Research Article
Soft $\alpha$-Open Sets and Soft $\alpha$-Continuous Functions

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We introduce soft $\alpha$-sets on soft topological spaces and study some of their properties. We also investigate the concepts of soft $\alpha$-continuous and soft $\alpha$-open functions and discuss their relationships with soft continuous and other weaker forms of soft continuous functions. Also counterexamples are given to show the noncoincidence of these functions.

1. Introduction

Molodtsov [1] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory to several directions such as smoothness of functions, game theory, Riemann Integration, and theory of measurement. In recent years, development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parameterization expressed by a soft set. Shabir and Naz [2] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later, Zorlutuna et al. [3], Aygunoglu and Aygun [4], and Hussain et al. continued to study the properties of soft topological space. They got many important results in soft topological spaces. Weak forms of soft open sets were first studied by Chen [5]. He investigated soft semipen open sets in soft topological spaces and studied some properties of them. Arockiarani and Arokialancy defined soft $\beta$-open sets and continued to study weak forms of soft open sets in soft topological space.

In the present paper, we introduce some new concepts in soft topological spaces such as soft $\alpha$-open sets, soft $\alpha$-closed sets, and soft $\alpha$-continuous functions. We also study relationship between soft continuity [6], soft semicontinuity [7], and soft $\alpha$-continuity of functions defined on soft topological spaces. With the help of counterexamples, we show the noncoincidence of these various types of mappings.

2. Preliminaries

Definition 1 (see [1]). Let $X$ be an initial universe and let $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and let $A$ be a nonempty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$.

Definition 2 (see [8]). A soft set $(F, A)$ over $X$ is called a null soft set, denoted by $\Phi$, if $e \in A$, $F(e) = \emptyset$.

Definition 3 (see [8]). A soft set $(F, A)$ over $X$ is called an absolute soft set, denoted by $\tilde{A}$, if $e \in A$, $F(e) = X$.

Definition 4 (see [8]). The union of two soft sets $(F, A)$ and $(G, B)$ over the common universe $X$ is the soft set $(H, C)$, where $C = A \cup B$ and, for all $e \in C$,

$$H(e) = \begin{cases}
F(e), & \text{if } e \in A - B, \\
G(e), & \text{if } e \in B - A, \\
F(e) \cup G(e), & \text{if } e \in A \cap B. 
\end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$.

Definition 5 (see [8]). The intersection $(H, C)$ of two soft sets $(F, A)$ and $(G, B)$ over a common universe $X$, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for all $e \in C$. 

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Definition 6 (see [8]). Let \((F, A)\) and \((G, B)\) be two soft sets over a common universe \(X\). \((F, A)\subset (G, B)\), if \(A \subset B\), and \(H(e) = F(e) \subset G(e)\), for all \(e \in A\).

Definition 7 (see [2]). Let \(\tau\) be the collection of soft sets over \(X\); then \(\tau\) is said to be a soft topology on \(X\) if it satisfies the following axioms:

1. \(\Phi\) and \(X\) belong to \(\tau\),
2. the union of any number of soft sets in \(\tau\) belongs to \(\tau\),
3. the intersection of any two soft sets in \(\tau\) belongs to \(\tau\).

The triplet \((X, \tau, \mathcal{E})\) is called a soft topological space over \(X\). Let \((X, \tau, \mathcal{E})\) be a soft topological space over \(X\); then the members of \(\tau\) are said to be soft open sets in \(X\). The relative complement of a soft set \((F, A)\) is denoted by \((F, A)^c\) and is defined by \((F, A)^c = (F^c, A)\), where \(F^c : A \rightarrow \mathcal{P}(X)\) is a mapping given by \(F^c(e) = X - F(e)\) for all \(e \in A\). Let \((X, \tau, \mathcal{E})\) be a soft topological space over \(X\). A soft set \((F, A)\) over \(X\) is said to be a soft closed set in \(X\) if its relative complement \((F, A)^c\) belongs to \(\tau\). If \((X, \tau, \mathcal{E})\) is a soft topological space with \(\tau = \{\Phi, \widehat{X}\}\), then \(\tau\) is called the soft indiscrete topology on \(X\) and \((X, \tau, \mathcal{E})\) is said to be a soft indiscrete topological space. If \((X, \tau, \mathcal{E})\) is a soft topological space with \(\tau\) is the collection of all soft sets which can be defined over \(X\), then \(\tau\) is called the soft discrete topology on \(X\) and \((X, \tau, \mathcal{E})\) is said to be a soft discrete topological space.

