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Research Article

Nonlinear Fluctuation Behavior of Financial Time Series Model by Statistical Physics System

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We develop a random financial time series model of stock market by one of statistical physics systems, the stochastic contact interacting system. Contact process is a continuous time Markov process; one interpretation of this model is as a model for the spread of an infection, where the epidemic spreading mimics the interplay of local infections and recovery of individuals. From this financial model, we study the statistical behaviors of return time series, and the corresponding behaviors of returns for Shanghai Stock Exchange Composite Index (SSECI) and Hang Seng Index (HSI) are also comparatively studied. Further, we investigate the Zipf distribution and multifractal phenomenon of returns and price changes. Zipf analysis and MF-DFA analysis are applied to investigate the natures of fluctuations for the stock market.

1. Introduction

The analysis of market index and return is an active topic to understand and model the distribution of financial price fluctuation, which has long been a focus of economic research. As the stock markets are becoming deregulated worldwide, the modelling of the dynamics of the forwards prices is becoming a key problem in the physical assets valuation, risk management, and derivatives pricing [1–6]. By applying the theory of interacting particle systems [7–11], some research has been made by applying the statistical physics systems to study the behaviors of fluctuations of price changes in the stock market and the corresponding valuation and hedging of contingent claims [12–23]. The motivation of modelling stock price by using a contact model is to uncover the empirical laws in stock price and better understand the dynamics of financial systems. Stauffer [15] and Yu and Wang [21] developed financial price models by lattice percolation system and lattice-oriented percolation system [8, 9], respectively, the local interaction or influence among market participants in a stock market is constructed, and an open cluster of percolation is applied to define the cluster of investors sharing the same opinion about the market. In these financial models, the main assumption is that the stock price fluctuation is influenced by the information in a stock market, and the investors decide the investment opinions by other investors' attitudes, so the investors investment attitudes of the stock market lead to the stock price fluctuation. Zhang and Wang [22] invented the finite-range contact particle system to model a stock price process for studying the behaviors of returns by statistical analysis and computer simulation. The epidemic spreading of the contact model is considered as the spreading of the investors investment attitudes towards the stock market, and we suppose that the investment attitudes are represented by the viruses of the contact model. These attitudes make the investors take buying stock positions, selling stock positions, or holding stock positions.

The contact process, a model for epidemic spreading in a continuous time Markov process, is one of interacting particle systems [8, 10, 11]. The contact process was introduced by Harris [24] and one interpretation of the contact process is often regarded as a crude model for the spread of a biological population or a disease. Healthy individuals become infected at a rate proportional to the number of infected neighbors, where infected individuals recover at a constant rate. To be specific, let η_s denote the contact process on the configuration space $\{0,1\}^{\mathbb{Z}^d}$; if $\eta_s(x) = 0$, the individual at the point x is healthy and will be infected at a rate equal to λ times

the number of infected neighbors; if $\eta_s(x) = 1$, one individual at the point x is thought of as being infected and recovers from its infection at rate one. In the present paper, we study the statistical behaviors of fluctuations of stock price changes by applying the contact system. We firstly construct the financial time series model of the stock market. Then we analyze the statistical behaviors of ensembles and specifics of returns for the financial model by Zipf analysis and multifractal detrended fluctuation analysis (MFDFA). Further, the statistical properties of returns of Shanghai Stock Exchange Composite Index (SSECI) and Hang Seng Index (HSI) are also studied for comparison between the actual time series and the simulated ones.

2. A Brief Description of Contact System

The contact system on $\mathbb{Z}^d$ with infection parameter λ is a continuous time Markov process η_s on the configuration space $\{0,1\}^{\mathbb{Z}^d}$ [8, 10, 11]. A configuration $\eta \in \{0,1\}^{\mathbb{Z}^d}$ is often identified with subsets A of $\mathbb{Z}^d$ via $A = \{x \in \mathbb{Z}^d : \eta(x) = 1\}$. Individuals in A are thought of as being infected, while the other individuals are regarded as being healthy. The transition rates for η_s are given by

(a) $A \to A \setminus \{x\}$ for all $x \in A$ at rate 1,

(b) $A \to A \cup \{x\}$ for all $x \in A$ at rate $\lambda|\{y \in A : |y-x| \le 1\}|$,

where $|A|$ denote the cardinality of a finite set A and $|y-x|$ is the minimal length of a path from x to y. More formally, the connection between the process η_s and the rate function $c(x,\eta)$ is made through the generator Ω of η_s. For functions f on $\{0,1\}^{\mathbb{Z}^d}$ that depend on finitely many coordinates, the generator has the form

$$\Omega f(\eta) = \sum_x c(x,\eta)\left[f(\eta^x) - f(\eta)\right], \tag{1}$$

where $\eta^x(y) = \eta(y)$ if $y \ne x$ and $\eta^x(y) = 1-\eta(x)$ if $y = x$, for $x, y \in \mathbb{Z}^d$. And $c(x,\eta)$ is given by (see [10])

$$c(x,\eta) = \begin{cases} 1, & \text{if } \eta(x) = 1, \\ \lambda \sum\limits_{y:|y-x|\le 1} \eta(y), & \text{if } \eta(x) = 0. \end{cases} \tag{2}$$

Let η_s^A denote the state at time s with the initial state $\eta_0^A = A$, and let $\eta_s^{\{0\}}$ be the state of $x \in \mathbb{Z}^d$ at time s with the initial point $\{0\}$. The most important concept of weak complete convergence for the contact model is given as $\eta_s^A \Rightarrow \alpha_A \overline{\upsilon} + [1-\alpha_A]\delta_\emptyset$, where $\alpha_A = P(\eta_s^A \ne \emptyset, \text{for all } s \ge 0)$ is the survival probability and $\overline{\upsilon}$ and $\delta_\emptyset$ are the biggest invariant measure and the smallest invariant measure, respectively, in the sense of the partial order [10]. The key characteristic of the contact process is that extinction and survival can be both occur, whether extinction occurs or survival occurs depends on the value of λ. There is a critical value λ_c, if $\lambda < \lambda_c$, the contact process is said to become extinct or die out; that is, $P(\eta_s^{\{0\}} \ne \emptyset, \text{for all } s \ge 0) = 0$; otherwise (for $\lambda > \lambda_c$) it is said to survive. For any finite lattice graph $(-L,L)^d$ (where $L \ge 1$ is a large positive integer) and for every finite A and every constant $C \ge 1$, there exists $\lim_{s\to\infty}\lim_{L\to\infty} P(|\eta_s^A| \ge C) = P(\eta_s^A \ne \emptyset, \text{for all } s \ge 0)$ [11]. Furthermore, we define the edge processes $r_s = \max\{y : \eta_s^{\{0\}}(y) = 1\}$ and $l_s = \min\{y : \eta_s^{\{0\}}(y) = 1\}$ on $\{0,1\}^{\mathbb{Z}}$, $r_0 = l_0 = 0$. And let $\tau = \inf\{s \ge 0 : \eta_s^{\{0\}} = \emptyset\}$. For $\lambda > \lambda_c$, $\lim_{s\to\infty}[\sum_{y=l_s}^{r_s} \eta_s^{\{0\}}(y)]/s = 2\alpha(\lambda)\rho(\lambda)$ (a.s. on $\{\tau = \infty\}$), where $\rho(\lambda) \ge 0$ is a nondecreasing function of λ and $\alpha(\lambda) \ge 0$. And if $\lambda < \lambda_c$, for some positive $\overline{\rho}(\lambda)$, we have $P(\eta_s^{\{0\}} \ne \emptyset) \le e^{-\overline{\rho}s}$, then the process dies out exponentially fast.

