for the multivariable closed-loop system. Finally, the presented method is applied to implement the speed control and reject the multiclass nonharmonic disturbances for a two-input motor drive system. The simulation results testify correctness and effectiveness of the presented algorithm.

1. Introduction

Due to its importance, the problem of disturbance rejection generated by the exosystem has been paid great attention in the world over the past few decades [1–3]. Especially, this problem in nonlinear system has been concerned in a recent period of time [4–6]. In terms of internal model principle (IMP), one of the key technologies of rejection of exogenous disturbance is to construct reasonable internal model equations, which is able to duplicate the characteristics of the external disturbance.

In addition, many industrial applications, such as servo system, wind power system, photovoltaic system, and other modern systems, which rely on power electronic equipments to work, [7–9], are frequently influenced by the external disturbances generated by exosystem [10, 11]. In the circumstance of the existence of external disturbances, the conventional PID controller with constant parameters is difficult to satisfy the requirement of high precision control for these systems [12, 13]. Therefore, rejection of external disturbances by means of new ways [14, 15] is an important work to improve the working performance of these industrial systems. From the state-of-the-art technologies in disturbance rejection, the researchers frequently assume the following:

1. the external disturbances are frequently sinusoidal; that is to say, the exosystem generally satisfies the condition of neutral stability such as references [16, 17] discussing the suppression of sinusoidal interference with known and unknown frequency, respectively;
2. currently, the focus of the research on disturbance rejection is gradually converting from linear systems to nonlinear systems; nevertheless, the disturbance rejections with semiglobal stability [18] and single-input and single-output (SISO) [19] systems are concerned.
Therefore, the disturbance rejection problem with nonharmonic interferences and multivariable nonlinear systems has not received adequate attention in current studies. However, the nonharmonic disturbances produced by nonlinear exosystem can induce noise and reduce accuracy to these industrial systems [20]. In addition, just a few articles on nonharmonic interference suppression aim at one class of disturbance [21]. Therefore, the research of multiclass disturbances rejection in multivariable nonlinear system can extend the application of present control theory to a generalized range [22]. Hence, in [23], a global multivariable control algorithm is proposed to reject multiclass external nonharmonic disturbances.

In view of the issues described above, the purpose of the paper is to propose an improved internal model principle (IMP) based multivariable nonlinear control algorithm with multiclass nonharmonic disturbances and utilize the proposed method to control a motor drive system to run stably and suppress multiclass external nonharmonic disturbances. As written in [23], the critical points of the problem in disturbance rejection are to model the nonlinear exosystem and present a corresponding control method to suppress the disturbance. Therefore, the essential distinction with the work in [23], this paper addresses the problem of multivariable disturbances rejection with other multiclass exosystems, additionally, which are provided by matrix form. For the difference of external exosystems, the entirely different internal model and a matching control method should be proposed to model and reject the external disturbances in terms of the intrinsic characteristics of exosystem. In practical terms, the construction of internal model replicating with the key of the study is to convert multivariable nonlinear system into multiple single-input systems [25].

Assumption 1. For the multivariable nonlinear disturbance-free system shown below:

\[
\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i, \quad 1 \leq i \leq m,
\]

there exists control law of state feedback \(\alpha_i(x)\), which can make the system

\[
\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) \alpha_i(x)
\]

asymptotically converge to the origin. In consequence, a Lyapunov function \(V(x)\) exists and makes the following inequalities hold [20, 23]:

\[
\frac{d}{dt} (\|x\|) \leq V(x) \leq \overline{d} (\|x\|),
\]

\[
\partial V(x) \frac{\partial V(x)}{\partial x} \left( f(x) + \sum_{i=1}^{m} g_i(x) \alpha_i(x) \right) \leq -d_0 (\|x\|),
\]

\[
\left| \frac{\partial V(x)}{\partial x} \sum_{i=1}^{m} g_i(x) \right|^2 \leq d_0 (\|x\|),
\]

where \(d_0, \overline{d}\), and \(d_0\) belong to class \(K_{\infty}\) functions.

