Research Article

Robust Exponential Stability of Impulsive Stochastic Neural Networks with Leakage Time-Varying Delay

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This paper investigates mean-square robust exponential stability of the equilibrium point of stochastic neural networks with leakage time-varying delays and impulsive perturbations. By using Lyapunov functions and Razumikhin techniques, some easy-to-test criteria of the stability are derived. Two examples are provided to illustrate the efficiency of the results.

1. Introduction

In recent years, stability for neural networks with time delay has been extensively studied due to their great applications in some practical engineering problems such as signal processing, associate memory, and combinatorial optimization (see [1–3]). In particular, the leakage delay, which exists in the negative feedback terms (known as forgetting or leakage terms) of the system, has great impact on the dynamical behavior of neural networks (see [4–10]). Gopalsamy [4] initially discussed the problem of bidirectional associative memory neural networks with constant delays in the leakage term by using model transformation technique. Then, many results of stability of neural networks with delays in the leakage terms are obtained (see, [5–12]).

However, besides time delay, neural networks are often subject to impulsive perturbation—the abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change, or other sudden noise (see [13, 14]). The impulsive effect can affect the dynamical behaviors of the system. Now there are many results on stability of the neural networks with time delays in the leakage term and impulsive perturbations under the corresponding delayed neural networks without impulses must be stable themselves (see [5–8]). To best of the authors’ knowledge, this is the first attempt to investigate the stability of the systems under the corresponding delayed neural networks without impulses which are unstable themselves.

On the other hand, in real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [15]. It is well known that a neural network could be stabilized or destabilized by certain stochastic inputs. Therefore, noise disturbances have an important effect on the stability of neural networks. Recently, many interesting results on stochastic effect to the stability of neural networks with delays have been reported (see [11, 16, 17]). Moreover, uncertainties are unavoidable in practical implementation of neural networks due to modeling errors and parameter fluctuation, which also cause instability and poor performance [12]. Hence, we can obtain a more perfect model of this situation if we include parameter uncertainties and stochastic effects in neural networks.

Motivated by the above, it is of practical and theoretical importance to study the stability problem of impulsive neural networks with time-varying delays in the leakage term. In this paper, we will investigate stability for a class of stochastic neural networks with time-varying delay in the leakage term and impulses. By using Razumikhin techniques [18–21], some new robust mean-square exponential stability criteria will be given under the corresponding delayed neural networks without impulses which are stable and unstable, respectively.
2. Problem Formulation

Consider the following uncertain neural networks model with impulses and leakage time-varying delay

\[
\begin{aligned}
dx(t) &= \left[-C(t)x(t - \sigma(t)) + A(t)f(x(t)) + B(t)f(x(t - \tau(t)))\right]dt \\
&\quad + h(x(t), x(t - \tau(t)), x(t - \sigma(t)))d\omega(t),
\end{aligned}
\]

\[t \geq t_0, \ t \neq t_k;\]
\[
\Delta x(t) = I_k(x(t^-), x(t^- - \tau(t)), x(t^- - \sigma(t))),
\]
\[t = t_k, \ k \in \mathbb{N},
\]
\[x(t_0 + s) = \varphi(s), \ s \in [-\rho, 0],
\]

where \(x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in \mathbb{R}^n\) is the neuron state vector of the neural networks, \(C(t) = \text{diag}(c_i(t))_{n \times n} > 0\), \(f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T\) represents the neuron activation function, and \(A(t), B(t) \in \mathbb{R}^{n \times n}\) are the connection weight matrix. \(\tau(t)\) is the time-varying delay and \(\sigma(t)\) is the leakage time-varying delay satisfying \(0 \leq \tau(t) \leq \tau, 0 \leq \sigma(t) \leq \sigma, \rho = \max(\tau, \sigma)\). \(\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T\) is a \(n\)-dimensional Brownian motion defined on complete probability space \((\Omega, \mathcal{F}, \mathbb{P})\). \(t_k \in J = \{t_k\} : 0 = t_0 < t_1 < \cdots < t_k < \cdots, \lim_{k \to \infty} t_k \to \infty\) is impulsive time. \(\Delta x(t) = x(t_k^-) - x(t_k^+)\) represents the jump in the state \(x\) at \(t_k\) with \(I_k\) determining the size of the jump and \(x(t_k^+) = x(t_k^-)\). The initial conditions \(\varphi(t) \in PC_{[-\rho, 0]; \mathbb{R}^n}\), where \(PC_{[-\rho, 0]; \mathbb{R}^n}\) denotes the family of all bounded \([-\rho, 0]\) measurable and \(PC([-\rho, 0]; \mathbb{R}^n)\) valued random variable \(\varphi\), satisfying \(E[\|\varphi\|^2] = \sup_{\varphi \in PC([-\rho, 0]; \mathbb{R}^n)} E[\|\varphi(t)\|^2] < +\infty\). \(E\) denotes the mathematical expectation. \(PC(f; \mathbb{R}^n) = \{\varphi : J \to \mathbb{R}^n\}\) is piecewise continuous.