3. Financial Price Model Form Contact System

In the following, we adopt the notations and settings of Sections 1-2. Consider a model of auctions for a stock in a stock market. Suppose that each trader can trade the stock several times at each day $t \in \{1,2,\ldots,T\}$, but at most one unit number of the stock at each time. Let l be the time length of trading time in each trading day; we denote the stock price at time s in the tth trading day by $\mathscr{P}_t(s)$, where $s \in [0,l]$. Suppose that this stock consists of $M+1$ (M is large enough) investors, who are located in a line $\{-M/2,\ldots,-1,0,1,\ldots,M/2\} \subset \mathbb{Z}$ (similarly for d-dimensional lattice $\mathbb{Z}^d$). At the beginning of trading in each day, suppose that the investor at the origin receives some news. We define a random variable ξ_t for this investor; suppose that this investor takes buying positions ($\xi_t = 1$), selling positions ($\xi_t = -1$), or neutral positions ($\xi_t = 0$) with probability p_1, p_{-1}, or $1-(p_1+p_{-1})$, respectively. Then this investor sends bullish, bearish, or neutral signal to his neighbors. According to the contact dynamic system, investors can affect each other or the news can be spread, which is assumed as the main factor of price fluctuations. Moreover, here the investors can change their buying positions or selling positions to neutral positions independently at a constant rate. More specifically, (i) when $\xi_t = 1$ and if $\eta_s^{\{0\}} = 1$, we say that the investor at x takes buying position at time s, and this investor recovers to neutral position at rate 1; if $\eta_s^{\{0\}} = 0$, we think that the investor at x takes neutral position at time s, and this investor is changed to take buying position by his neighbors at rate $\lambda\sum_{y:|y-x|\le 1}\eta_s^{\{0\}}(y)$. In this case, the more investors who take buying positions, the more possibility that stock price goes up. (ii) When $\xi_t = -1$ and if $\eta_s^{\{0\}} = 1$, we say that the investor at x takes selling position at time s, and also this investor recovers to neutral position at rate 1; if $\eta_s^{\{0\}} = 0$, the investor is changed to take selling position by his neighbors at rate $\lambda\sum_{y:|y-x|\le 1}\eta_s^{\{0\}}(y)$. (iii) When the initial random variable $\xi_t = 0$, the process $\eta_s^{\{0\}}$ is ignored; this means that the investors do not affect the fluctuation of the stock price.

For a fixed $s \in [0,l]$ (l large enough), the aggregate excess demand for the asset at time t is defined by

$$\mathscr{B}_t(s) = \frac{\xi_t\left|\eta_s^{\{0\}}\right|}{M}, \tag{3}$$

where M may depend on the trading days T. From the above definitions and [3, 6], we define the formula of a discrete time stock price as follows:

$$\mathscr{P}_t(s) = e^{\alpha \mathscr{B}_t(s)} \mathscr{P}_{t-1}(s), \quad t = 1, 2, \ldots, T, \tag{4}$$

where $\alpha > 0$, represents the depth parameter of the market. Then the stock price of the present paper is supposed to follow the form

$$\mathscr{P}_t(s) = \mathscr{P}_0 \exp\left\{\alpha \sum_{k=1}^{t} \mathscr{B}_k(s)\right\}, \tag{5}$$

where $\mathscr{P}_0$ is the initial stock price at time 0. The formula of the single-period stock logarithmic returns from t to $t+1$ is given as follows:

$$\mathscr{R}(t) = \ln \mathscr{P}_{t+1}(s) - \ln \mathscr{P}_t(s). \tag{6}$$

In this paper, we analyze the logarithmic returns for the daily price changes. In the light of theory of the contact process and the above definitions, if $\lambda < \lambda_c$, the virus will die out at last; namely, the influence on the stock price by the investors is limited. If $\lambda > \lambda_c$, the virus will not die out; namely, the news will spread widely, so this will affect the investors' positions and finally will affect the fluctuation of the stock price. The contact system is a statistical physics system, and it is comprised of a large number of interacting units, which has the similarity with the financial markets consisting of a large number of interacting "agents." There are random behaviors ingrained in the contact model, and then we consider the financial market as a complicated evolving system. Modeling stock price by using a contact model has the motivation to uncover the empirical laws in real stock markets and understand better the dynamics of financial systems.

4. Zipf Analysis for Financial Model and Real Stock Markets

4.1. Zipf Analysis of Time Series. Zipf analysis, as a way for quantifying time series correlations, has been widely applied to the literature, stock market, computer science, network, economic management, and many other fields [25–29]. The technique is based on translating a given time series into a sequence of symbols and counting the frequency of any word, that is, pattern of consecutive symbols. Ranking these words by their frequencies from the most common to the least common and plotting the logarithm of frequencies versus the logarithm of rank give us a Zipf plot, which was firstly introduced by George Kingsley Zipf, in order to study the statistical occurrences in different languages. For the lowest ranks, the plotted points usually appear to fall along a line. The gradient of the best line fit corresponds to the Zipf exponent β, which characterizes correlations in time series. Let $(x_1, x_2, \ldots, x_n)$ denote a set of n observations on a random variable x, the corresponding cumulative distribution function is $F(x)$, and assume that the observations are ordered from the largest to the smallest so that the index i is the rank of x_i. The Zipf plot of the sample is the graph of $\ln x_i$ against $\ln i$. Because of the ranking, then $i/n = 1 - F(x_i)$ $(i = 1, \ldots, n)$ and $\ln i = \ln[1 - F(x_i)] + \ln n$. Thus, the log of the rank is simply a transformation of cumulative distribution function. In studying English word occurrence frequency, Zipf's law reveals that while only a few words are used very often, many or most are used rarely. It is found that if the words have the descending orders of frequency, the frequency of occurrence of each word and its symbol ranking has simple inverse relations; that is $P(i) = ci^{-\beta}$. Making a transformation, the above equation can be converted into $\ln P(i) = \ln c - \beta \ln i$, where $P(i)$ is the frequency of the word whose rank is i and c is some positive constant. Plotting the graph by $\ln P(i)$ against $\ln i$, the graph is close to a line with the slope of $-\beta$.