Definition 8. Let \((X, \tau, \mathcal{E})\) be a soft topological space over \(X\) and let \((A, \mathcal{E})\) be a soft set over \(X\).

1. [3] The soft interior of \((A, \mathcal{E})\) is the soft set \(\text{int}(A, \mathcal{E}) = \bigcup\{(O, \mathcal{E}) : (O, \mathcal{E})\) is soft open and \(\mathcal{E}(A, \mathcal{E})\)\).
2. [2] The soft closure of \((A, \mathcal{E})\) is the soft set \(\text{cl}(A, \mathcal{E}) = \bigcap\{(F, \mathcal{E}) : (F, \mathcal{E})\) is soft closed and \(\mathcal{E}(A, \mathcal{E})\)\)\).

Clearly \(\text{cl}(A, \mathcal{E})\) is the smallest soft closed set over \(X\) which contains \((A, \mathcal{E})\) and \(\text{int}(A, \mathcal{E})\) is the largest soft open set over \(X\) which is contained in \((A, \mathcal{E})\).

Throughout the paper, the spaces \(X\) and \(Y\) (or \((X, \tau, \mathcal{E})\) and \((Y, \nu, \mathcal{K})\)) stand for soft topological spaces assumed unless otherwise stated.

### 3. Soft \(\alpha\)-Open Sets

Definition 9. A soft set \((A, \mathcal{E})\) of a soft topological space \((X, \tau, \mathcal{E})\) is called soft \(\alpha\)-open set if \((A, \mathcal{E})\subset \text{cl}(\text{int}(A, \mathcal{E}))\). The complement of soft \(\alpha\)-open set is called soft \(\alpha\)-closed set.

Definition 10. A soft set \((A, \mathcal{E})\) is called soft preopen set [9] (resp., soft semiopen [5]) in a soft topological space \(X\) if \((A, \mathcal{E})\subset \text{cl}(\text{int}(A, \mathcal{E}))\) (resp., \((A, \mathcal{E})\subset \text{cl}(\text{int}(A, \mathcal{E}))\)).

We will denote the family of all soft \(\alpha\)-open sets (resp., soft \(\alpha\)-closed sets and soft preopen sets) of a soft topological space \((X, \tau, \mathcal{E})\) by \(\text{SaOS}(X, \tau, \mathcal{E})\) (resp., \(\text{SaCS}(X, \tau, \mathcal{E})\) and \(\text{SPO}(X, \tau, \mathcal{E})\)).
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\[(F_{10}, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2\})\}, \]
\[(F_{11}, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2\})\}, \]
\[(F_{12}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2, x_4\})\}, \]
\[(F_{13}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_4\}), (e_3, \{x_2\})\}, \]
\[(F_{14}, E) = \{(e_1, \{x_3, x_4\}), (e_2, \{x_1, x_2\})\}, \]
\[(F_{15}, E) = \{(e_1, \{x_1\}), (e_2, \{x_2, x_3\}), (e_3, \{x_1\})\}. \]

Then \(\tau\) defines a soft topology on \(X\) and thus \((X, \tau, E)\) is a soft topological space over \(X\).

Clearly the soft closed sets are \(\overline{X}, \Phi, (F_1, E)^{c}, (F_2, E)^{c}, (F_3, E)^{c}, \ldots, (F_{15}, E)^{c}\).

Then, let us take \((B, E) = \{(e_1, \{x_1, x_2, x_3\}), (e_2, \{x_2, x_3, x_4\}), (e_3, \{x_1, x_2\})\}\), then \(\text{int}(B, E) = (F_{10}, E), \text{cl}(\text{int}(B, E)) = \overline{X}, \text{int}(\text{cl}(\text{int}(B, E))) = \overline{X}\), and so \((B, E)\) is a soft \(\alpha\)-closed set.

Now, let us take \((H, E) = \{(e_1, \{x_2\}), (e_2, \{x_1, x_3\})\}\) then \(\text{int}(H, E) = (F_1, E), \text{cl}(\text{int}(H, E)) = (F_2, E)^{c}\), and so \((H, E)\) is a soft \(\alpha\)-open set.

Finally, let us take \((G, E) = \{(e_1, \{x_2\}\) then \(\text{cl}(G, E) = (F_1, E)^{c}, \text{int}(\text{cl}(G, A)) = (F_2, E)^{c}\), and so \((G, E)\) is a soft \(\alpha\)-open set.