Throughout this paper, symmetric matrix \(M \geq 0\) (resp., \(M > 0\)) means that the matrix \(M\) is positive semidefinite (resp., positive definite). \(I\) denotes an identity matrix. The notation \(M^T\) represents the transpose of the matrix \(M\). The symmetric terms in asymmetric matrix are denoted by *. \(\lambda_{\text{max}}(A)\) and \(\lambda_{\text{min}}(A)\) mean the largest and the smallest eigenvalue of \(A\), respectively.

In this paper, we assume that \(x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T\) is the equilibrium point of the system (1). And we have the following assumptions.

(A1) The neuron activation function \(f_j(x)\) is continuous on \(\mathbb{R}\) and satisfies

\[
l_j \leq \frac{f_j(u) - f_j(v)}{u - v} \leq L_j,
\]

\[j = 1, 2, \ldots, n, \text{ for any } u, v \in \mathbb{R}, \ u \neq v,
\]

where \(l_j, L_j\) are some real constants and they may be positive, zero, or negative.

(A2) Consider

\[
I_k (x(t^-), x(t^- - \tau(t)), x(t^- - \sigma(t))) = (G_{ik}(t) - I) (x(t^-) - x^*) + G_{2k}(t) (x(t^- - \tau(t)) - x^*),
\]

\[
G_{3k}(t) (x(t^- - \sigma(t)) - x^*),
\]

where \(G_{ik}(t) \in \mathbb{R}^{n \times n}, i = 1, 2, 3, \text{ and } k \in \mathbb{N}.
\]

(A3) Consider \(h(x(t), x(t-\tau(t)), x(t-\sigma(t))) = H_1(t)(x(t) - x^*) + H_2(t)(x(t-\tau(t)) - x^*) + H_3(t)(x(t-\sigma(t)) - x^*).
\]

Let \(y(t) = x(t) - x^*\), and system (1) becomes

\[
\begin{aligned}
dy(t) &= \left[-C(t)y(t-\sigma(t)) + A(t)g(y(t)) + B(t)g(y(t-\tau(t)))\right]dt \\
&\quad + [H_1(t)y(t) + H_2(t)y(t-\tau(t)) + H_3(t)y(t-\sigma(t))]
\]

\[d\omega(t), \quad t \geq t_0, \ t \neq t_k;
\]
\[
\Delta y(t) = I_k(y(t^-), y(t^- - \tau(t)), y(t^- - \sigma(t))),
\]
\[t = t_k, \ k \in \mathbb{N},
\]
\[y(t_0 + s) = \phi(s), \ s \in [-\rho, 0],
\]

where \(g(y(t)) = f(x(t)) - f(x^*) = (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)))^T, \phi(s) = \phi(s) - x^*.
\]

(A4) We consider the parameter uncertainties expressed as

\[
A(t) = A + \Delta A(t), \quad B(t) = B + \Delta B(t),
\]

\[
C(t) = C + \Delta C(t),
\]

\[
H_i(t) = H_i + \Delta H_i(t), \quad \Delta G_{ik}(t) = \Delta G_{ik}(t),
\]

\[i = 1, 2, 3, \quad k \in \mathbb{N},
\]

where \(A, B, C, H_i, \text{ and } G_{ik}\) are known real constant matrices; \(\Delta A(t), \Delta B(t), \Delta C(t), \Delta H_i(t), \text{ and } \Delta G_{ik}(t)\) are unknown matrices representing the parameter uncertainties, which are assumed to be the following form:

\[
\begin{aligned}
\Delta A(t) = E_1F_1(t) [M_1 \ M_2 \ M_3],
\end{aligned}
\]

\[
\Delta H_i(t) = E_1F_2(t) [N_1 \ N_2 \ N_3],
\]

\[
\begin{aligned}
\Delta G_{ik}(t) = E_1F_3(t) [U_1 \ U_2 \ U_3],
\end{aligned}
\]

where \(U_1, E_1, M_i, N_i \ (i = 1, 2, 3)\) are known real constant matrices and \(F_i(t)\) are unknown real time-varying matrix functions satisfying \(F_i(t)F_i(t) \leq I, \ i = 1, 2, 3.
\]