Let $\mathscr{S}(t)$ $(t = 1, 2, \cdots, n)$ denote the time series of daily closing stock prices, and let τ be the given integer time scale; then the τ-step of logarithmic price changes in a stock market is defined by

$$\mathscr{R}_\tau(t) = \ln \mathscr{S}(t+\tau) - \ln \mathscr{S}(t), \tag{7}$$

where $t = 1, 2, \ldots, n - \tau$. Next we consider a new time series with a random environment which is derived from the original τ-step of logarithmic price changes of the model. In a real stock market, various kinds of information will affect the investing positions of the market participants, and the investing environment is also ceaselessly changing. So we introduce a random environment in the financial model as the following. Let θ be a nonnegative random variable on a probability space Ω (with the probability distribution $F_\theta(x)$), which is called a random threshold of the model. For example, θ can be a uniform on the interval $(0, 2)$, or θ can also be a random variable $|\xi|$, where ξ follows a normal distribution, and so forth. Then the new time series derived from the original stock prices is given as

$$y_\tau(t, \theta) = \begin{cases} u, & \text{if } \ln \mathscr{S}(t+\tau) - \ln \mathscr{S}(t) \geq \theta, \\ s, & \text{if } |\ln \mathscr{S}(t+\tau) - \ln \mathscr{S}(t)| < \theta, \\ d, & \text{if } \ln \mathscr{S}(t+\tau) - \ln \mathscr{S}(t) \leq -\theta, \end{cases} \tag{8}$$

where u, s, and d denote "price-up," "price-stable," and "price-down," respectively. In this model, the random threshold θ represents the expected returns for the market investors. Then, for the different parameters τ and θ, we investigate the fluctuation behaviors of the time series $y_\tau(t, \theta)$ $(t = 1, 2, \ldots, n - \tau)$.

Let $n_u(\tau, \theta)$, $n_s(\tau, \theta)$, and $n_d(\tau, \theta)$ denote the frequencies of occurrences for price-up, price-stable, and price-down, respectively. Then the corresponding absolute frequencies of the time series $y_\tau(t, \theta)$ for these case are given as follows:

$$\begin{aligned} f_u(\tau, \theta) &= \frac{n_u(\tau, \theta)}{n - \tau} \frac{1 - F_\theta(x)}{2}, \\ f_d(\tau, \theta) &= \frac{n_d(\tau, \theta)}{n - \tau} \frac{1 - F_\theta(x)}{2}, \\ f_s(\tau, \theta) &= \frac{n_s(\tau, \theta)}{n - \tau} F_\theta(x), \end{aligned} \tag{9}$$

where $n_u(\tau,\theta) + n_d(\tau,\theta) + n_s(\tau,\theta) = n - \tau$ and $F_\theta(x) = P(\theta \le x)$. In financial markets, the large fluctuation of daily price changes usually occurs with the small probability. Considering this property, the frequencies of occurrences defined in the above depend on the probability distribution $F_\theta(x)$. Next the corresponding relative frequencies of the time series $y_\tau(t,\theta)$ are given as follows:

$$g_u(\tau,\theta) = \frac{n_u(\tau,\theta)}{n_u(\tau,\theta) + n_d(\tau,\theta)}\left[1 - F_\theta(x)\right],$$
$$g_d(\tau,\theta) = \frac{n_d(\tau,\theta)}{n_u(\tau,\theta) + n_d(\tau,\theta)}\left[1 - F_\theta(x)\right]. \quad (10)$$

In the above definitions of the relative frequencies, we omit the occurrences of stable-price; thus $g_u(\tau,\theta)$ and $g_d(\tau,\theta)$ measure the total occurrences of price rising and price falling, respectively.

4.2. Results of Empirical Research. In this section, we study the statistical properties of absolute frequencies and relative frequencies of price changes for various values of two parameters τ and θ. Both the actual data of SSECI and the simulation data of the model will be considered. We select the data for the daily closing prices of SSECI in the 9-year period from August 23, 2002, to March 9, 2011; the database is from Shanghai Stock Exchange; see http://www.sse.com.cn/. And we also consider the simulation data of the model which is derived from the contact system with the infection rate $\lambda = 1.3$ (with which the simulation data has similar fluctuations with the real stock market). For the actual data and the simulation data, we make the empirical research for the absolute frequency and the relative frequency. By the computer simulation [30], we compute the absolute frequencies and the relative frequencies for different values of τ and θ, and the corresponding plots are plotted in Figures 1 and 2. Figure 1 displays the empirical results of the actual data for SSECI, the horizontal axis indicates the random expected return θ, and the vertical axis indicates the absolute frequencies of the time series $y_\tau(t,\theta)$. Figures 1(a) and 1(b) exhibit that the price-up function $f_u(\tau,\theta)$ and price-down function $f_u(\tau,\theta)$ are decreasing functions when θ is increasing. And for two τ-steps τ_1, τ_2, such that $\tau_1 > \tau_2$, the curve of $f_u(\tau_1,\theta)$ is over the curve of $f_u(\tau_2,\theta)$, and we have the similar results to the price-down function $f_d(\tau,\theta)$. But for the absolute frequency of price-stable, Figure 1(c) displays the opposite trend; that is, the function $f_s(\tau,\theta)$ is increasing with θ increasing. Figure 1 shows that, for a given step τ, the absolute frequencies reach their inflection point as θ increases. In addition, when τ increases, the corresponding value of θ (where the inflection of the absolute frequencies occur) becomes larger. We also consider the Zipf distributions of the absolute frequencies of price-up $f_u(\tau,\theta)$, price-down $f_u(\tau,\theta)$, and price-stable $f_s(\tau,\theta)$. For different interval times τ, we compute and plot the corresponding Zipf distributions in Figure 1 (the smaller plots in (a), (b), and (c)); Figure 1 exhibits the power-law distributions for the absolute frequencies. In Figure 2, we obtain the similar empirical results of the simulation data for the financial model with the infection rate $\lambda = 1.3$.