**Definition 15.** Let \((X, \tau, E)\) be a soft topological space and let \((A, E)\) be a soft set over \(X\).

1. Soft \(\alpha\)-closure of a soft set \((A, E)\) in \(X\) is denoted by \(\text{sc}(A, E) = \overline{\text{cl}(A, E)}\) which is a soft \(\alpha\)-closed set and \((A, E)(\overline{A}, E)\).
2. Soft \(\alpha\)-interior of a soft set \((A, E)\) in \(X\) is denoted by \(\text{si}(A, E) = \bigcup\{(O, E) : (O, E) is a soft \(\alpha\)-open set and \((A, E)(\overline{A}, E)\}\).

Clearly \(\text{sc}(A, E)\) is the smallest soft \(\alpha\)-closed set over \(X\) which contains \((A, E)\) and \(\text{si}(A, E)\) is the largest soft \(\alpha\)-open set over \(X\) which is contained in \((A, E)\).

**Proposition 16.** Let \((X, \tau, E)\) be a soft topological space and let \((A, E)\) be a soft set over \(X\); then

1. \((A, E) \in \text{SCS}(X, \tau, E) \iff (A, E) = \text{sc}(A, E)\);
2. \((A, E) \in \text{SOS}(X, \tau, E) \iff (A, E) = \text{si}(A, E)\).

**Proof.** 1. Let \((A, E) = \text{sc}(A, E) = \overline{\text{cl}(A, E)}\) then \((A, E)\) is a soft \(\alpha\)-closed set and \((A, E)(\overline{A}, E)\).

This shows that \((A, E) \in \{(F, E) : (F, E) is a soft \(\alpha\)-closed set and \((A, E)(\overline{A}, E)\}\).

Hence \((A, E)\) is soft \(\alpha\)-closed.

Conversely, let \((A, E)\) be soft \(\alpha\)-closed set. Since \((A, E)(\overline{A}, E)\) and \((A, E)\) is a soft \(\alpha\)-closed, \((A, E) \in \{(F, E) : (F, E) is a soft \(\alpha\)-closed set and \((A, E)(\overline{A}, E)\}\).

Further, \((A, E)(\overline{A}, E)\) for all such \((F, E)\)’s.

2. Similar to (1).  \(\square\)

**Proposition 17.** In a soft space \((X, \tau, E)\), the following hold for soft \(\alpha\)-closure.

1. \(\text{sc}(\Phi) = \Phi\);
2. \(\text{sc}(\text{cl}(A, E)\subset\text{sc}(\text{cl}(B, E)))\) if \((A, E)\subset(B, E)\).
3. \(\text{sc}(\text{cl}(A, E))\subset\text{sc}(\text{cl}(B, E))\) if \((A, E)\subset(B, E)\).
4. \(\text{sc}(\text{sc}(A, E)) = \text{sc}(A, E)\).

**Proof.** Easy.

**Theorem 18.** Let \((X, \tau, E)\) be a soft topological space and let \((G, E)\) and \((K, E)\) be two soft sets over \(X\); then

1. \(\text{sc}(\text{cl}(G, E)^{c}) = \text{sc}(\text{cl}(G, E)^{c})\);
2. \(\text{sc}(\text{int}(G, E)^{c}) = \text{sc}(\text{int}(G, E)^{c})\);
3. \(\text{sc}(\text{cl}(G, E)\subset\text{sc}(\text{cl}(K, E)))\) if \((A, E)\subset(B, E)\);
4. \(\text{sc}(\text{sc}(A, E)) = \text{sc}(A, E)\).

**Proof.** Let \((G, E)\) and \((K, E)\) be two soft sets over \(X\).

1. \(\text{sc}(\text{cl}(G, E)^{c}) = \text{sc}(\text{cl}(G, E)^{c})\);
2. \(\text{sc}(\text{int}(G, E)^{c}) = \text{sc}(\text{int}(G, E)^{c})\);
3. \(\text{sc}(\text{cl}(G, E)\subset\text{sc}(\text{cl}(K, E)))\) if \((A, E)\subset(B, E)\);
4. \(\text{sc}(\text{sc}(A, E)) = \text{sc}(A, E)\).

(2) Similar to (1).

(3) It follows from Definition 15.

(4) Since \(\Phi\) and \(\overline{X}\) are soft \(\alpha\)-closed sets so, \(\text{sc}(\Phi) = \Phi\) and \(\text{sc}(\overline{X}) = \overline{X}\).