Assumption 2. The flows of the vector field for nonlinear exosystem (2) are bounded.
Remark 3. Generally speaking, there exist numerous nonlinear systems satisfying Assumption 2, such as harmonic functions and limit cycles of nonlinear dynamic system. The well-known Van der Pol oscillator [20] is selected as the exosystem in the paper and can be expressed as below:

\[ \ddot{w}_1 = w_2, \]
\[ \ddot{w}_2 = -aw_1 + b\left(1 - w_3^2\right)w_2, \tag{6} \]

where a and b are constants greater than zero. Under such circumstances, the limit cycle generated by Van der Pol oscillator is stable. Consequently, (6) is written in the form of (2), and we can obtain

\[ \dot{w} = A_{11}w + A_{12}w_{12}(w), \tag{7} \]

where \( A_{11} = \begin{bmatrix} 0 & 1 \\ -a & b \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -b \end{bmatrix}, s_{12}(w) = w_1^2. \]

Assumption 4. There exist a constant \( r_i \) and a set of real numbers \( a_{i0}, a_{i1}, \ldots, a_{i(r_i-1)} \) satisfying the following equation:

\[ L'_{A_i,w}v_i(w) = a_{i0}v_i(w) + a_{i1}L_{A_i,w}v_i(w) \]
\[ + \cdots + a_{i(r_i-1)}L_{A_i,w}^{r_i-1}v_i(w), \tag{8} \]

where \( L_{A_i,w}v_i(w) = (\partial v_i(w)/\partial w)A_{i1}w, L_{A_i,w}^m v_i(w) = (\partial L_{A_i,w}^{m-1}v_i(w)/\partial w)A_{i1}w, m = 2, 3, \ldots, r_i, \) and \( L \) is an operator of Lee derivative.

Apparently, if \( v_i(w) \) denotes a linear expression of \( w, \) Assumption 4 will hold automatically.

Assumption 5. There exists a smooth function \( h_i(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) making

\[ \frac{\partial h_i(x)}{\partial x} g_i(x) = G_i, \quad 1 \leq i \leq m, \tag{9} \]

where \( G_i \) is a nonzero constant vector defined in \( \mathbb{R}^n. \)

The questions resolved in the paper can be depicted as below.

Definition 6. For an arbitrarily given compact subset \( D_w \in \mathbb{R}^n, \) a state feedback controller \( u \) can always be found to guarantee the solution of original multivariable closed-loop system (1) existence and boundedness under arbitrary initial conditions for all \( w(0) \in D_w, \) when \( t \geq 0, \) and even \( \lim_{(0)} - e_i x(t) = C, \) where \( C \) is a constant vector representing the reference value.

3. Design of Multiclass Nonlinear Internal Models

Let

\[ \tau_i(w) = \text{col}\left(v_i(w), L_{A_i,w}v_i(w), \ldots, L_{A_i,w}^{r_i-1}v_i(w)\right); \tag{10} \]

then there exist matrices \( \Phi_i = \begin{bmatrix} 0 \vdots \\ a_{i0} \vdots a_{i(r_i-1)} \end{bmatrix} \) and \( \psi_i = [1, 0, \ldots, 0] \) making

\[ \frac{\partial \tau_i(w)}{\partial w} A_{ij}w = \Phi_i \tau_i(w), \]
\[ v_i(w) = \psi_i \tau_i(w), \tag{11} \]

where matrix pair \( (\Phi_i, \psi_i) \) is observable and \( I_{r_i-1} = (r_i - 1) \times (r_i - 1) \) unit matrix.

For the sake of establishment of nonlinear internal model equation, the following assumption is brought up.