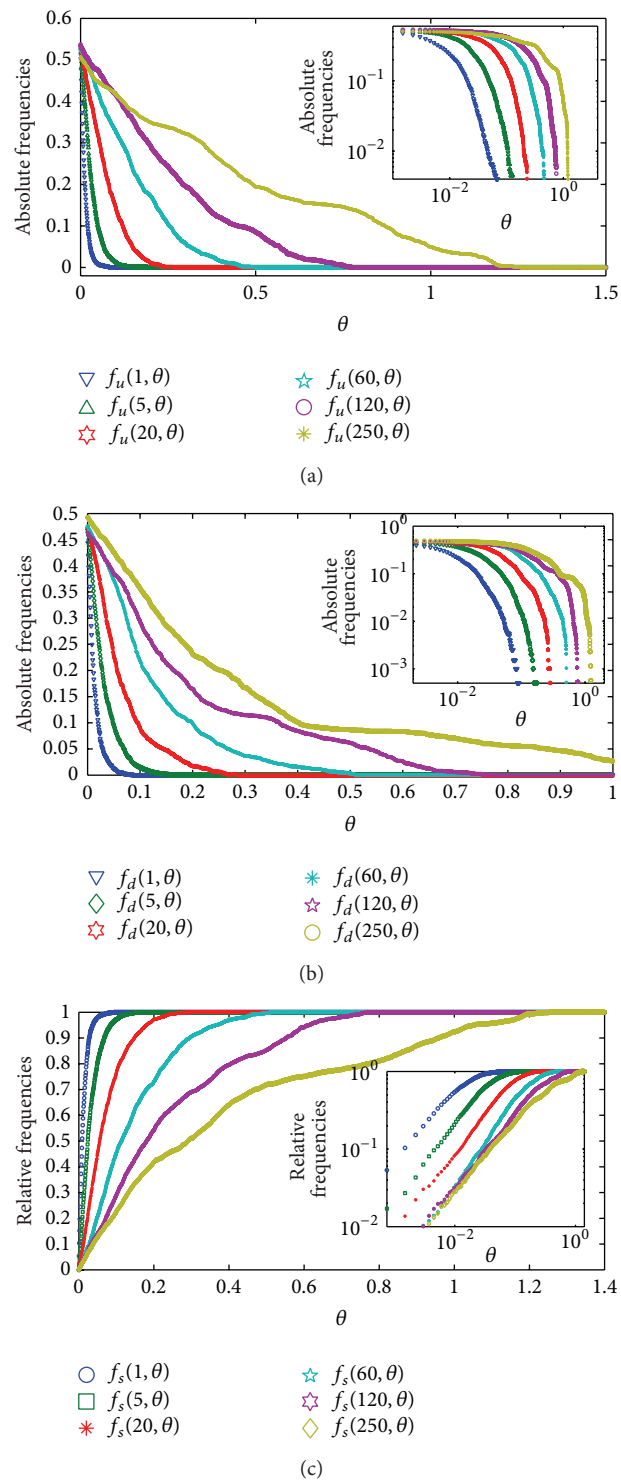


Figure 1: The evolution trends of absolute frequencies of the actual data for SSECI with the random variable θ. Plots (a), (b), and (c) are the absolute frequencies of price-up, price-down, and price-stable, respectively. The insets are the corresponding double logarithmic presentations.

Figures 3 and 4 show the distributions of the relative frequencies for different time scales and different expected returns. In Figure 3, when $\theta \in (0, 0.1]$, the relative frequencies

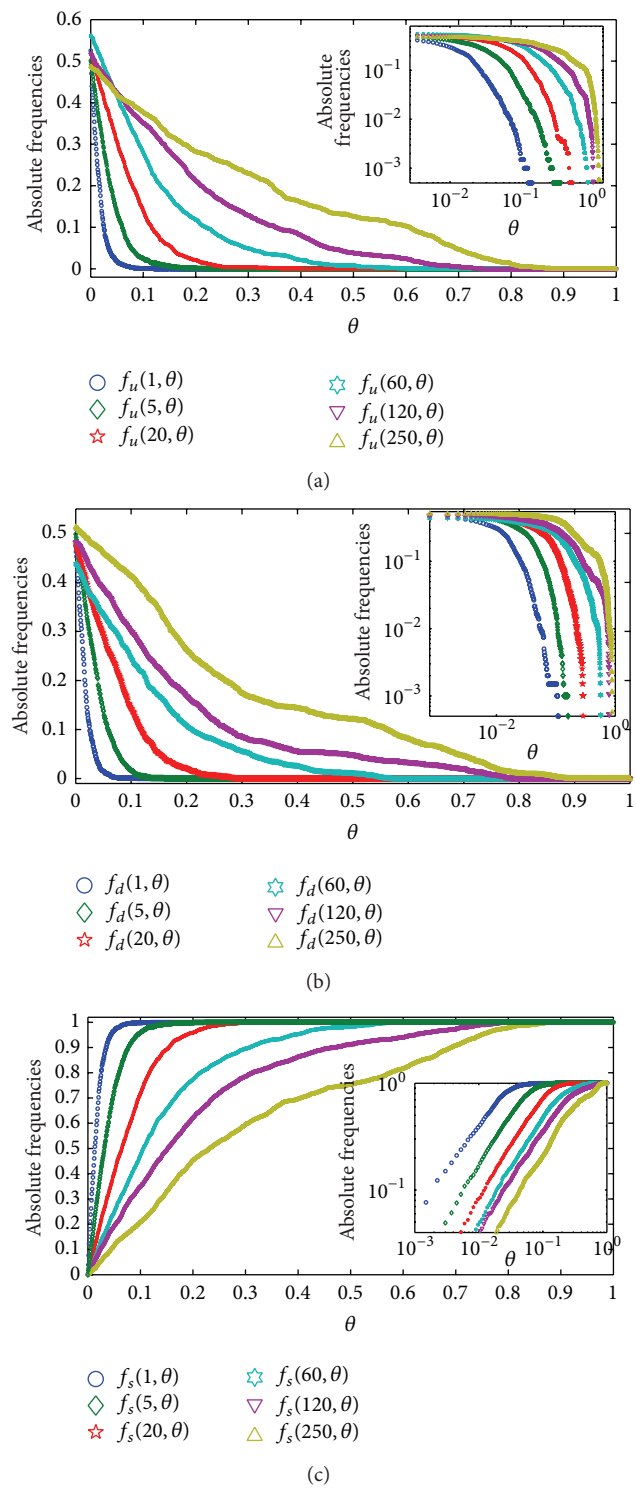


Figure 2: The evolution trends of absolute frequencies of the actual data for the simulation data for the financial model with the random variable θ. Plots (a), (b), and (c) are the absolute frequencies of price-up, price-down, and price-stable, respectively. The insets are the corresponding double logarithmic presentations.

are approximately equal to 0.5; when θ becomes larger, the relative frequencies depart from the value 0.5 rapidly. Note that the daily price fluctuation is limited in Chinese stock markets; that is, the changing limits of the daily returns (i.e., $\tau = 1$) for stock prices and stock indices are between −10% and 10%. This means that we should let $\theta \in [0, 0.1]$ for the Chinese stock markets. According to the above discussion, the relative frequencies are near to 0.5 for $\theta \in (0, 0.1]$; then we can reach a conclusion that θ ($\theta \in (0, 0.1]$) is a low risk expected return. If a market participant hopes to obtain a return which is larger than 0.1, he will face a high investing risk. In Figure 4, according to the simulation data of the model, we obtain the similar empirical results to those of SSECI in Figure 3.