Remark 1. Assumptions (A1)–(A4) imply that system (4) satisfies the local Lipschitz condition and linear growth condition. Thus there exists a unique solution of system (4).
**Definition 2.** The equilibrium point $x^*$ of system (1) is said to be robustly exponentially stable in the mean square, if there exists scalars $\gamma > 0$ and $\delta > 0$ such that for any $\epsilon > 0$ and initial condition $\phi$ satisfying $E\|\phi\| \leq \delta$ which implies $E[|x(t; t_0, \phi) - x^*| < \epsilon e^{-\gamma(t-t_0)}, \ t \geq t_0].$

**Lemma 3** (see [22]). Given matrices $D, E, \text{and } F$ with $F^TF \leq I$ and a scalar $\epsilon > 0$, then

$$DFE + E^TF^TD^T \leq \epsilon DD^T + \epsilon^{-1}E^T E.$$  

**Proof.** Since the matrix inequality (9) holds, we can choose small enough scalars $\eta > 0, h > 0$ satisfying $\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2h} < 1$ and $h < 1 - \mu_1 - \mu_2 e^{2\eta} - \mu_3 e^{2h}$, such that

$$\begin{bmatrix}
-\mu_1 P + \epsilon_1 U_1^T U_1 & \epsilon_1 U_1^T U_2 & \epsilon_1 U_1^T U_3 & G_{1k}^T P & 0 \\
* & -\mu_2 P + \epsilon_1 U_2^T U_2 & \epsilon_1 U_2^T U_3 & G_{2k}^T P & 0 \\
* & * & -\mu_3 P + \epsilon_1 U_3^T U_3 & G_{3k}^T P & 0 \\
\end{bmatrix} < 0, \quad (8)$$

where $\Omega_{11} = [((\alpha + \beta)/(\mu_1 + \mu_2 + \mu_3) + \ln(\mu_1 + \mu_2 + \mu_3)/\Delta_{max})] P - W_1 K_2 + \epsilon_3 N_1^T N_2, \Omega_{22} = -\alpha P - W_2 K_2 + \epsilon_3 N_2^T N_3, \Omega_{33} = -\beta P + \epsilon_2 M_1^T M_2 + \epsilon_3 N_3^T N_3, K_1 = \text{diag}(l_1 + L_1/2), (l_1 + L_2/2), \ldots, (l_n + L_n/2)], K_2 = \text{diag}(l_1 L_1, l_2 L_2, \ldots, l_n L_n)$.

$$\begin{bmatrix}
\Omega_{11} & \epsilon_3 N_1^T N_2 & -PC + \epsilon_3 N_1^T N_3 & PA + W_1 K_1 & PB & H_1^T P & PE_1 & 0 \\
* & \Omega_{22} & \epsilon_3 N_2^T N_3 & 0 & W_2 K_1 & H_2^T P & 0 & 0 \\
* & * & \Omega_{33} & -\epsilon_2 M_1^T M_2 & -\epsilon_2 M_1^T M_3 & H_3^T P & 0 & 0 \\
* & * & * & -W_1 + \epsilon_2 M_2^T M_2 & \epsilon_2 M_2^T M_3 & 0 & 0 & 0 \\
* & * & * & * & -W_2 + \epsilon_2 M_3^T M_3 & 0 & 0 & 0 \\
* & * & * & * & * & -P & 0 & PE_2 \\
* & * & * & * & * & * & -\epsilon_2 I & 0 \\
* & * & * & * & * & * & * & -\epsilon_3 I \\
\end{bmatrix} < 0, \quad (9)$$