(5) Since \(\Phi\) and \(\overline{X}\) are soft \(\alpha\)-open sets so, \(\text{sc}(\Phi) = \Phi\) and \(\text{sc}(\overline{X}) = \overline{X}\).

(6) We have \((G, E)\subset\text{sc}(G, E)(\overline{K}, E)\) and \((K, E)\subset\text{sc}(G, E)(\overline{K}, E)\).

Then by Proposition 17(3), \(\text{sc}(G, E)\subset\text{sc}(G, E)(\overline{K}, E)\) and \(\text{sc}(K, E)\subset\text{sc}(G, E)(\overline{K}, E)\) implies \(\text{sc}(G, E)(\overline{K}, E)\subset\text{sc}(G, E)(\overline{K}, E)\). That is, \(\text{sc}(G, E)(\overline{K}, E)\subset\text{sc}(G, E)(\overline{K}, E)\).
But $\sigma cl((G,E) \cup (K,E))$ is the smallest soft $\alpha$-closed set containing $(G,E) \cup (K,E)$.

Hence $\sigma cl((G,E) \cap (K,E)) \subseteq (G,E)$ is a soft $\alpha$-closed set containing $(G,E) \cup (K,E)$.

So, $\sigma cl((G,E) \cap (K,E)) = \sigma cl((G,E) \cup (K,E))$.

(7) Similar to (6).

(8) We have $((G,E) \cap (K,E)) \subseteq (G,E)$ and $((G,E) \cap (K,E)) \subseteq (G,E)$.

$\Rightarrow \sigma cl((G,E) \cap (K,E)) \subseteq \sigma cl((G,E) \cup (K,E)) \subseteq (G,E)$.

$\Rightarrow (G,E) \cap (K,E) \subseteq \sigma cl((G,E) \cup (K,E))$.

(9) Similar to (8).

(10) Since $(\sigma cl(G,E)) \in \mathcal{S}_{\alpha}(X,\tau,E)$ so by Proposition 16(1), $(\sigma cl(G,E)) = (\sigma cl(G,E))$.

(11) Since $(\sigma cl(G,E)) \in \mathcal{S}_{\alpha}(X,\tau,E)$ so by Proposition 16(2), $(\sigma cl(G,E)) = (\sigma cl(G,E))$.

(12) If $(G,E)$ is soft $\alpha$-closed set, then $\sigma cl((G,E)) \subseteq (G,E)$.

(13) $(G,E)$ is soft $\alpha$-open set.

Proof. (1)$\Rightarrow$(2) If $(G,E)$ is soft $\alpha$-closed set, then $\sigma cl((G,E)) \subseteq (G,E)$.

(2)$\Rightarrow$(3) $(\sigma cl((G,E)) \subseteq (G,E))$.

(3)$\Rightarrow$(4) It is obvious from Definition 9.

(4)$\Rightarrow$(1) It is obvious from Definition 9.

4. Soft $\alpha$-Continuity

Definition 20 (see [10]). Let $(X,E)$ and $(Y,K)$ be soft classes.

Let $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Then a mapping $f : (X,E) \rightarrow (Y,K)$ is defined as follows: for a soft set $(F,A)$ in $(X,E)$, $f((F,A)) = (f(F),A)$, where $f(F)$ is defined as $f(F) = \beta \in K$ for which $\beta \in f^{-1}(K,B)$.

Definition 21 (see [10]). Let $f : (X,E) \rightarrow (Y,K)$ be a mapping from a soft class $(X,E)$ to another soft class $(Y,K)$ and $(G,C)$ a soft set in soft class $(Y,K)$, where $C \subseteq K$. Let $u : X \rightarrow Y$ and $p : E \rightarrow K$ be mappings. Then $f^{-1}(G,C)$ is defined as $f^{-1}(G,C) = \alpha \in D \subseteq E$, where $f^{-1}(G,C)$ is called a soft inverse image of $(G,C)$.

Theorem 22 (see [10]). Let $f : (X,E) \rightarrow (Y,K)$, $u : X \rightarrow Y$, and $p : E \rightarrow K$ be mappings. Then for soft sets $(F,A)$ and $(G,B)$, one has:

(1) $f(\Phi) = \Phi$,

(2) $f(\overline{X}) = \overline{Y}$,

(3) $f((F,A) \cup (G,B)) = f((F,A) \cup (G,B))$ in general $f((\cup_{i=1}^{n}(F_i,A_i))) = \cup_{i=1}^{n}(F_i,A_i)$,

(4) $f((F,A) \cap (G,B)) = f((F,A) \cap (G,B))$ in general $f((\cap_{i=1}^{n}(F_i,A_i))) = \cap_{i=1}^{n}(F_i,A_i)$,

(5) $f((F,A) \cap (G,B))$, then $f((F,A) \cap (G,B))$.