Table 1 gives the values of the inflection points for the absolute frequencies and the relative frequencies. We can find that the inflection points are larger when τ is increasing, corresponding with the empirical results of Figures 1–4. Furthermore, according to the definitions of $f_u(\tau,\theta)$, $f_d(\tau,\theta)$, $f_s(\tau,\theta)$, $g_u(\tau,\theta)$, and $g_d(\tau,\theta)$, it can be easily known that the inflection points of $f_s(\tau,\theta)$, $f_d(\tau,\theta)$, and $f_s(\tau,\theta)$ are interrelated. Suppose that θ_1 is the inflection point of $f_s(\tau,\theta)$, then $f_u(\tau,\theta_1) = 0$ and $f_d(\tau,\theta_1) = 0$, and θ_1 is also inflection point of $f_u(\tau,\theta)$ or $f_d(\tau,\theta)$. We also obtain that the inflection points of $g_u(\tau,\theta)$ and $g_d(\tau,\theta)$ are the same from the definitions, the values are related to inflection points of absolute frequencies, and all the estimates are displayed in Table 1.

5. MF-DFA Analysis for Financial Time Series

5.1. MF-DFA Analysis. Recently, some research work has been made for the multifractal characterization of nonstationary time series, which is based on a generalization of detrended fluctuation analysis (DFA): multifractal detrended fluctuation analysis (MF-DFA) [12, 13, 31–34]. It has been concluded that MF-DFA should be recommended for a global detection of multifractal behavior [35, 36].

MF-DFA method can be summarized as follows [32]. (i) Starting with a correlated time series $\{x_i, i = 1, \ldots, N\}$, where N is the length of the series, the corresponding profile is determined by integration

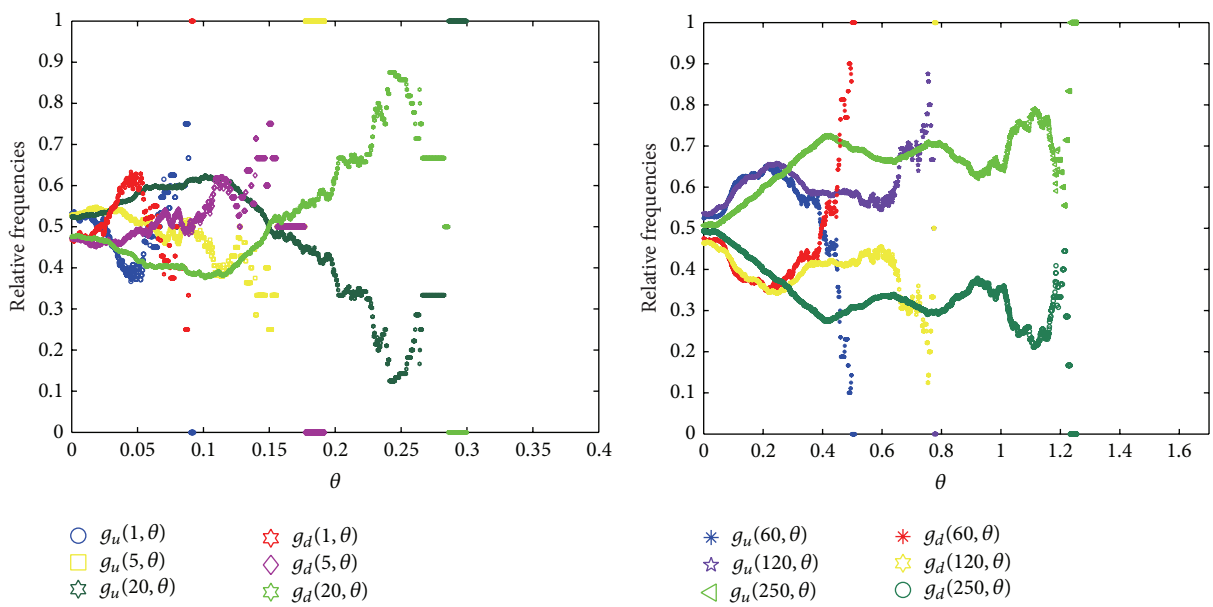
$$Y(i) = \sum_{k=1}^{i} \left(x_k - \langle x_k \rangle\right), \quad i = 1, 2, \ldots, N, \tag{11}$$

where $\langle x \rangle$ denotes the averaging over the whole time series. (ii) The profile $Y(i)$ is divided into $N_s \equiv \mathrm{int}(N/s)$ nonoverlapping windows of equal length s. Since the record length N does not need to be a multiple of the considered time scale s, a short part at the end of the profile will remain in most cases. In order to have into account this part of the record, the same procedure is repeated, starting from the other end of the record. Thus, $2N_s$ windows are obtained. (iii) Calculate the local trend for each of the $2N_s$ segments by a leastsquare fit of the series and determine the variance

$$\begin{aligned} F^2(s,\nu) &= \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[(\nu-1)s+i\right] - y_\nu(i)\right\}^2, \\ F^2(s,\nu) &= \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[N-(\nu-N_s)s+i\right] - y_\nu(i)\right\}^2 \end{aligned} \tag{12}$$

Table 1: The values of the inflection points.

SSECI	$f_u(\tau,\theta)$	$f_d(\tau,\theta)$	$f_s(\tau,\theta)$	$g_u(\tau,\theta)$	$g_d(\tau,\theta)$
$\tau = 1$	0.0907	0.0930	0.0930	0.0907	0.0907
$\tau = 5$	0.1928	0.1777	0.1928	0.1777	0.1777
$\tau = 20$	0.0300	0.2865	0.0300	0.2865	0.2865
$\tau = 60$	0.4988	0.5100	0.5100	0.4988	0.4988
$\tau = 120$	0.7763	0.7830	0.7830	0.7763	0.7763
$\tau = 250$	1.2555	1.2337	1.2555	1.2337	1.2337
Simulation data	$f_u(\tau,\theta)$	$f_d(\tau,\theta)$	$f_s(\tau,\theta)$	$g_u(\tau,\theta)$	$g_d(\tau,\theta)$
$\tau = 1$	0.0660	0.0720	0.0720	0.0660	0.0660
$\tau = 5$	0.1732	0.2055	0.2055	0.1732	0.1732
$\tau = 20$	0.4402	0.3465	0.4402	0.3465	0.3465
$\tau = 60$	0.7155	0.5062	0.7155	0.5062	0.5062
$\tau = 120$	0.9210	0.7620	0.9210	0.7620	0.7620
$\tau = 250$	0.9547	1.0328	1.0328	0.9547	0.9547



(a) Relative frequency for $\tau = 1, 5, 20$

(b) Relative frequency for $\tau = 60, 120, 250$

Figure 3: The evolution trends of relative frequencies of the actual data for SSECI with the random variable θ.

