Finite-Time Chaos Control of a Complex Permanent Magnet Synchronous Motor System

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1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in various industrial fields because of their simple structure, high efficiency, high power density, low manufacturing cost, and large torque to inertia ratio [1]. It is well known that the existing mathematical models of PMSMs are nonlinear, multivariable, and strongly coupled; therefore, these systems can exhibit complex behaviors, such as Hopf bifurcations, limit cycles, and even chaos [2]. Furthermore, the operation of PMSMs in industrial environment can be affected by many uncertain factors such as unknown system parameters, external load disturbance, friction force, and unmodeled uncertainties. These uncertain factors can seriously degrade the performance quality of PMSM systems [3–5]. So, it is indispensable to study methods of controlling or suppressing chaos in PMSM systems. Up till now, there are numerous methods to control chaos in PMSM systems [6–15]. In [9], a nonlinear feedback control method was proposed to control the chaos in a PMSM system. Loría [10] achieved both set-point and tracking output regulation of PMSM systems via a simple linear output feedback controller. Based on synchronization characteristics, a continuous feedback control method was proposed to eliminate chaotic oscillations in a PMSM system [11]. Hou [12] investigated the guaranteed cost control of chaos problem in a PMSM system via Takagi-Sugeno fuzzy method approach. Choi [13] proposed a simple adaptive controller design method for a chaotic PMSM system based on the sliding mode control theory.

The existing methods stabilize chaotic systems asymptotically; that is, the trajectories of chaotic systems converge to zero with infinite settling time. However, from the practical engineering point of view, it is more crucial to stabilize chaotic systems in a finite time. Therefore, it is important to consider the problem of finite-time stabilization of chaotic systems. Finite-time control is a very useful technique to achieve faster convergence speed in control systems. In addition, the finite-time control technique has demonstrated better robustness and disturbance rejection properties [14]. In [15], Wei and Zhang presented a nonlinear controller to achieve finite-time chaos control in a PMSM system based on the finite-time stability theory.

Since Fowler et al. [16] generalized the real Lorenz model to a complex one, the complex modeling of phenomena in nature and society has been intensively investigated. The complex systems appear in physics and engineering fields, such as detuned lasers, rotating fluids, disk dynamos, electronic circuits, and particle beam dynamics in high energy...
accelerators [17, 18]. Wang and Zhang proposed a complex PMSM system by modifying cross-coupled term in [19], for complex number voltage and complex number current exist widely in motor systems and it is easier to analyze motor systems with complex systems.

Motivated by the above discussion, in the present paper, we construct controllers to stabilize a complex PMSM system. Based on the finite-time stability theorem, two control strategies are proposed to realize chaos control in a finite time. Numerical simulation results show that the proposed controllers are very effective.

2. System Descriptions

A PMSM system in a field-oriented rotor can be described by the following equation [20]:
\[
\frac{d i_d}{dt} = \frac{1}{L_d} (u_d - R_i i_d + \omega L_q i_q),
\]
\[
\frac{d i_q}{dt} = \frac{1}{L_q} (u_q - R_i i_q - \omega L_d i_d + \psi),
\]
\[
\frac{d \omega}{dt} = \frac{1}{J} (n_p \psi r_i + n_p (L_d - L_q) i_d i_q - T_L - \beta \omega),
\]
where \( i_d \), \( i_q \), and \( \omega \) are the state variables, which represent currents and motor angular frequency, respectively; \( u_d \) and \( u_q \) are the direct-axis stator and quadrature-axis stator voltage components, respectively; \( J \) is the polar moment of inertia; \( T_L \) is the external load torque; \( \beta \) is the viscous damping coefficient; \( R_i \) is the stator winding resistance; \( L_d \) and \( L_q \) are the direct-axis stator inductors and quadrature-axis stator inductors, respectively; \( \psi \) is the permanent magnet flux, \( n_p \) is the number of pole-pairs, and the parameters \( L_d, L_q, J, T_L, \) \( R_i, \psi \), and \( \beta \) are all positive. When the air gap is even, and the motor has no load power and outage, then the dimensionless equations of a PMSM system can be modeled by
\[
\dot{z}_1 = a (z_2 - z_1),
\]
\[
\dot{z}_2 = b z_1 - z_2 - z_1 z_3,
\]
\[
\dot{z}_3 = z_1 z_2 + z_1 z_2 + \mu_5,
\]
where \( a \) and \( b \) are both positive parameters. If the current in the system (1) is plural and the variables \( z_1 \) and \( z_2 \) in the system (2) are complex numbers, by changing cross-coupled terms \( z_1 \) and \( z_2 \) to conjugate form, a complex PMSM system is constructed as follows [20]:
\[
\dot{z}_1 = a (z_2 - z_1),
\]
\[
\dot{z}_2 = b z_1 - z_2 - z_1 z_3,
\]
\[
\dot{z}_3 = \frac{1}{2} (\bar{z}_1 z_2 + z_1 \bar{z}_2) - z_3 + \mu_5,
\]
where \( z_1 = u_1 + i u_2 \) and \( z_2 = u_3 + i u_4 \) are complex variables, \( i = \sqrt{-1}; u_i \) \( (i = 1, 2, 3, 4) \) and \( z_3 = u_5 \) are real variables. \( \bar{z}_1 \) and \( \bar{z}_2 \) are the conjugates of \( z_1 \) and \( z_2 \), respectively, and \( a \) and \( b \) are positive parameters determining the chaotic behaviors and bifurcations of system (3). When the parameters satisfy \( 1 \leq a \leq 11, 10 \leq b \leq 20 \), there is one positive Lyapunov exponent, two zero Lyapunov exponents, and two negative Lyapunov exponents for system (4), which means system (3) is chaotic [20].