Assumption 7. There exists a matrix \( \Phi_{ik}, k = 2, \ldots, n, \) making

\[ \frac{\partial \tau_i(w)}{\partial w} A_{ik}w = \Phi_{ik} \tau_i(w). \tag{12} \]

Assume \( \tilde{\tau}_i(w) = T_i \tau_i(w) \) and \( T_i \in \mathbb{R}^{r_i*}, \) to be nonsingular matrices. With derivative of \( \tilde{\tau}_i(w) \) along with (2), we can obtain

\[ \frac{\partial \tilde{\tau}_i(w)}{\partial w} s_i(w) = \frac{\partial \tilde{\tau}_i(w)}{\partial w} \left(A_{i1}w + \sum_{k=2}^{n} A_{ik}w s_{ik}(w)\right) \]
\[ = T_i \Phi_i T_i^{-1} \tilde{\tau}_i(w) + \sum_{k=2}^{n} T_i \Phi_i s_{ik}(w) T_i^{-1} \tilde{\tau}_i(w) \]
\[ = T_i \Phi_i(w) T_i^{-1} \tilde{\tau}_i(w) + \psi_i T_i^{-1} \tilde{\tau}_i(w), \tag{13} \]

where \( \phi_i(w) = \Phi_i + \phi_i(w), \psi_i(w) = \sum_{k=2}^{n} \Phi_{ik}, \) and \( \phi_i(0) = 0, \phi_i(0) = \Phi_i. \)

In terms of linear observer theory, a Hurwitz matrix \( F_i \) is chosen to make matrix pair \( (F_i, G_i) \) controllable for nonzero constant vector \( G_i \) defined in (9). Due to the observability of matrix pair \( (\Phi_i, \psi_i) \) and the fact that \( F_i \) and \( \Phi_i \) have nonintersecting frequency spectrum, hence Sylvester equation \( T_i \Phi_i - F_i T_i = G_i \psi_i \) has a unique nonsingular solution \( T_i. \)

Let \( q_i = \psi_i T_i^{-1}, \) the nonlinear exosystem can immerse into the system shown as below:

\[ \dot{\eta}_i = \left(F_i + T_i \phi_i(w) T_i^{-1}\right) \eta_i + G_i q_i \eta_i; \tag{14} \]
\[ v_i(w) = q_i \eta_i; \]

In consequence, multiclass nonlinear internal models can be designed as follows:

\[ \dot{\xi}_i = \left(F_i + T_i \phi_i(w) T_i^{-1}\right) \left(\xi_i + h_i(x)\right) \]
\[ - \frac{\partial h_i(x)}{\partial x} \left(\xi_i + h_i(x) + g_i(x) u_i\right). \tag{15} \]

Define an auxiliary error \( e_i \) as

\[ e_i = \eta_i - \xi_i - h_i(x), \tag{16} \]
and with derivative of (16) along with (1), (14), and (15), we can get

\[ e_i = \left( F_i + T_i q_i (w) T_i^{-1} \right) \dot{\eta}_i + G_i q_i \eta_i - \left( F_i + T_i q_i (w) T_i^{-1} \right) (\xi_i + h_i (x)) + \frac{\partial h_i (x)}{\partial x} (f_i (x) + g_i (x) u_i) \]

\[ - \frac{\partial h_i (x)}{\partial x} (f_i (x) + g_i (x) (u_i + v_i (w))) = \left( F_i + T_i q_i (w) T_i^{-1} \right) (\eta_i - \xi_i - h_i (x)) = \left( F_i + T_i q_i (w) T_i^{-1} \right) e_i. \]

(17)

4. Design of Multivariable Nonlinear State Feedback Controller

On the basis of the established multiclass nonlinear internal models (15) and Assumption 1, the multivariable nonlinear state feedback controller can be designed as follows:

\[ u_i = \alpha_i (x) - q_i (\xi_i + h_i (x)). \]

(18)

Construct a Lyapunov function \( W \) as below:

\[ W = V (x) + \sum_{i=1}^{m} y e_i^T e_i, \]