**Proof.** Since the matrix inequality (9) holds, we can choose small enough scalars $\eta > 0, h > 0$ satisfying $\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2h} < 1$ and $h < 1 - \mu_1 - \mu_2 e^{2\eta} - \mu_3 e^{2h}$, such that

$$\begin{bmatrix}
\Omega_{11} & \epsilon_3 N_1^T N_2 & -PC + \epsilon_3 N_1^T N_3 & PA + W_1 K_1 & PB & H_1^T P & PE_1 & 0 \\
* & \Omega_{22} & \epsilon_3 N_2^T N_3 & 0 & W_2 K_1 & H_2^T P & 0 & 0 \\
* & * & \Omega_{33} & -\epsilon_2 M_1^T M_2 & -\epsilon_2 M_1^T M_3 & H_3^T P & 0 & 0 \\
* & * & * & -W_1 + \epsilon_2 M_2^T M_2 & \epsilon_2 M_2^T M_3 & 0 & 0 & 0 \\
* & * & * & * & -W_2 + \epsilon_2 M_3^T M_3 & 0 & 0 & 0 \\
* & * & * & * & * & -P & 0 & PE_2 \\
* & * & * & * & * & * & -\epsilon_2 I & 0 \\
* & * & * & * & * & * & * & -\epsilon_3 I \\
\end{bmatrix} < 0, \quad (10)$$
where $\Pi_{11} = [2\eta + (\alpha + \beta)/(\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2\eta}) + \ln(\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2\eta} + h)/\Delta_{\max}] P - W_i K_2 + \xi_1 N_i^T N_1, \Delta_{\max} = -\alpha e^{2\eta} P - W_i K_2 + \xi_1 N_i^T N_1, \Delta_{\max} = -\alpha e^{2\eta} P - W_i K_2 + \xi_1 N_i^T N_1$.

Let $q = 1/(\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2\eta})$, $\lambda = -\ln(\mu_1 + \mu_2 e^{2\eta} + \mu_3 e^{2\eta} + h)/\Delta_{\max}$, and then $\lambda > 0, q > 1, e^{\lambda \Delta_{\max}} < q$.

Define Lyapunov function

$$V(t, y(t)) = 2\eta P + \frac{\alpha}{2} y(t) + y(t) + \beta y(t) - y(t - \sigma(t)) \leq 0, \quad \lambda \Delta_{\max} < q.$$  

From assumption (A1), the following inequalities hold for any diagonal matrices $W_i > 0, W_2 > 0$,

$$e^{2\eta t} \left[ g(y(t)) W_i g(y(t)) - 2y^T(t) W_i K_1 g(y(t)) + y^T(t) W_i K_2 y(t) \right] \leq 0,$$

and

$$e^{2\eta t} \left[ g(y(t - \tau(t)) - 2y^T(t) W_i K_2 y(t) \right] \leq 0.$$

Set

$$\xi^T(t) = \left( y^T(t), y^T(t - \tau(t)), y^T(t - \sigma(t)), g^T(y(t)) \right).$$

Combining (15)–(16) together, we have

$$W(t) \leq e^{2\eta t} \xi^T(t) \Psi \xi(t),$$

where

$$W(t) = \left[ \begin{array}{cccc} \Gamma_{11} & 0 & -PC(t) & PA(t) + W_i K_1 & PB(t) \\ \Gamma_{22} & 0 & 0 & 0 & W_2 K_2 \\ * & * & -\beta P & 0 & 0 \\ * & * & * & -W_1 & 0 \\ * & * & * & * & -W_2 \end{array} \right]$$

and

$$\Psi = \left[ \begin{array}{c} H_1^T(t) P \\ H_2^T(t) P \\ H_3^T(t) P \\ 0 \\ 0 \end{array} \right] \left[ \begin{array}{c} H_1^T(t) P \\ H_2^T(t) P \\ H_3^T(t) P \\ 0 \\ 0 \end{array} \right]$$

and

$$\Gamma_{11} = 2\eta P + \alpha q P - \beta q P - W_i K_2 - \lambda P, \quad \Gamma_{22} = -\alpha P - W_2 K_2.$$
First, for \( t \in [t_0 - \rho, t_0] \),
\[
EV(t) = E \left( e^{2\eta t} y^T(t) Py(t) \right) 
\leq \lambda_1 E \| \phi \|^2 \leq \lambda_1 \delta^2 < \frac{1}{q} \lambda_0 \epsilon^2 < \lambda_0 \epsilon^2.
\] (22)

Then we will prove that
\[
EV(t) < \lambda_0 \epsilon^2, \quad t \in [t_0, t_1). \quad (23)
\]

