Editorial

Analytical and Numerical Methods for Solving Partial Differential Equations and Integral Equations Arising in Physical Models

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Mathematical modelling of real-life problems usually results in functional equations, like ordinary or partial differential equations, integral and integrodifferential equations, and stochastic equations. Many mathematical formulations of physical phenomena contain integrodifferential equations; these equations arise in many fields like fluid dynamics, biological models, and chemical kinetics. Partial differential equations (PDEs) have become a useful tool for describing the natural phenomena of science and engineering models. In addition, most physical phenomena of fluid dynamics, quantum mechanics, electricity, ecological systems, and many other models are controlled within their domain of validity by PDEs. Therefore, it becomes increasingly important to be familiar with all traditional and recently developed methods for solving PDEs and the implementations of these methods. Leaving aside quantum mechanics, which remains to date an inherently linear theory, most real-world physical systems, including gas dynamics, fluid mechanics, elasticity, relativity, ecology, neurology, and thermodynamics, are modelled by nonlinear partial differential equations.

The aim of this special issue is to bring together the leading researchers of dynamics, quantum mechanics, ecology, and neurology area including applied mathematicians and allow them to share their original research work. Analytical and numerical methods with advanced mathematical and real physical modelling, recent developments of PDEs, and

integral equations in physical systems are included in the main focus of the issue.

Accordingly, various papers on partial differential equations and integral equations have been included in this special issue after completing a heedful, rigorous, and peer-review process. In particular, the nonlinear hydroelastic waves propagating beneath an infinite ice sheet floating on an inviscid fluid of finite depth are investigated analytically in one of the papers. In this paper, the approximate series solutions for the velocity potential and the wave surface elevation are derived, respectively, by an analytic approximation technique named homotopy analysis method (HAM) and are presented for the second-order components.

In another paper, a domain decomposition method is proposed for the coupled stationary Navier-Stokes and Darcy equations with the Beavers-Joseph-Saffman interface condition in order to improve the efficiency of the finite element method. The physical interface conditions are directly utilized to construct the boundary conditions on the interface and then decouple the Navier-Stokes and Darcy equations. Newton iteration is used to deal with the nonlinear systems.

Another paper proposes a pressure-stabilized Lagrange-Galerkin method in a parallel domain decomposition system in which the new stabilization strategy is proved to be effective for large Reynolds number and Rayleigh number simulations. The symmetry of the stiffness matrix enables

the interface problems of the linear system to be solved by the preconditioned conjugate method, and an incomplete balanced domain preconditioner is applied to the flow-thermal coupled problems.

One of the papers is of use of Sumudu transform on fractional derivatives for solving some interesting nonhomogeneous fractional ordinary differential equations. Then spectral and spectral element methods have been discussed with Legendre-Gauss-Lobatto nodal basis for general 2nd-order elliptic eigenvalue problems. A priori and a posteriori error estimates for spectral and spectral element methods have been proposed. In the another paper, a generalized double sinh-Gordon equation has many more applications in various fields such as fluid dynamics, integrable quantum field theory, and kink dynamics has been solved by Exp-function method to obtain new exact solutions for this generalized double sinh-Gordon equation. A semianalytical method called the optimal homotopy asymptotic method has been also applied for solving the linear Fredholm integral equations of the first kind in another paper. In one of the papers, two strategies for inverting the open boundary conditions with adjoint method are compared by carrying out semi-idealized numerical experiments. In the first strategy, the open boundary curves are assumed to be partly space varying and are generated by linearly interpolating the values at feature points and, in the second strategy, the open boundary conditions are assumed to be fully space varying and the values at every open boundary points are taken as control variables. Another paper contains the use of a relatively new analytical method like homotopy decomposition method to solve the 2D and 3D Poisson equations and biharmonic equations. The method does not require the linearization or assumptions of weak nonlinearity, the solutions are generated in the form of general solution, and it is more realistic compared to the method of simplifying the physical problems.