(19)

where \( y \) is a positive real constant. With derivative of function \( W \) along with originally nonlinear system (1) and auxiliary error (17), we can obtain

\[ \dot{W} = \frac{\partial V (x)}{\partial x} \left( f (x) + \sum_{i=1}^{m} g_i (x) (u_i + v_i (w)) \right) \]

\[ + \sum_{i=1}^{m} \left( y e_i^T \left( F_i + T_i q_i (w) T_i^{-1} + F_i^T \right) \right. \]

\[ \left. + T_i^{-T} q_i^T (w) T_i^T \right) e_i) \]

\[ = \frac{\partial V (x)}{\partial x} \left( f_i (x) \right. \]

\[ \left. + \sum_{i=1}^{m} g_i (x) (\alpha_i (x) - q_i (\xi_i + h_i (x)) + q_i \eta_i) \right) \]

\[ + \sum_{i=1}^{m} y e_i^T (S (w)) e_i \]

\[ = \frac{\partial V (x)}{\partial x} \left( f_i (x) + \sum_{i=1}^{m} g_i (x) \alpha_i (x) \right) \]

\[ + \frac{\partial V (x)}{\partial x} \sum_{i=1}^{m} g_i (x) q_i e_i + \sum_{i=1}^{m} y e_i^T (S (w)) e_i \]

\[ \leq - d_0 (\|x\|) + \left| \frac{\partial V (x)}{\partial x} \sum_{i=1}^{m} g_i (x) \right| \|q_i e_i\| \]

\[ + \sum_{i=1}^{m} y e_i^T (S (w)) e_i, \]

(20)

where \( S (w) = F_i + T_i q_i (w) T_i^{-1} + F_i^T + T_i^{-T} q_i^T (w) T_i^T \). By application of inequality \( 2ab \leq ca^2 + c^{-1}b^2 \) (choose \( c = 1 \)) into the second item of (20), we can get

\[ \left| \frac{\partial V (x)}{\partial x} \sum_{i=1}^{m} g_i (x) \right| \|q_i e_i\| \]

\[ \leq \frac{1}{2} \left[ \frac{\partial V (x)}{\partial x} \sum_{i=1}^{m} g_i (x) \right]^2 + \sum_{i=1}^{m} \|q_i\|^2 \|e_i\|^2 \]

\[ \leq \frac{1}{2} d_0 (\|x\|) + \sum_{i=1}^{m} \|q_i\|^2 \|e_i\|^2. \]

(21)

Assume that there exists a compact set \( S (w) \) making \( S (w) \) be a symmetric matrix of negative definiteness for all \( w \in S (w) \). Hence, there exists a positive real number \( f \) satisfying the following inequality for all \( w \in S (w) \):

\[ e_i^T (S (w)) e_i \leq - f \|e_i\|^2. \]

(22)

Substitute (21) and (22) into (20); we can get

\[ W \leq - d_0 (\|x\|) + \frac{1}{2} \left[ \frac{\partial V (x)}{\partial x} \sum_{i=1}^{m} g_i (x) \right] \|q_i e_i\| \]

\[ + \sum_{i=1}^{m} y e_i^T (S (w)) e_i \]

\[ \leq - d_0 (\|x\|) + \frac{1}{2} d_0 (\|x\|) + \sum_{i=1}^{m} \|q_i\|^2 \|e_i\|^2 \]

\[ \leq - \frac{1}{2} d_0 (\|x\|) + \sum_{i=1}^{m} \left( \frac{1}{2} \|q_i\|^2 - y f \right) \|e_i\|^2. \]

Choose appropriate \( y \) and \( f \) to satisfy

\[ \theta = \frac{1}{2} \|q_i\|^2 - y f < 0; \]

(24)
then, we can obtain
\[ \dot{W} \leq -\frac{1}{2} \theta (\|x\|) + \sum_{i=1}^{m} \theta \|e_i\|^2 < 0. \]  
(25)

Hence, all variables are bounded. Combined with the utilization of invariant set theorem, we can get \( \lim_{t \to \infty} x(t) = C \) and \( \lim_{t \to \infty} e_i = 0 \). The following theorem can be obtained.

**Theorem 8.** For the multivariable nonlinear system (1) and nonlinear ecosystem (2) satisfying Assumption 1 to Assumption 7, multiclass nonlinear internal models (15) and multivariable nonlinear state feedback controller (18) are able to make the closed-loop system globally bounded, and \( \lim_{t \to \infty} x(t) = C \).