(6) $f^{-1}(\Phi) = \Phi$,

(7) $f^{-1}(Y) = X$,

(8) $f^{-1}((F,A) \cup (G,B)) = f^{-1}((F,A) \cup (G,B))$ in general $f^{-1}((\cup_{i=1}^{n}(F_i,A_i))) = \cup_{i=1}^{n}(F_i,A_i)$,

(9) $f^{-1}((F,A) \cap (G,B)) = f^{-1}((F,A) \cap (G,B))$ in general $f^{-1}((\cap_{i=1}^{n}(F_i,A_i))) = \cap_{i=1}^{n}(F_i,A_i)$.

Definition 23. A mapping $f : (X,E) \rightarrow (Y,K)$ is said to be soft mapping if $(X,E)$ and $(Y,K)$ are soft topological spaces and $u : X \rightarrow Y$ and $p : E \rightarrow K$ are mappings.

Throughout the paper, the spaces $X$ and $Y$ (or $(X,E)$ and $(Y,K)$) stand for soft topological spaces assumed unless otherwise stated.

Definition 24. A soft mapping $f : X \rightarrow Y$ is said to be soft $\alpha$-continuous if the inverse image of each soft open subset of $Y$ is a soft $\alpha$-open set in $X$.

Example 25. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3\}, E = \{e_1, e_2, e_3\}, K = \{k_1, k_2, k_3\}, \tau = \{\Phi, X, (F,E)\}, \sigma = \{\{x_1\}, \{x_2\}, \{x_3\}\}$, and $(G,K) = \{(k_1, \{y_1, y_2\}, (k_2, \{y_1\}), (k_3, \{y_2\})\}$ and let $(X,E)$ and $(Y,K)$ be soft topological spaces.

Define $u : X \rightarrow Y$ and $p : E \rightarrow K$ as

$u(x_1) = y_1, u(x_2) = y_2, u(x_3) = y_3$,

$p(e_1) = \{k_1\}, p(e_2) = \{k_2\}, p(e_3) = \{k_3\}$.

Let $f_{pu} : (X,E) \rightarrow (Y,K)$ be a soft mapping. Then $(G,K)$ is a soft open in $Y$ and $f_{pu}^{-1}((G,K)) = (F,E)$ is a soft $\alpha$-open in $X$. Therefore, $f_{pu}$ is a soft $\alpha$-continuous function.

Theorem 26. Let $f : X \rightarrow Y$ be a mapping from a soft space $X$ to soft space $Y$. Then the following statements are true:

(1) $f$ is soft $\alpha$-continuous;

(2) for each soft singleton $(P,E)$ in $X$ and each soft open set $(O,K)$ in $Y$ and $f((P,E)) \subseteq (O,K)$, there exists a soft $\alpha$-open set $(U,E)$ in $X$ such that $(P,E) \subseteq (U,E)$ and $f((U,E)) \subseteq (O,K)$.

(3) the inverse image of each soft closed set in $Y$ is soft $\alpha$-closed in $X$;

(4) $f(cl(cl(int(A,E)))) \subseteq cl(f(A,E))$, for each soft set $(A,E)$ in $X$;

(5) $cl(cl(f^{-1}(B,K))) \subseteq f^{-1}(cl(B,K))$, for each soft set $(B,K)$ in $Y$.
Proof. (1)⇒(2) Since \((O, K)\) is soft open in \(Y\) and \(f((P, E)) \subseteq (O, K)\), so \((P, E) \subseteq f^{-1}((O, K))\) and \(f^{-1}((O, K))\) is a soft \(\alpha\)-open set in \(X\). Put \((U, E) = f^{-1}((O, K))\). Then \((P, E) \subseteq (U, E) \subseteq f((U, E))\). Hence \(f^{-1}((O, K)) \subseteq S-aos(X)\).