If the above inequality does not hold, then there exist \( t^* = \inf \{ t \in (t_0, t_1) : EV(t) \geq \lambda_0 \epsilon^2 \} \), such that \( EV(t^*) = \lambda_0 \epsilon^2 \). Set \( \hat{t} = \sup \{ t \in [t_0, t^*) : EV(t) \leq (1/q) \lambda_0 \epsilon^2 \} \); then \( \hat{t} \in (t_0, t^*) \) and \( EV(\hat{t}) = (1/q) \lambda_0 \epsilon^2 \). Hence for all \( t \in (\hat{t}, t^*) \),
\[
q EV(t) > EV(t - \tau(t)), \quad q EV(t) > EV(t - \sigma(t))
\quad \text{for } t \in (\hat{t}, t^*). \quad (24)
\]

It follows from (20) that for any \( \alpha > 0, \beta > 0 \) and \( t \in (\hat{t}, t^*) \), we have
\[
D^T EV(t) = E D^T V(t) < E D V(t)
+ \alpha [q EV(t) - EV(t - \tau(t))] 
+ \beta [q EV(t) - EV(t - \sigma(t))] < \lambda EV(t),
\] (25)

which leads to \( EV(t^*) \leq EV(\hat{t}) e^{\lambda(t^* - \hat{t})} \leq EV(\hat{t}) e^{\lambda \Delta \max} = (1/q) \lambda_0 \epsilon^2 < \lambda_0 \epsilon^2 \). This is a contradiction.

Thus (23) holds.

Now we assume that for some \( m \in \mathbb{N} \), \( EV(t) < \lambda_0 \epsilon^2, t \in [t_0 - \rho, t_m) \); we will prove that
\[
EV(t) < \lambda_0 \epsilon^2, \quad t \in [t_m, t_{m+1}). \quad (26)
\]

From (13), we have
\[
EV(t_m) \leq \mu_1 EV(t_k) + \mu_2 e^{2\eta \tau} EV(t_k - \tau(t_k)) 
+ \mu_3 e^{2\eta \tau} EV(t_k - \sigma(t_k)) 
+ (\mu_4 + \mu_5 e^{2\eta \tau} + \mu_6 e^{2\eta \tau}) \lambda_0 \epsilon^2
\] (27)

Suppose (26) does not hold; then there exists \( t^* = \inf \{ t \in (t_m, t_{m+1}) : EV(t) \geq \lambda_0 \epsilon^2 \} \) and \( EV(t^*) = \lambda_0 \epsilon^2 \). Set \( \tilde{t} = \sup \{ t \in [t_m, t^*) : EV(t) \leq (1/q) \lambda_0 \epsilon^2 \} \), and then from (27), \( \tilde{t} \in (t_m, t^*) \) and \( EV(\tilde{t}) = (1/q) \lambda_0 \epsilon^2 \). In the sequel, the proof is very similar with the proof of (23). Therefore (26) holds. By mathematical induction, inequality (21) holds. This together with \( EV(t) \geq \lambda_0 \epsilon^2 \) and \( EV(t) \leq (1/q) \lambda_0 \epsilon^2 \), we have
\[
E \| y(t) \| < \epsilon e^{-\eta t}.
\] (28)

This completes the proof of Theorem 4. \( \Box \)

Remark 5. Theorem 4 shows that robustly exponential stability of system (1) can be achieved by adjusting suitable impulsive control and appropriate impulsive intervals even if the given networks without impulses may be unstable or chaotic themselves.

Remark 6. In [5], the authors investigated the stability of neural networks with delayed leakage term and impulsive control and appropriate impulsive intervals even if the given networks without impulses may be unstable or chaotic themselves.