5. Application to Speed Control of a Two-Input Nonlinear System

Speed control of a permanent magnet synchronous motor drive system is widely utilized in various industrial fields. Hence, consider a two-input variable motor drive system shown as below [25], which is not able to tackle with a single-input method of previous disturbance rejection:

\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{R_d}{L_d} i_d + p i_q \omega_m + \frac{1}{L_d} u_d, \\
\frac{d\omega_m}{dt} &= \frac{\rho \phi_f}{J_m} i_q - \frac{B_m}{J_m} \omega_m - \frac{1}{J_m} T_L, \\
\frac{di_q}{dt} &= -\frac{R_q}{L_q} i_d - p i_d \omega_m + \frac{\rho \phi_f}{L_q} \omega_m + \frac{1}{L_q} u_q,
\end{align*}
\]

(26)

where \( i_d \) and \( i_q \) represent the current in \( d \)-axis and \( q \)-axis, respectively, \( u_d \) and \( u_q \) indicate the voltage in \( d \)-axis and \( q \)-axis, respectively, \( R_d \) and \( \phi_f \) denote the resistance of stator and flux linkage of permanent magnet, \( \omega_m \) is the speed, \( p \) and \( J_m \) denote number of pole pairs and moment of inertia of rotor, respectively, and \( T_L \) is an increasing load.

With conversion of (26) into an affine form as (1), we can get

\[
\dot{x} = f(x) + \sum_{i=1}^{2} g_i(x) (u_i + v_i(w)),
\]

(27)

where \( x = [x_1 \ x_2 \ x_3]^T = [i_d \ \omega_m \ i_q]^T \),

\[
f(x) = \begin{bmatrix}
-\frac{R_d}{L_d} x_1 + p x_2 x_3 \\
\frac{\rho \phi_f}{J_m} x_3 - \frac{B_m}{J_m} x_2 - \frac{1}{J_m} T_L \\
\frac{R_q}{L_q} x_3 - p x_2 x_1 - \frac{\rho \phi_f}{L_q} x_2
\end{bmatrix},
\]

(28)

\[
g_1(x) = \begin{bmatrix}
\frac{1}{L_d} \\
0 \\
\frac{1}{L_q}
\end{bmatrix}^T, \quad g_2(x) = \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix},
\]

and control inputs \( u = [u_1 \ u_2]^T = [u_d \ u_q]^T \).

For the sake of understandability, the external disturbances inputs, \( v_1 \) and \( v_2 \), which denote different nonlinear nonharmonic disturbance input signals, are both produced by Van der Pol oscillator shown in (6), and let \( a = b = 1 \). Hence, \( A_{11} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \), \( A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \). Evidently, \( A_{21} = A_{11} \) and \( A_{22} = A_{12} \). It is important to note that \( A_{11}, A_{21}, A_{12}, \) and \( A_{22} \) are not elements in matrix \( A_1 \) or \( A_2 \), which are all independent matrices defined in (2). For the sake of convenience, let \( A_{11} = A_{21} = A_{12} = A_{22} = A_1 = A_2 \), and \( s_1(w) = s_2(w) = w_c^2 \). The bounded limit cycle will be generated by the oscillator; hence, Assumption 2 holds.

Let \( c_1, c_2, \) and \( c_3 \) all be certain positive constants; the control law of disturbance-free system for (27) can be obtained by means of backstepping control and can be shown as follows:

\[
\alpha(x) = \begin{bmatrix}
\alpha_1(x) \\
\alpha_2(x)
\end{bmatrix} = \begin{bmatrix}
-L_d (c_1 x_1 + \frac{L_d}{L_d} \rho \phi_f x_3) \\
-c_2 (x_2 - \omega_{ref}) + \frac{R_q}{L_q} x_3 \\
-L_d (c_3 x_3 - \frac{L_m}{L_m} \rho \phi_f (\frac{\phi_f}{J_m} \omega_{ref} + \frac{1}{J_m} T_L))
\end{bmatrix},
\]

(29)

where \( \omega_{ref} \) is the reference speed. It can be verified that disturbance-free system (3) can be stabilized by \( \alpha(x) \). Owing to its unimportance for the study of the paper, the derivation process of the controller for the disturbance-free system is omitted.