(2)⇒(1) Let \((O, K)\) be a soft open set in \(Y\) such that \((P, E) \subseteq f^{-1}((O, K))\) and thus there exists \((U, E) \subseteq S-aos(X)\) such that \((P, E) \subseteq f((U, E))\). Then \((P, E) \subseteq U, E \subseteq f^{-1}((O, K))\) = \(U, E \subseteq S_u(X)\). Hence \(f^{-1}((O, K)) \subseteq S-aos(X)\) and therefore \(f\) is soft \(\alpha\)-continuous.

(3)⇒(4) Let \((S, K)\) be a soft set in \(X\). Then \(cl_s(f((S, K)))\) is a soft closed set in \(X\), so that \(f^{-1}(cl_s((S, K)))\) is a soft \(\alpha\)-closed in \(X\).

- Therefore, we have \(f^{-1}(cl_s((S, K))) \subseteq cl_s(f^{-1}((S, K)))\).
- (4)⇒(5) Since \((B, K)\) is a soft set in \(X\), then \(f^{-1}((B, K))\) is a soft set in \(X\); thus by hypothesis we have \(cl_s(f^{-1}((B, K))) \subseteq cl_s((B, K))\).
- (5)⇒(1) Let \((O, K)\) be soft open in \(Y\). Let \((U, E) = O, K \subseteq f^{-1}((U, E))\). By (5) we have \(cl_s(f^{-1}((U, E))) \subseteq cl_s((U, E))\). Hence \(f^{-1}((O, K))\) is a soft \(\alpha\)-open set in \(X\); hence \(f\) is a soft \(\alpha\)-continuous function.

**Corollary 27.** Let \(f : X \rightarrow Y\) be a soft \(\alpha\)-continuous mapping. Then

1. \(cl_s(f((A, E))) \subseteq cl_s(f((A, E)))\), for each \((A, E) \in S-p比如X);\)
2. \(f^{-1}((B, K)) \subseteq cl_s((B, K))\), for each \((B, K) \in S-p比如Y).\)

**Proof.** Since for each \((A, E) \in S-p比如X), \(cl_s((A, E)) = cl_s(f((A, E)))\), therefore the proof follows directly from statements (4) and (5) of Theorem 26.

**Definition 28.** A soft mapping \(f : X \rightarrow Y\) is called soft precontinuous (resp., soft semicontinuous [7]) if the inverse image of each soft open set in \(Y\) is soft preopen (resp., soft semiopen) in \(X\).

**Remark 29.** It is clear that every soft \(\alpha\)-continuous map is soft semicontinuous and soft precontinuous. Every soft continuous map is soft \(\alpha\)-continuous. Thus we have implications as shown in Figure 2.

The converses of these implications are not true, which is clear from the following examples.

**Example 30.** Let \(X = \{x_1, x_2, x_3, x_4\}, Y = \{y_1, y_2, y_3, y_4\}, E = \{e_1, e_2, e_3, e_4\}, K = \{k_1, k_2, k_3\}\) and \((X, r, E)\) and let \((Y, v, K)\) be soft topological spaces.

Define \(u : X \rightarrow Y\) and \(p : E \rightarrow K\) as

\[
\begin{align*}
u(x_1) &= \{y_2\}, \quad u(x_2) = \{y_4\}, \quad u(x_3) = \{y_1\}, \quad u(x_4) = \{y_3\}, \\
p(e_1) &= \{k_2\}, \quad p(e_2) = \{k_1\}, \quad p(e_3) = \{k_3\}.
\end{align*}
\]

Let us consider the soft topology \(\tau\) given in Example 14; that is,

\[
\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}, \quad v = \{\Phi, Y, (F, K)\}, \quad \text{and} \quad (F, K) = \{(k_1, \{y_1, y_3, y_4\}, (k_2, \{y_1, y_2, y_3\}, (k_3, \{y_2, y_4\})\}
\]

and let mapping \(f_{up} : (X, r, E) \rightarrow (Y, v, K)\) be a soft mapping. Then \((F, K)\) is a soft open in \(Y\) and \(f_{up}((F, K))\) = \(\{(e_1, \{x_2, x_3\}, \{e_2, \{x_2, x_3, x_4\}, \{e_3, \{x_1, x_2, x_3\}\}\}\) is a soft \(\alpha\)-open but not soft open in \(X\). Therefore, \(f_{up}\) is a soft \(\alpha\)-continuous function but not soft continuous function.

**Example 31.** Let \(X = \{x_1, x_2, x_3, x_4\}, Y = \{y_1, y_2, y_3, y_4\}, E = \{e_1, e_2, e_3, e_4\}, K = \{k_1, k_2, k_3\}\) and \((X, r, E)\) and let \((Y, v, K)\) be soft topological spaces.