One of the papers has shown that a strong solution of the Degasperis-Procesi equation possesses persistence property in the sense that the solution with algebraically decaying initial data and its spatial derivative must retain this property. In another paper, the fractional complex transformation has been used to transform nonlinear partial differential equations to nonlinear ordinary differential equations. The improved (G'/G) -expansion method has suggested solving the space and time fractional foam drainage and KdV equations. Integral equation has been one of the essential tools for various areas of applied mathematics. For solving nonlinear Fredholm integrodifferential equations, the method based on hybrid functions approximate has been proposed in one of the papers. The properties of hybrid of block pulse functions and orthonormal Bernstein polynomials have been presented and utilized to reduce the problem to the solution of nonlinear algebraic equations. Another paper contains many numerical methods, namely, B-Spline wavelet method, Wavelet Galerkin method, and quadrature method, for solving Fredholm integral equations of second kind. A peer-review of different numerical methods for solving both linear and nonlinear Fredholm integral equations of second kind has been presented. This paper has more emphasized on

the importance of interdisciplinary effort for advancing the study on numerical methods for solving integral equations. Also one of the papers has used a numerical method like function approximation to determine the numerical solution of system of linear Volterra integrodifferential equations using Bezier curves. Two-dimensional Volterra integral equations have also been solved using more recent semianalytic method like the reduced differential transform method and also compared with the differential transform method. One of the papers has presented a numerical method to achieve the approximate solutions in a generalized expansion form of two-dimensional fractional-order Legendre functions (2D-FLFs). The operational matrices of integration and derivative for 2D-FLFs have been derived.

Then a mixed finite element method has been introduced for an elliptic equation modelling of Darcy flow in porous media. In present mixed finite element, the approximate velocity is continuous and the conservation law holds locally. In order to assess the rotational potential vorticity-conserved equation with topography effect and dissipation effect, the multiple-scale method has been studied to describe the Rossby solitary waves in deep rotational fluids. A one step optimal homotopy analysis method has been applied numerically to harmonic wave propagation in a nonlinear thermoelasticity under influence of rotation, thermal relaxation times, and magnetic field. The problem has been solved in one-dimensional elastic half-space model subjected initially to a prescribed harmonic displacement and the temperature of the medium. In one of the papers, the analytical and multishaped solitary wave solutions have been presented for extended reduced Ostrovsky equation. The exact solitary (traveling) wave solutions are also expressed by three types of functions which are hyperbolic function solution, trigonometric function solution, and rational solution. In order to classify the exact solutions, including solitons and elliptic solutions, of the generalized $K(m, n)$ equation by the complete discrimination system a polynomial method has been obtained. To examine the possible approximate solutions of both integer and noninteger systems of nonlinear differential equations which describe tuberculosis disease population dynamics, the relatively new analytical technique like homotopy decomposition method has been proposed. In one of the papers, a relatively new operator called the triple Laplace transform has been introduced and to make use of the operator some kind of third-order differential equation called Mboutata equations has been solved.

Another paper investigates the effect of boundary slip on the transient pulsatile fluid flow through a vessel with body acceleration. To describe the non-Newtonian behavior, the modified second-grade fluid model has been analyzed in which the viscosity and the normal stresses have been represented in terms of the shear rate. One of the papers proves the existence of global solutions for nonlinear wave equations with damping and source terms by constructing a stable set and also obtaining the asymptotic stability of global solutions through the use of a difference inequality. In order to assess the spatial dynamical behavior of a predator-prey system with Allee effect, the bifurcation analyses have been used in which the exact Turing domain has been found

in the parameters space. According to the operator theory, the temperature dependence of the solution to the BCS gap equation has been connected with superconductivity. When the potential is a positive constant, the BCS gap equation reduces to the simple gap equation. The solution to the BCS gap equation has been indeed continuous with respect to both the temperature and the energy under a certain condition when the potential is not a constant. This study represents that there is a unique nonnegative solution to the simple gap equation, which is continuous and strictly decreasing and is of class C^2 with respect to the temperature.

At present, the use of partial differential equation and integral equation in real physical systems is commonly encountered in the fields of science and engineering. Analysis and numerical approximate of such physical models are required for efficient computational tools. The present issue has addressed recent trends and developments regarding the analytical and numerical methods that may be used in the dynamical system. Eventually, it may be expected that the present special issue would certainly helpful to explore the researchers with their new arising problems and elevate the efficiency and accuracy of the solution methods in use nowadays.

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