3. Basic Conception of Finite-Time Stability Theory

Finite-time stability means that the states of the dynamic system converge to a desired target in a finite time.

Definition 1 (see [21]). Consider the nonlinear dynamical system modeled by
\[
\dot{x} = f(x),
\]
where the state variable \( x \in \mathbb{R}^n \). If there exists a constant \( T > 0 \) \((T > 0 \) may depend on the initial state \( x(0) \)), such that
\[
\lim_{t \to T} \|x(t)\| = 0,
\]
and \( \|x(t)\| \neq 0 \), if \( t \geq T \), then system (1) is finite-time stable.

Lemma 2 (see [22]). Assume that a continuous, positive-definite function \( V(t) \) satisfies the following differential inequality:
\[
\dot{V}(t) \leq -\alpha V^\lambda(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0,
\]
where \( \alpha > 0 \) and \( 0 < \lambda < 1 \) are constants. Then, for any given \( t_0 \), \( V(t) \) satisfies the following inequality:
\[
V(t) \leq V^{1-\lambda}(t_0) - \alpha (1 - \lambda) (t - t_0), \quad t_0 \leq t \leq t_1,
\]
\[
V(t) \equiv 0, \quad \forall t \geq t_1,
\]
with \( t_1 \) given by
\[
t_1 = t_0 + \frac{V^{1-\lambda}(t_0)}{\alpha (1 - \lambda)}
\]

Lemma 3 (see [23]). For any real number \( \alpha_i, i = 1, 2, \ldots, k \) and \( 0 < r < 1 \), the following inequality holds:
\[
(\alpha_1 + \alpha_2 + \cdots + \alpha_k)^r \leq (\alpha_1)^r + (\alpha_2)^r + \cdots + (\alpha_k)^r.
\]

4. Finite-Time Chaos Control of a Complex PMSM System

In order to control chaotic oscillation in the complex PMSM system (3), we add controllers to system (3) and then the controlled system can be expressed by
\[
\dot{z}_1 = a (z_2 - z_1) + \mu_1 + i \mu_2,
\]
\[
\dot{z}_2 = b z_1 - z_2 - z_1 z_3 + \mu_3 + i \mu_4,
\]
\[
\dot{z}_3 = \frac{1}{2} (\bar{z}_1 z_2 + z_1 \bar{z}_2) - z_3 + \mu_5,
\]
where \( \mu_i \) \((i = 1, 2, \ldots, 5)\) are controllers to be designed to achieve finite-time control.

By separating the real and imaginary parts, we have the following real system:

\[
\begin{align*}
\dot{u}_1 &= a(u_3 - u_1) + \mu_1, \\
\dot{u}_2 &= a(u_4 - u_2) + \mu_2, \\
\dot{u}_3 &= bu_1 - u_3 - u_1u_5 + \mu_3, \\
\dot{u}_4 &= bu_2 - u_4 - u_2u_5 + \mu_4, \\
\dot{u}_5 &= u_1u_3 + u_2u_4 - u_5 + \mu_5.
\end{align*}
\]  

(11)

Next, we apply the finite-time stability theory to design controllers to globally stabilize the unstable equilibrium \(O(0,0,0,0,0)\) in a finite time. Two control strategies are proposed to fulfill this goal.