for each segment $\nu = 1, \ldots, N_s$. Here, $y_\nu(i)$ is the fitting polynomial in segment ν. Linear, quadratic, cubic, or higher order polynomials can be used in the fitting procedure. In this part we denote the algorithm as MF-DFA-1. (iv) Averaging all subsets and obtaining the qth order fluctuation function $F_q(s)$,

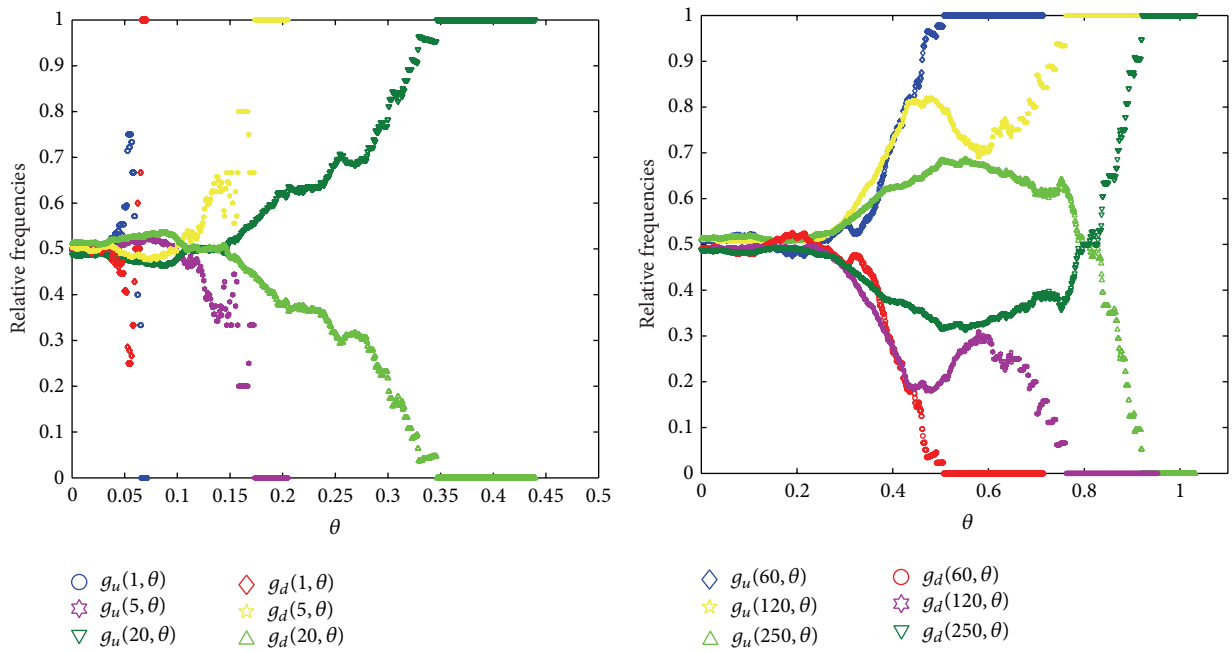
$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \left[F^2(\nu, s) \right]^{q/2} \right\}^{1/q}. \tag{13}$$

(v) Determine the scaling behavior of the fluctuation functions by analyzing log-log plots $F_q(s)$ versus s for each value of q. If the series x_i are long-range power-law correlated, $F_q(s)$ increases, for large values of s, as a power-law $F_q(s) \sim q^{h(q)}$, where $h(q)$ is called the generalized Hurst exponent. The value of $h(0)$, which corresponds to the limit $h(q)$ for $q \to 0$, cannot be determined directly using the averaging procedure because of the diverging exponent. Instead, a logarithmic averaging procedure has to be employed,

$$F_0(s) \equiv \exp\left\{ \frac{1}{4N_s} \sum_{\nu=1}^{2N_s} \ln\left[F^2(\nu, s) \right] \right\} \sim s^{h(0)}. \tag{14}$$

Note that $h(0)$ cannot be defined for time series with fractal support, where $h(q)$ diverges for $q \sim 0$.

For monofractal time series with compact support, $h(q)$ is independent of q, since the scaling behavior of the variance $F_s^2(\nu)$ is identical for all segments ν, and the averaging procedure will give just this identical scaling behavior for all values of q. Only if small and large fluctuations scale



(a) Relative frequency for $\tau = 1, 5, 20$

(b) Relative frequency for $\tau = 60, 120, 250$

FIGURE 4: The evolution trends of relative frequencies of the simulation data for financial model with the random variable θ.

differently, there will be a significant dependence of $h(q)$ on q. For positive values of q, $h(q)$ describes the scaling behavior of the segments with large fluctuations. For negative values of q, $h(q)$ describe the scaling behavior of the segments with small fluctuations. Obviously, richer multifractality corresponds to higher variability of $h(q)$. Then, the multifractality degree can be quantified by $\Delta h = h(q_{\min}) - h(q_{\max})$. As large fluctuations are characterized by smaller scaling exponent $h(q)$ than small fluctuations, $h(q)$ for $q < 0$ are larger than those for $q > 0$, and Δq is positively defined. The analytical relation between the generalized Hurst exponent introduced previously and the scaling exponent $\tau(q)$ defined by the standard partition function multifractal formalism is given as [32]

$$\tau(q) = qh(q) - 1. \tag{15}$$

The singularity spectrum $f(\alpha)$ is another way to characterize the multifractality of the series. The parameter α is the Holder exponent or singularity strength, while $f(\alpha)$ is the fractal dimension of the subset of the time series with singularities of strength equal to α. The spectrum $f(\alpha)$ is related with $\tau(q)$ via a Legendre transformation

$$\alpha = \tau'(q), \quad f(\alpha) = q\alpha - \tau(q). \tag{16}$$

Then it is straightforward to relate these quantities to the generalized Hurst exponent

$$\alpha = h(q) + qh'(q), \quad f(\alpha) = q[\alpha - h(q)] + 1. \tag{17}$$

An alternative quantifier for the multifractality degree is the width $\Delta\alpha$ of the singularity spectrum $f(\alpha)$ [14, 22]. $\Delta\alpha$ is the width of the multifractal spectrum, $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$. $\Delta\alpha$ can statistically represent the variation ranges under scaling invariance. While the variation of the stock price is larger, the width of the multifractal spectrum is wider and the $\Delta\alpha$ is larger, and vice verse.

5.2. Multifractal Analysis of Real Market and Financial Model. In this part, we investigate the multifractal behaviors of time series for SSECI, HSI, and the simulation data of the financial model. We select the daily closing prices of SSECI from August 23, 2002, to March 9, 2011, and the daily closing prices of HSI from February 27, 2004, to February 17, 2012, and the total number of observed data is about 2000 for Shanghai Composite Index and Hang Seng Index, respectively. We also consider the financial model with infection rate $\lambda = 1.3$, and the corresponding simulation data with the same time length of SSECI is selected. As mentioned above, the generalized Hurst exponent $h(q)$ can be obtained by analyzing log-log plots of $F_q(s)$ versus s for each q. To show this procedure, we plot the log-log graphs of $F_q(s)$ versus s for SSECI, HSI, and the simulation data in Figure 5. Table 2 displays the generalized Hurst exponents $h(q)$, via the MF-DFA-2 procedure. In this analysis, q ranges from -4 to 4 with a step length of 0.2, the window lengths of s, are between 10 and $N/1$ with a step of 50, and N is the length of the time series. From $h(q)$ and the equation in the above part, we can obtain the corresponding singularity spectrum $f(\alpha)$ versus α in Figure 7(b), and the values of $\Delta\alpha$ and Δf are exhibited in Table 2.