Let

\[
V(x) = \frac{1}{2} x_1^2 + \frac{1}{2} c_1(x_2 - \omega_{ref})^2 + \frac{1}{2} (x_3 - \frac{L_m}{L_m} \rho \phi_f (\frac{\phi_f}{J_m} \omega_{ref} + \frac{1}{J_m} T_L))^2.
\]

(30)

By means of some calculations and simplifications, we can get

\[
\frac{\partial V(x)}{\partial x} (f(x) + \sum_{i=1}^{2} g_i(x) \alpha_i) = -c_1 (x_1 + \frac{R_q}{L_d} x_2 - \omega_{ref})^2 - c_3 (x_3 - \frac{L_m}{L_m} \rho \phi_f (\frac{\phi_f}{J_m} \omega_{ref} + \frac{1}{J_m} T_L))^2.
\]

(31)

\[
\frac{\partial V(x)}{\partial x} \sum_{i=1}^{2} g_i(x) = \frac{1}{L_d} x_1 + \frac{1}{L_q} \left( x_3 - \frac{L_m}{L_m} \rho \phi_f (\frac{\phi_f}{J_m} \omega_{ref} + \frac{1}{J_m} T_L) \right).
\]
To be more specific, the relevant parameters of the motor drive system are depicted as follows: the rated torque of motor $T_e = 5.0$ N·m, number of rotor pole pairs $p = 4$, flux linkage of permanent magnet $\Psi_p = 0.18$ Wb, resistance of stator $R_s = 1.95$ $\Omega$, inductances of stator in $d$-axis and $q$-axis $L_d = L_q = 0.0115$ H, moment of inertia of rotor $J_m = 0.008$ kg·m², and damping coefficient of motor $B_m = 0.01$ N·m/s.

In addition, we assume $x^T = [x_1, (x_2 - \omega_{rel})]$, $(x_3 - (J_m/\rho_p)(\psi_1/J_m\omega_{rel} + (1/J_m)T_1))$, by application of (30) and (31) and selection of $c_1 = 8000$, $c_2 = 40$, and $c_3 = 8000$, we can get

$$\frac{1}{2}\|x\|^2 \leq V(x) \leq 20\|x\|^2.$$  

(32)

Hence, Assumption 1 is satisfied.

Let

$$h_1(x) = [0 \ 3L_d x_1]^T,$$

$$h_2(x) = [0 \ 3L_d x_3]^T.$$  

(33)

Hence,

$$G_1 = \frac{\partial h_1(x)}{\partial x} g_1(x) = [0 \ 3]^T,$$

$$G_2 = \frac{\partial h_2(x)}{\partial x} g_2(x) = [0 \ 3]^T.$$  

(34)

Assumption 5 holds.

Assume that $v_1$ and $v_2$ denote different nonlinear disturbance input signals and act on the currents of $d$-axis and $q$-axis, respectively. Choose $v_1 = w_1$ and $v_2 = w_2$. In other words, $v_1$ and $v_2$ represent that the system is immersed into two different external disturbances.

For $v_1 = w_1$, by means of some mathematical calculations, it can be obtained that

$$L_{A_1 w} v_1 (w) = \frac{\partial v_1 (w)}{\partial w} A_1 w = [1 \ 0] \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = w_2,$$

$$L^2_{A_1 w} v_1 (w) = \frac{\partial L_{A_1 w} v_1 (w)}{\partial w} A_1 w = [0 \ 1] \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_1 + w_2 = -a_{10} v_1 (w) + a_{11} L_{A_1 w} v_1 (w).$$  

(35)

Hence, there exist constants $r_1 = 2$, $a_{10} = -1$, and $a_{11} = 1$ satisfying Assumption 4.