Define \(u : X \rightarrow Y\) and \(p : E \rightarrow K\) as

\[
\begin{align*}
u(x_1) &= \{y_2\}, \quad u(x_2) = \{y_4\}, \quad u(x_3) = \{y_1\}, \quad u(x_4) = \{y_3\}, \\
p(e_1) &= \{k_2\}, \quad p(e_2) = \{k_1\}, \quad p(e_3) = \{k_3\}.
\end{align*}
\]

Let us consider the soft topology \(\tau\) given in Example 14; that is,

\[
\tau = \{\Phi, \bar{X}, (F_1, E), (F_2, E), (F_3, E), \ldots, (F_{15}, E)\}, \quad v = \{\Phi, Y, (F, K)\}, \quad \text{and} \quad (F, K) = \{(k_1, \{y_1, y_3, y_4\}, (k_2, \{y_1, y_2, y_3\}, (k_3, \{y_2, y_4\})\}
\]

and let mapping \(f_{up} : (X, r, E) \rightarrow (Y, v, K)\) be a soft mapping. Then \((F, K)\) is a soft open in \(Y\) and \(f_{up}((F, K))\) = \(\{(e_1, \{x_2\}, \{e_2, \{x_1, x_2\}, \{e_3, \{x_1, x_3\}\}\}\) is a soft precontinuous function but not soft \(\alpha\)-continuous function.
Therefore, $f_{pa}$ is a soft semicontinuous function but not soft $\alpha$-continuous function.

**Theorem 33.** Let $(A, E) \in SPO(X)$ and $(B, E) \in S\alpha OS(X)$. Then $(A, E) \overline{\cap} (B, E) \in S\alpha OS(X)$.

**Proof.** Let $(A, E) \in SPO(X)$ and $(B, E) \in S\alpha OS(X)$. Then $(A, E) \overline{\cap} (B, E) = \overline{(A, E) \cap (B, E)}$.

5. **Soft $\alpha$-Open and Soft $\alpha$-Closed Mappings**

**Definition 37.** A soft mapping $f : X \to Y$ is called soft $\alpha$-open (resp., soft $\alpha$-closed) mapping if the image of each soft open (resp., soft closed) set in $X$ is a soft $\alpha$-open set (resp., soft $\alpha$-closed set) in $Y$.

**Definition 38.** A soft mapping $f : X \to Y$ is called soft pre-open (resp., soft semiopen [7]) if the image of each soft open set in $X$ is soft preopen (resp., soft semiopen) in $Y$.

Clearly a soft open map is soft $\alpha$-open and every soft $\alpha$-open map is soft preopen as well as soft $\alpha$-open. Similar implications hold for soft closed mappings.

**Theorem 39.** A soft mapping $f : X \to Y$ is soft $\alpha$-closed if and only if $\alpha cl(f((A, E))) \subseteq f(cl((A, E)))$ for every soft set $(A, E) \subseteq X$.

**Proof.** Let $f : X \to Y$ be a soft mapping. Then $f^{-1}(f((A, E))) \subseteq f^{-1}(cl((A, E)))$.

**Theorem 40.** A soft mapping $f : X \to Y$ is soft $\alpha$-open if and only if $\alpha cl(f((A, E))) \subseteq f(cl((A, E)))$ for every soft set $(A, E) \subseteq X$.

**Proof.** Let $f : X \to Y$ be a soft mapping. Then $f^{-1}(f((A, E))) \subseteq f^{-1}(cl((A, E)))$.

**Theorem 36.** A soft function $f : X \to Y$ is soft $\alpha$-continuous if and only if $f^{-1}(\alpha cl((H, K))) \subseteq \alpha int(f^{-1}((H, K)))$ for every soft set $(H, K) \subseteq Y$.

**Proof.** Let $f : X \to Y$ be soft $\alpha$-continuous. Now for any soft set $(G, E) \subseteq Y$, int$(f((G, E)))$ is a soft open set in $Y$; since $f$ is soft $\alpha$-continuous, then $f^{-1}(\alpha cl((G, E))) \subseteq f^{-1}(cl((G, E)))$.