In Figure 7(a), we can see that there is a significant dependence of $h(q)$ on q for both the real data and the simulation data, $h(q)$ for $q < 0$ are larger than those for $q > 0$, and the values of Δh are displayed in Table 2. Since richer

Table 2: $\Delta h(q)$ and $\Delta\alpha$ for SSECI, HSI, simulation data, and their shuffled series.

	Original data			Shuffled data		
	SSECI	HSI	Simulation data	SSECI	HSI	Simulation data
$h(q)_{\max}$	0.7049	0.6277	0.6195	0.7490	0.6124	0.6253
$h(q)_{\min}$	0.6238	0.4637	0.5195	0.5486	0.3980	0.4282
$\Delta h(q)$	0.0811	0.1640	0.1000	0.2004	0.2144	0.1971
$\alpha_{\max}$	0.7562	0.6732	0.6538	0.7736	0.6314	0.6717
$\alpha_{\min}$	0.5793	0.3304	0.4555	0.4466	0.2636	0.3476
$\Delta\alpha$	0.1768	0.3428	0.1983	0.3270	0.3678	0.3241

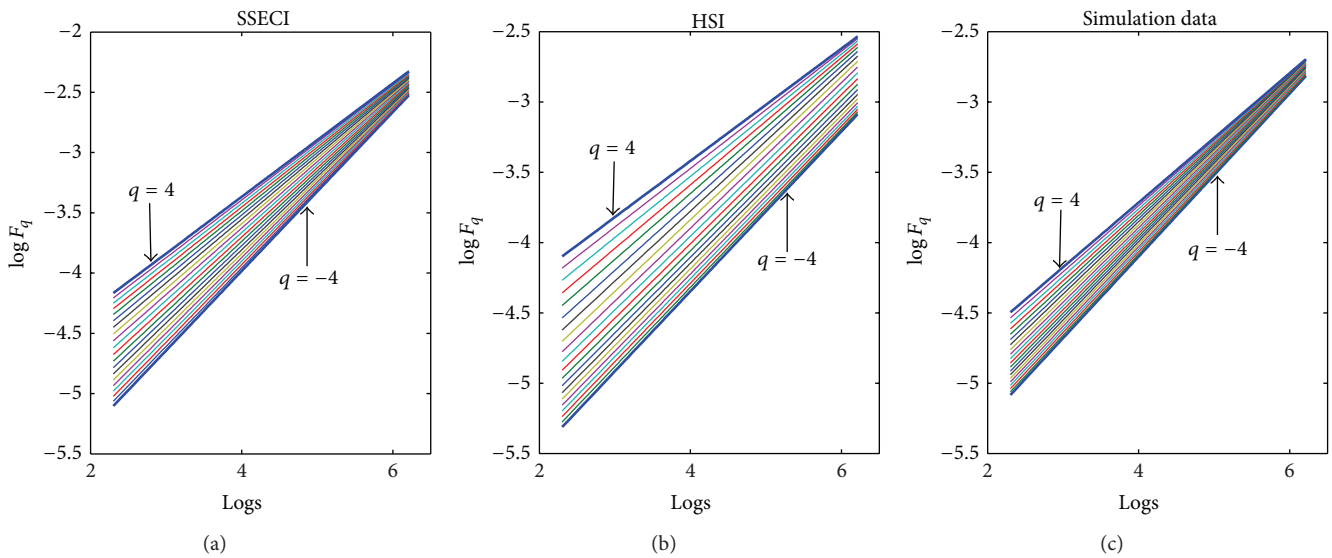


Figure 5: Log-log plots of $F_q(s)$ versus s for SSECI, HSI, and the simulation data with different q between -4 (bottom line) and 4 (top line).

multifractality corresponds to higher variability of $h(q)$, in Table 2, we find that HSI has the multifractality degree with $\Delta h = 0.1640$, where SSECI is of $\Delta h = 0.0811$, which indicate that there is multifractal phenomenon in both Hang Seng Index and Shanghai Composite Index, and the multifractality in SHI is richer. We also compute the multifractality degree for the simulation data of the model. From Figure 7(a) and Table 2, we can see that $h(q)$ of the simulation data is changing with q; this shows that the simulation data has the multifractal characteristic. The similar empirical results can be found from the study of the corresponding singularity spectrum $f(\alpha)$ calculated from $h(q)$. Figure 7(b) gives the plot of $f(\alpha)$ via α, and Table 2 displays the corresponding values. It is known that, when the volatility of stock prices is larger, the width of the multifractal spectrum is wider and the $\Delta\alpha$ is larger, and vice verse. HSI has the multifractal spectrum with $\Delta\alpha = 0.3428$, while $\Delta\alpha = 0.1768$ for SSECI and $\Delta\alpha = 0.1983$ for the simulation data. This indicates that there is the multifractality in the real data and the simulation data, corresponding with the study of $h(q)$.

Evidence in many existed works shows that Chinese stock markets are multifractal. The empirical results [37–39] have indicated that the long-range nonlinear correlations generate multifractality. To give further evidences, we will compare the multifractality between original series and randomly shuffled series. We shuffle the return time series by Zipf's law, which is introduced in Section 4.1 of the present paper, and comparatively study the multifractality on the original return time series and the shuffled return time series (by Zipf's law). For an original return time series $r = \{r_1, r_2, \ldots, r_k, \ldots\}$, we shuffle this return time series into a new corresponding return time series according to the ranges of small and large fluctuations. In the details, let $R_1 = \{r_{1,1}, r_{1,2}, \ldots r_{1,k}, \ldots\}$ such that $|r| < 0.01$, R_1 is a subsequence of r, and assume that the order of elements in R_1 follows the order of the corresponding elements in r. The length of the subsequence R_1 is the number of elements in R_1. Similarly, we consider the subsequence R_2 with $0.01 \le |r| < 0.015$, R_3 with $0.015 \le |r| < 0.03$, and R_4 with $|r| \ge 0.03$. In the above procedure, we divide the original return time series into four subsequences, and then we arrange these four sequences by their length from the longest to the shortest. Thus we invent the new time series, which is shuffled by the method (or idea) of Zipf's law. For example, an original return time series of SSECI is plotted in Figure 6(a), and the shuffled return time series of SSECI by the above method is exhibited in Figure 6(b), that is, R_1, R_3, R_2, and R_4.