For $v_2 = w_2$, by means of similar mathematical calculations, we can get

$$L_{A_2 w} v_2 (w) = \frac{\partial v_2 (w)}{\partial w} A_1 w = [0 \ 1] \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_1 + w_2,$$

$$L^2_{A_2 w} v_2 (w) = \frac{\partial L_{A_2 w} v_2 (w)}{\partial w} A_1 w = [0 \ 1] \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = -w_1 + a_{20} v_1 (w) + a_{21} L_{A_1 w} v_1 (w).$$  

(36)

Therefore, there exist constants $r_1 = 2$, $a_{20} = 1$, and $a_{21} = 1$ making Assumption 4 be satisfied.

In terms of (35), let

$$\tau_1 (w) = \operatorname{col} (v_1(w), L_{A_1 w} v_1 (w)) = \operatorname{col} (w_1, w_2).$$  

(37)

Then, there exist matrices $\Phi_1 = \begin{bmatrix} 0_{0_{12-11}} & l_{11-1} \\ a_{10} & a_{11-1} \end{bmatrix}$ and $\psi_1 = [1, 0]$ making (II) hold and rank $[\psi_1, \psi_1 \Phi_1]^T = 2$.

According to (36), let

$$\tau_2 (w) = \operatorname{col} (v_2(w), L_{A_1 w} v_2 (w)) = \operatorname{col} (w_2, -w_1 + w_2).$$  

(38)

In that way, there exist matrices $\Phi_2 = \begin{bmatrix} 0_{0_{12-11}} & l_{22-1} \\ a_{20} & a_{21-1} \end{bmatrix}$ and $\psi_2 = [1, 0]$ making (II) satisfy and rank $[\psi_2, \psi_2 \Phi_2]^T = 2$.

Choose $\Phi_{12} = A_2$; after some calculations it can be gotten that

$$\frac{\partial \tau_1 (w)}{\partial w} A_2 w = \Phi_{12} \tau_1 (w).$$  

(39)

Select $\Phi_{22} = [-1 \ 0]$; after some calculations we can get

$$\frac{\partial \tau_2 (w)}{\partial w} A_2 w = \Phi_{22} \tau_2 (w).$$  

(40)

Consequently, Assumption 7 is satisfied.

Assume $\tilde{r}_1 (w) = T_1 \tau_1 (w)$, where $T_1 \in \mathbb{R}^{2 \times r_1}$ is a nonsingular matrix, and by derivation of $\tilde{r}_1 (w)$ along with (2), we can get

$$\frac{\partial \tilde{r}_1 (w)}{\partial w} s_1 (w) = T_1 \psi_1 (w) T_1^{-1} \tau_1 (w),$$  

(41)

where $\psi_1 (w) = \Phi_1 + \phi_1 (w)$, $\phi_1 (w) = \Phi_1 s_1 (w)$.

Suppose $\tilde{r}_2 (w) = T_2 \tau_2 (w)$, where $T_2 \in \mathbb{R}^{2 \times r_2}$ is a nonsingular matrix and by derivation of $\tilde{r}_2 (w)$ along with (2), we can get

$$\frac{\partial \tilde{r}_2 (w)}{\partial w} s_2 (w) = T_2 \phi_2 (w) T_2^{-1} \tau_2 (w),$$  

(42)

where $\phi_2 (w) = \Phi_2 + \phi_2 (w)$, $\phi_2 (w) = \Phi_2 s_2 (w)$. 

Choose $F_1 = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$; we can verify rank $[G_1 \ G_1F_1]^T = 2$. Solving Sylvester equation $T_1\Phi_1 - F_1T_1 = G_1\psi_1$, it can be gotten that

$$T_1 = \begin{bmatrix} 0.1993 & -0.0897 \\ 0.4884 & -0.0698 \end{bmatrix}. \quad (43)$$

Select $F_2 = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$. Similarly, we are able to obtain rank $[G_2 \ G_2F_2]^T = 2$. Solving Sylvester equation $T_2\Phi_2 - F_2T_2 = G_2\psi_2$, we can get

$$T_2 = \begin{bmatrix} 0.3220 & -0.1317 \\ 0.5122 & -0.0732 \end{bmatrix}. \quad (44)$$