Conversely, take a soft open set $(G, K) \subseteq Y$. Then $f^{-1}(\alpha cl((G, K))) \subseteq \alpha int(f^{-1}((G, K))) \Rightarrow f^{-1}((G, K)) \subseteq \alpha int(f^{-1}((G, K)))$.
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Corollary 42. If $f : X \to Y$ is a soft $\alpha$-open mapping, then

1. $f^{-1}(\text{cl}(\text{int}(\text{cl}(B,K)))) \subseteq \text{cl}(f^{-1}(\text{cl}(B,K)))$, for every soft set $(B, K)$ in $Y$;
2. $f^{-1}(\text{cl}(C,K)) \subseteq \text{cl}(f^{-1}(C,K))$, $(C, K) \in \text{SPO}(Y)$.

Proof. (1) $\text{cl}(f^{-1}(B,K))$ is a soft closed set in $X$, containing $f^{-1}(B,K)$, for a soft set $(B, E)$ in $Y$.

By Theorem 44, there exists a soft $\alpha$-closed set $(F, K)$ in $Y$, and $(B, K) \subseteq (F, K)$ such that $f^{-1}(F,K) \subseteq \text{cl}(f^{-1}(B,K))$.

Thus $f^{-1}(\text{cl}(\text{int}(\text{cl}(B,E)))) \subseteq \text{cl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(F,E))))$, $(B, K) \subseteq (B, K)$.

Therefore $f^{-1}(B,K)$ is a soft preopen set in $X$.

Theorem 43. If $f : X \to Y$ is a soft precontinuous and soft $\alpha$-open mapping, then $f^{-1}(\text{cl}(B,K)) \in \text{SPO}(X)$ for each $(B, K) \in \text{SPO}(Y)$.

Proof. We have $f^{-1}(\text{cl}(B,K)) \subseteq f^{-1}(\text{cl}(\text{int}(\text{cl}(B,K)))) \subseteq f^{-1}(\text{int}(\text{cl}(\text{cl}(B,K)))) \subseteq f^{-1}(\text{int}(\text{cl}(\text{cl}(B,K))))$.

Since $f$ is a soft $\alpha$-open mapping, by Corollary 42,

$f^{-1}(B, K) \subseteq \text{cl}(\text{int}(f^{-1}(\text{cl}(B,K))))$ and $f^{-1}(B, K) \subseteq \text{cl}(\text{int}(\text{int}(f^{-1}(B,K)))) = \text{int}(\text{int}(f^{-1}(B,K)))$.

Hence $f$ is a soft $\alpha$-continuous mapping.

Theorem 44. If $f : X \to Y$ is a soft precontinuous and soft semi-continuous, then $f$ is soft $\alpha$-continuous.

Proof. Let $(B, K)$ be any soft open set in $X$. Then $f^{-1}(B,K)$ is a soft preopen set as well as a soft semiopen set in $X$.

We have $f^{-1}(B, K) \subseteq \text{cl}(\text{int}(f^{-1}(B,K)))$ and $f^{-1}(B, K) \subseteq \text{cl}(\text{int}(\text{int}(f^{-1}(B,K))))$.

Hence $f$ is a soft $\alpha$-continuous mapping.

Theorem 45. If $f : X \to Y$ is a soft preopen mapping, then, for each soft set $(B, K)$ in $Y$, $f^{-1}(\text{int}(\text{cl}(B,K))) \subseteq \text{cl}(f^{-1}(B,K))$.

Proof. It follows immediately from Corollary 42.

Theorem 46. If $f : X \to Y$ is soft $\alpha$-continuous and soft preopen, then the inverse image of each soft $\alpha$-set is a soft $\alpha$-open set.

Proof. Let $(B, K)$ be any soft $\alpha$-open set in $Y$.

Then $f^{-1}(B,K) \subseteq f^{-1}(\text{int}(\text{cl}(B,K))) \subseteq \text{int}(\text{cl}(\text{int}(\text{cl}(B,K)))) \subseteq \text{int}(\text{cl}(f^{-1}(\text{cl}(B,K))))$.

By Theorem 44 we have $f^{-1}(B,E) \subseteq \text{int}(\text{cl}(\text{int}(B,E)))$.

Since $f$ is a soft $\alpha$-continuous mapping, by Theorem 22(5), $f^{-1}(B,K) \subseteq f^{-1}(\text{int}(\text{cl}(B,K)))$.

Hence $f^{-1}(B,K)$ is a soft $\alpha$-open set.

Corollary 47. If $f : X \to Y$ is soft $\alpha$-continuous and soft preopen mapping, then one has the following:

1. the inverse image of each soft $\alpha$-closed set is soft $\alpha$-closed.

Conflict of Interests

There is no conflict of interests regarding the publication of this paper.

References

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