From the empirical research in Figures 7(a) and 7(b), they exhibit that there is the multifractility in the shuffled return time series. In Table 2, we find that the multifractality in shuffled time series is richer than that in the original ones, which is probably that the long-range dependence of small

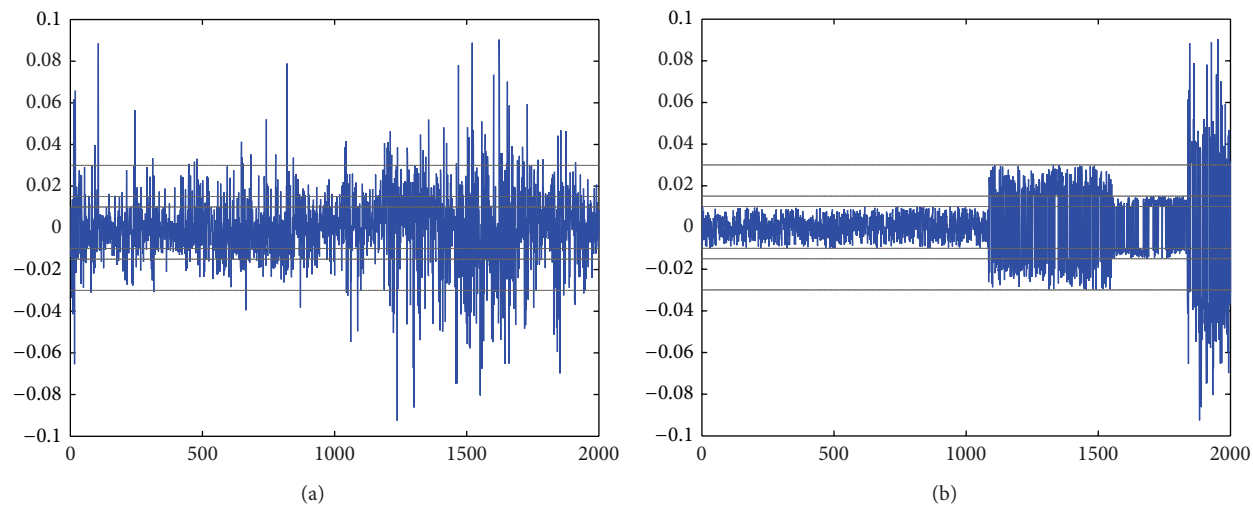


FIGURE 6: (a) The plot of an original return time series of SSECI; (b) the plot of the shuffled series return time of SSECI, that is, R_1, R_3, R_2, and R_4.

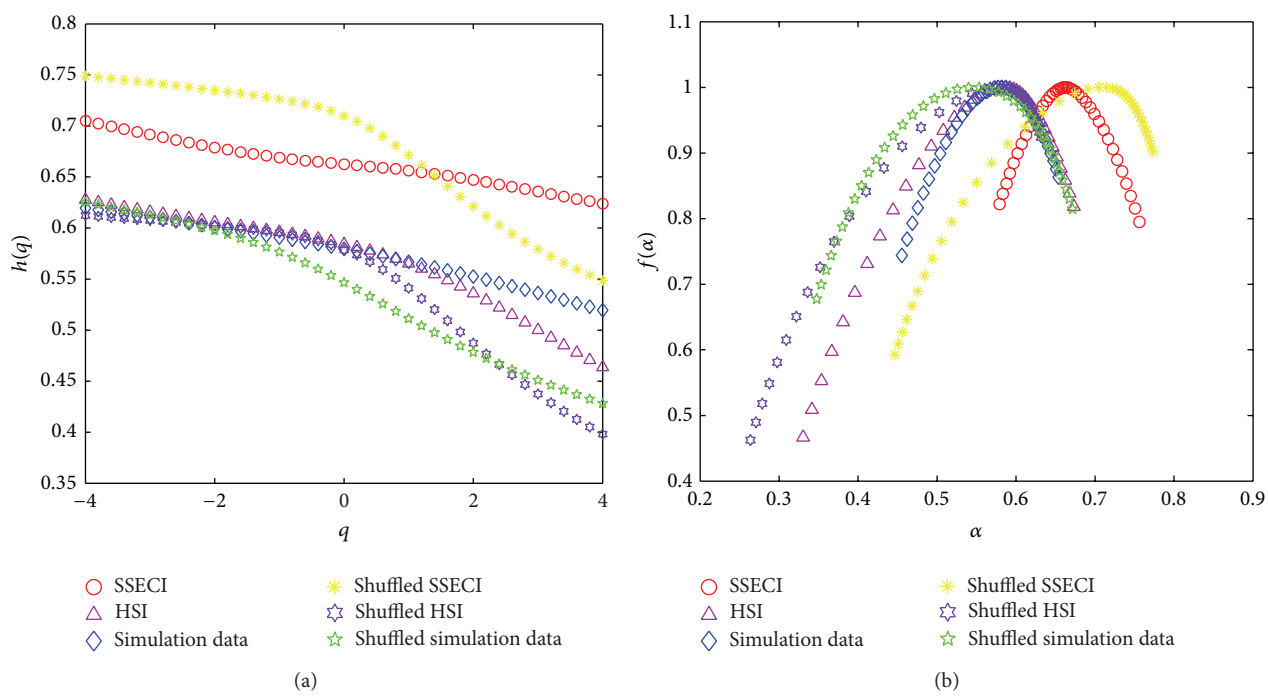


FIGURE 7: (a) The plot of Hurst exponent $H(q)$ versus q for SSECI, HSI, the simulation data, and their shuffled series; (b) the plot of the corresponding singularity spectrum $f(\alpha)$ versus α.

and large fluctuations in the series shuffled by Zipf's law is stronger. From these results, it may suggest that Zipf's law contributes the long-range dependence of small and large fluctuations of return time series.

6. Conclusion

In this present paper, we develop a random stock price model by the interacting contact process. We apply MF-DFA analysis and Zipf analysis to investigate the multifractal characteristic and Zipf distribution of returns and price changes for the financial model. Moreover, we also consider the corresponding behaviors of daily returns for Shanghai Composite Index and Hang Seng Index, and the comparisons of statistical behaviors of returns between the actual data and the simulation data are exhibited. We analyze and show the statistical properties of ensembles and specifics of returns by Zipf method for different values of the parameters. And the empirical research shows an evidence that there is the multifractal character for both the real market

and the financial model by analyzing the Hurst exponent and the singularity spectrum of returns time series. We also show the multifractality of shuffled return time series by Zipf's law. We find that the shuffled series have richer multifractality than the original series. These all indicate that there are multifractal phenomenon and Zipf distribution in both Chinese stock markets and the financial model, which also implies that the financial model of the present paper is reasonable for the real stock market to some extent.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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