Theorem 7. Suppose that assumptions (A_1)-(A_4) hold and for prescribed scalars \( \Delta_{\max} > 0 \), choose positive scalars \( \alpha, \beta, \mu_1, \mu_2, \mu_3, \) satisfying \( \mu_1 + \mu_2 + \mu_3 \geq 1 \) and \( \mu_1 + \mu_2 + \mu_3 \leq 1 \). Then the equilibrium point of system (1) is robustly exponentially stable in the mean square over any impulse time sequences satisfying \( \inf \{ t_k - t_{k-1} : k = 1, 2, \ldots \} \geq \Delta_{\min} \), if there exist positive definite matrix \( P \) and positive definite diagonal matrices \( W_1, W_2 \) and positive constants \( \epsilon_1, \epsilon_2, \epsilon_3 \) such that (8) and the following LMI hold:
\[
\begin{bmatrix}
\Omega_{11} & \epsilon_3 N_1^T N_2 - PC + \epsilon_3 N_1^T N_3 & PA + W_1 K_1 & PB & H^T P & PE_1 & 0 \\
* & \Omega_{22} & \epsilon_3 N_2^T N_3 & 0 & W_2 K_1 & H_1^T P & 0 & 0 \\
* & * & \Omega_{33} & -\epsilon_2 M_1^T M_2 & -\epsilon_2 M_1^T M_3 & H_2^T P & 0 & 0 \\
* & * & * & -W_1 + \epsilon_2 M_2^T M_2 & \epsilon_2 M_2^T M_3 & 0 & 0 & 0 \\
* & * & * & * & -W_2 + \epsilon_2 M_3^T M_3 & 0 & 0 & 0 \\
* & * & * & * & * & -P & 0 & PE_2 \\
* & * & * & * & * & * & -\epsilon_3 I & 0 \\
* & * & * & * & * & * & * & -\epsilon_3 I 
\end{bmatrix} < 0,
\] (29)
where \( \Omega_{11} = [(\alpha + \beta)(\mu_1 + \mu_2 e^t + \mu_3 e^{\sigma t}) + \ln(\mu_1 + \mu_2 e^t + \mu_3 e^{\sigma t})/\Delta_{\min}] P - W_i K_2 + e_3^T T_{11} N_1 \) and \( \Omega_{22} = -\alpha P - W_i K_2 + e_3^T T_{11} N_2, \Omega_{33} = -\beta P + e_3^T T_1 M_1 + e_3^T T_{11} N_3. \)

Proof. Since the matrix inequality (29) holds, we can choose small enough scalars \( \eta > 0, h > 0 \) satisfying \( \ln(\mu_1 + \mu_2 e^t + \mu_3 e^{\sigma t} + 2h)/\Delta_{\min} + 2\eta \leq c, \) such that the following matrix inequality holds:

\[
\begin{bmatrix}
\Omega_{11} & e_3^T T_{11} N_3 & PA + W_i K_1 & PB & H_i^T P & PE_i & 0 \\
* & \Omega_{22} & e_3^T T_{11} N_2 & 0 & W_i K_2 & H_i^T P & 0 & 0 \\
* & * & \Omega_{33} & -e_2^T T_2 M_2 & -e_2^T T_2 M_3 & e_2^T T_2 M_2 & 0 & 0 & 0 \\
* & * & * & -W_1 + e_2^T T_2 M_2 & -e_2^T T_2 M_2 & e_2^T T_2 M_3 & 0 & 0 & 0 \\
* & * & * & * & * & * & -P & 0 & PE_2 \\
* & * & * & * & * & * & * & -e_3 I & 0 \\
* & * & * & * & * & * & * & * & -e_3 I
\end{bmatrix} < 0, \tag{30}
\]

Then we will prove that

\[
EV(t) < \lambda_0 e^t, \quad t \in [t_0, t_1). \tag{37}
\]

If (37) does not hold, there exists \( t^* = \inf \{t \in [t_0, t_1) : EV(t) \geq \lambda_0 e^t \}. \) From (36), \( t^* \in (t_0, t_1) \) and \( EV(t^*) = \lambda_0 e^{t^*}. \) Let \( \tilde{t} = \sup \{t \in [t_0, t^*) : EV(t) \leq (1/\eta)\lambda_0 e^t \}; \) then \( \tilde{t} \in (t_0, t^*). \)

So for \( t \in (\tilde{t}, t^*) \) and any \( \theta \in [-\rho, 0], \) we have \( qEV(t) \geq \lambda_0 e^t > EV(t + \theta). \) It follows from (34) that for any \( \alpha > 0, \beta > 0\) and \( t \in (\tilde{t}, t^*), \)

\[
D^+ EV(t) = E\mathcal{L}V(t) \leq E\mathcal{L}V(t) + \alpha [qEV(t) - EV(t - \tau(t))]
+ \beta [qEV(t) - EV(t - \sigma(t))] + \lambda EV(t). \tag{38}
\]

Then we have \( \lambda_0 e^{t^*} = EV(t^*) < EV(\tilde{t})e^{-\lambda_0 \tilde{t}} \leq EV(\tilde{t}) = (1/\eta)\lambda_0 e^{\tilde{t}}. \) This is a contradiction. Thus (37) holds.