It has already been verified that all assumed conditions required for systems (27) and (6) are satisfied. Following the design procedure given in Sections 3 and 4, the corresponding nonlinear internal models and state feedback controller are able to be designed as below:

$$\begin{align*}
\dot{\xi}_1 &= \left(-2 - 1.4651\omega_1^2\right)\xi_1 + \left(1 + 0.598\omega_1^2\right)(\xi_2 + 0.0345x_1), \\
\dot{\xi}_2 &= -1.1395\omega_1^2\xi_1 + \left(-6 + 0.4651\omega_1^2\right)(\xi_2 + 0.0345x_1) \\
&\quad + 5.85x_1 - 0.138x_2x_3 - 3u_1, \\
\dot{\xi}_3 &= \left(-2 + 0.3171\omega_1^2\right)\xi_3 + \left(1 - 0.5707\omega_1^2\right)(\xi_4 + 0.0345x_1), \\
\dot{\xi}_4 &= 0.7317\omega_1^2\xi_3 + \left(-6 - 1.3171\omega_1^2\right)(\xi_4 + 0.0345x_1) \\
&\quad - 5.85x_3 - 0.138x_2x_3 - 2.16x_2 + 3u_2, \\
u_1 &= -92.046x_1 - 0.046x_2x_3 + 2.3333\xi_1 - 3\xi_2, \\
u_2 &= -413.28x_1 - 90.096x_3 + 249550 + 0.046x_2x_1 \\
&\quad + 127.7778T_L + 1.6667\xi_3 - 3\xi_4. \quad (45)
\end{align*}$$

In order to evaluate the validity of the proposed method, the numerical simulations are performed in Matlab software with the sampling interval $\Delta t = 0.001$ s and initial conditions $x(0) = [0.1 \\ 0.1 \ 0]$, $\eta(0) = [0 \\ 0 \\ 0]$, and $w(0) = [1 \\ -1]$. In addition, the reference values of $d$-axis current $i_d$ and speed $\omega_m$ are set to zero and 600 r/min, respectively, and $q$-axis current $i_q$ tracks the linear variation value of load $T_L$. Ultimately, the simulation results are depicted in Figures 1, 2, and 3. Figure 1 describes the multiclass nonlinear nonharmonic disturbances $\nu_1$ and $\nu_2$, in which we can see that the exogenous disturbances immersing into the system are bounded and different, and they can originate from different exosystems so long as the exosystems can be written in the form of (2). Figure 2 demonstrates the control inputs $u_d$ and $u_q$, which matches the external disturbances and reference values; and their combined actions make the different disturbances be rejected and ensure the system outputs to track the reference values. Figure 3 indicates the state outputs of the system, in which we can see that the $d$-axis current $i_d$ and speed $\omega_m$ are able to track their own reference values in high precision. In other words, the simulation results signify that the proposed algorithm is able to reject the multiclass external disturbance effectively and ensures that the closed-loop system rapidly converge to the reference values. In consequence, the designed internal models can replicate with the different exogenous disturbances and the proposed controller has an excellent control performance.

6. Conclusions

In the paper, an IMP based multivariable nonlinear control algorithm with multiclass nonharmonic disturbances is proposed to suppress the differently external nonharmonic disturbances and control the closed-loop system to track the reference values. The major conclusions of the research are summarized as follows:

1. In light of the multiclass nonharmonic disturbances, a class of new multiclass nonlinear internal models can be constructed on the basis of the definitions of expanded steady state generator and IMP, and
the characteristics of which depend on the structural information of exosystem and state information of closed-loop system;

(2) a multivariable nonlinear state feedback controller associated with the designed internal model equations is designed; the application of Lyapunov theory can justify the convergence of the proposed control algorithm;

(3) the application of the algorithm into a two-input variable motor drive system demonstrates that the presented algorithm is able to suppress the multiclass disturbances and guarantee the multivariable closed-loop system global uniform convergence.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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