Suppose that for some \( m \in \mathbb{N}, EV(t) < \lambda_0 e^t, t \in [t_0 - \rho, t_m). \) We will prove that

\[
EV(t) < \lambda_0 e^t, \quad t \in [t_m, t_{m+1}). \tag{39}
\]

We first claim that \( EV(t_m) < (1/\eta)\lambda_0 e^t. \) The proof is very similar to [18, 19], so we omit it here. Next, we show that

\[
EV(t_{m} - \sigma(t_m)) \leq \frac{1}{q} e^{\lambda_0 e^t}. \tag{40}
\]
Suppose not; then we have $\mathbb{E}V(t_m - \sigma(t_m)) > (1/q)e^{\lambda_0 t_m e^2}$. Without loss of generality, we assume $t_m - \sigma(t_m) \in (t_{k-1}, t_k], l \in \mathbb{N}, l \leq m$. There are two cases to be considered.

**Case 1.** $EV(t) > (1/q)e^{\lambda_0 t} e^2$ for any $t \in [t_{k-1}, t_k - \sigma(t_m))]$. Then we have

$$qEV(t) > e^{\lambda_0 t} e^2 \geq \lambda_0 e^2 > EV(t + \theta), \quad \forall \theta \in [-\rho, 0], \quad t \in [t_{k-1}, t_k - \sigma(t_m)),$$

(41)

It follows from (34) that for any $\alpha > 0, \beta > 0$ and $t \in (t_{k-1}, t_k - \sigma(t_m))$ (38) holds, which leads to

$$EV(t_m - \alpha - \sigma(t_m)) < EV(t_{k-1})e^{-\lambda_0 e^2} < 1 \quad \forall \theta \in [-\rho, 0].$$

(42)

This is a contradiction.

**Case 2.** There exist some $t \in [t_{k-1}, t_k - \sigma(t_m))]$, such that $EV(t) \leq (1/q)e^{\lambda_0 t_m e^2}$. Set $\bar{t} \supset \{t \in [t_{k-1}, t_m - \sigma(t_m)) : EV(t) \leq (1/q)e^{\lambda_0 t_m e^2}\}$.

Then we have $\bar{t} \subset [t_{k-1}, t_m - \sigma(t_m))$ and $EV(\bar{t}) = (1/q)e^{\lambda_0 t_m e^2}$. Hence for $t \in (\bar{t}, t_m - \sigma(t_m))$, we have

$$qEV(t) \geq e^{\lambda_0 t_m e^2} \geq \lambda_0 e^2 > EV(t + \theta), \quad \forall \theta \in [-\rho, 0].$$

(43)

It follows from (34) that for any $\alpha > 0, \beta > 0$ and $t \in (\bar{t}, t_m - \sigma(t_m))$ (38) holds, which leads to

$$EV(t_m - \alpha - \sigma(t_m)) < EV(\bar{t}) e^{-\lambda_0 e^2} \leq EV(\bar{t}) \quad \forall \theta \in [-\rho, 0].$$

(44)

This is a contradiction. 

Therefore (40) holds. By the same methods, we can prove $EV(t_m - \tau(t_m)) \leq (1/q)e^{\lambda_0 t_m e^2}$. Then from (33), we get

$$EV(t_m) \leq \mu_1 \mathbb{E}V(t_m) + \mu_2 e^{2\eta t_m} \mathbb{E}V(t_m) + \mu_3 e^{2\eta t_m} \mathbb{E}V(t_m - \tau(t_m))$$

$$= \mu_1 \mathbb{E}V(t_m) + \mu_2 e^{2\eta t_m} \mathbb{E}V(t_m - \tau(t_m))$$

$$\leq \left(\mu_1 + \mu_2 e^{2\eta \lambda_0} + \mu_3 e^{2\eta \lambda_0}\right) \frac{1}{q} \lambda_0 e^2$$

(45)

$$\leq \mu_1 e^{2\eta t_m} + \mu_3 e^{2\eta t_m} \frac{1}{q} \lambda_0 e^2 \leq \lambda_0 e^2,$$

Suppose (39) does not hold; then there exist $t^* = \inf \{t \in (t_m, t_{m+1}) : EV(t) \geq \lambda_0 e^2\}$ and $EV(t^*) = \lambda_0 e^2$. If $EV(t) > (1/q)\lambda_0 e^2$ for all $t \in [t_{m+1}, t^*)$, set $\bar{t} = t_m$. Otherwise let $\bar{t} = \sup \{t \in (t_m, t^*) : EV(t) \leq (1/q)\lambda_0 e^2\}$. Then for $t \in (\bar{t}, t^*)$ and any $\theta \in [-\rho, 0]$, we have $qEV(t) \geq \lambda_0 e^2 > EV(t + \theta)$. It follows from (34) that for any $\alpha > 0, \beta > 0$, and $t \in (\bar{t}, t^*)$, (38) holds. Then, $\lambda_0 e^2 = EV(t^*) < EV(\bar{t}) e^{-\lambda_0 e^2} \leq EV(\bar{t}) = (1/q)\lambda_0 e^2 < \lambda_0 e^2$.

This is a contradiction. Thus (39) holds.

The next proof is very similar to Theorem 4. This completes the proof.

**Remark 8.** Theorem 7 shows that the system will remain exponentially stable on the condition that the impulses, which may destabilize the system, do not occur too frequently.

**Remark 9.** If there is no leakage delay, that is, $\sigma(t) = 0$, $H_k(t) = 0, G_{k1}(t) = 0, k \in \mathbb{N}, t \geq 0$, then the system (4) is the one investigated in [19]. If there is no stochastic perturbation either, then system (4) is the one investigated in [18, 20].

### 4. Examples

In this section, we present some examples to verify the effectiveness of the theoretical results.

**Example 1.** Consider (4) with two neurons. The uncertain parameters satisfy assumption (A1), where $C = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$, $A = \begin{bmatrix} 0.6 & -0.2 \\ 0.7 & -0.3 \end{bmatrix}$, $H_1 = H_2 = H_3 = 0.03I, M_1 = M_2 = M_3 = 0.03I, G_{1k} = G_{2k} = G_{3k} = 0.1I, k \in \mathbb{N}, N_1 = N_2 = N_3 = 0.1I, U_1 = U_2 = U_3 = 0.03I, E_1 = E_2 = E_3 = 0.03I, \tau < +\infty, \sigma < +\infty, g_1(y) = g_2(y) = |y + 1| - |y - 1|$, and it is obvious that $K_1 = I, K_2 = 0$. Choose $\mu_1 = 0.05, \mu_2 = 0.1, \mu_3 = 0.1, \alpha = 0.5, \beta = 0.5$, for $t_k - t_{k-1} \leq \Delta_{\max} = 0.01$, then the LMIs in Theorem 4 have the following feasible solution via MATLAB LMI toolbox: $\varepsilon_1 = 1.2054, \varepsilon_2 = 11.5508, \varepsilon_3 = 0.7167, P = \begin{bmatrix} 1.2516 & 0.0492 \\ 0.0492 & 1.1285 \end{bmatrix}, W_1 = \begin{bmatrix} 9.7101 \\ 0.0092 \end{bmatrix}, W_2 = \begin{bmatrix} 0.3534 \\ 0.0244 \end{bmatrix}$. Thus from Theorem 4, the equilibrium $(0, 0)^T$ of system (4) is robustly exponentially stable in the mean square.

**Example 2.** We consider neural network shown in [5] as follows:

$$\dot{x}(t) = \begin{bmatrix} -9 & 0 \\ 0 & -9 \end{bmatrix} x(t - \sigma) + \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} f(x(t)), \quad t > 0,$$

(46)

where $f_1(s) = f_2(s) = \tanh(s)$. As shown in [5], the system (46) is stable when $\sigma = 0$, and it becomes unstable when $\sigma = 0.2$. We consider that the system has the following impulsive perturbation at times $t_k$:

$$\Delta x(t) = (G_{1k}(t) - I) x(t^-), \quad t = t_k, \quad k \in \mathbb{N},$$

(47)

where $G_{1k} = 0.3I, k \in \mathbb{N}$. It is obvious that $K_1 = 0.5I, K_2 = 0$. For $t_k - t_{k-1} \leq \Delta_{\max} = 0.01$, choosing $\mu_1 = 0.1, \mu_2 = 0.01, \mu_3 = 0.05, \alpha = 0.5, \beta = 0.5$, by using the LMI toolbox in MATLAB, a feasible solution of Theorem 4 is

$$P = \begin{bmatrix} 0.0210 & 0 \\ 0 & 0.0210 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.6321 \\ 0 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.0289 \\ 0 \end{bmatrix}, \quad \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.6748.$$
5. Conclusion
Robust exponential stability of stochastic neural networks with time-varying delay in the leakage term under impulsive perturbations is investigated. The leakage delay is time varying and the impulsive perturbations depend not only on the current state of neurons at impulse times but also on the state of neurons in its recent history. Based on Lyapunov functions and Razumikhin techniques, some new criteria are derived. Some examples have been given to demonstrate that, even though the corresponding delayed neural networks without impulses are unstable, impulses may compensate the deviating trend.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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