**Control Strategy 1**

**Theorem 4.** If the controllers are designed as

\[
\begin{align*}
\mu_1 &= -au_3 - u_1^k, \\
\mu_2 &= -au_4 - u_2^k, \\
\mu_3 &= -bu_1 - u_3^k, \\
\mu_4 &= -bu_2 - u_4^k, \\
\mu_5 &= -u_5^k,
\end{align*}
\]  

(12)

where \(k = q/p\) is a proper rational number, \(p\) and \(q\) are positive odd integers, and \(p > q\), the chaos in the complex PMSM system (10) will be controlled; that is, the complex PMSM system (10) will be asymptotically stabilized at the equilibrium \(O(0,0,0,0,0)\) in a finite time.

**Proof.** Construct the following Lyapunov function:

\[
V = \frac{1}{2} \left( u_1^2 + u_2^2 + u_3^2 + u_4^2 + u_5^2 \right).
\]  

(13)

By differentiating the function \(V\) along the trajectories of system (11), we have

\[
\dot{V} = u_1 \dot{u}_1 + u_2 \dot{u}_2 + u_3 \dot{u}_3 + u_4 \dot{u}_4 + u_5 \dot{u}_5 = u_1 \left[ a(u_3 - u_1) + \mu_1 \right] + u_2 \left[ a(u_4 - u_2) + \mu_2 \right] + u_3 \left( bu_1 - u_3 - u_1u_5 + \mu_3 \right) + u_4 \left( bu_2 - u_4 - u_2u_5 + \mu_4 \right) + u_5 \left( u_1u_3 + u_2u_4 - u_5 + \mu_5 \right).
\]  

(14)

Substituting the controllers (12) into the above equation yields

\[
\dot{V} = u_1 \left[ a(u_3 - u_1) - au_3 - u_1^k \right] + u_2 \left[ a(u_4 - u_2) - au_4 - u_2^k \right] + u_3 \left( bu_1 - u_3 - u_1u_5 - bu_1 - u_1^k \right) + u_4 \left( bu_2 - u_4 - u_2u_5 - bu_2 - u_2^k \right) + u_5 \left( u_1u_3 + u_2u_4 - u_5 - u_1^k \right) = -au_1^2 - au_2^2 - u_1^k + u_2^k - u_3^k - u_4^k - u_5^k \leq -\left( \frac{1}{2} \right)^{-(k+1)/2} \left[ \left( \frac{1}{2} u_1^2 \right)^{k+1} + \left( \frac{1}{2} u_2^2 \right)^{k+1} + \left( \frac{1}{2} u_3^2 \right)^{k+1} + \left( \frac{1}{2} u_4^2 \right)^{k+1} + \left( \frac{1}{2} u_5^2 \right)^{k+1} \right].
\]  

(15)

In light of Lemma 3, we have

\[
\dot{V} \leq -\left( \frac{1}{2} \right)^{-(k+1)/2} \left( \frac{1}{2} u_1^2 + \frac{1}{2} u_2^2 + \frac{1}{2} u_3^2 + \frac{1}{2} u_4^2 + \frac{1}{2} u_5^2 \right)^{k+1/2} = -\left( \frac{1}{2} \right)^{-(k+1)/2} (V)^{k+1/2}.
\]  

(16)

Then from Lemma 2, the controlled system (11) is finite-time stable. This implies that there exists a \(T > 0\) such that \(u_i(t) \equiv 0\) \((i = 1, 2, \ldots, 5)\) if \(t \geq T\). □

**Control Strategy 2**

**Theorem 5.** If the controllers are designed as

\[
\begin{align*}
\mu_1 &= -au_3 - u_1^k, \\
\mu_2 &= -au_4 - u_2^k, \\
\mu_3 &= -u_3^k, \\
\mu_4 &= -u_4^k, \\
\mu_5 &= -u_5^k,
\end{align*}
\]  

(17)

where \(k = q/p\) is a proper rational number, \(p\) and \(q\) are positive odd integers, and \(p > q\), the chaos in the complex PMSM system (10) will be controlled; that is, the complex PMSM system (10) will be asymptotically stabilized at the equilibrium \(O(0,0,0,0,0)\) in a finite time.
Proof. The design procedure is divided into two steps.

Step 1. Substituting the controllers \(\mu_1\) and \(\mu_2\) into the first two parts of system (11) yields

\[
\dot{u}_1 = -au_1 - u_1^k,
\]

\[
\dot{u}_2 = -au_2 - u_2^k.
\]

(18)

Choose the following candidate Lyapunov function:

\[
V_1 = \frac{1}{2} (u_1^2 + u_2^2).
\]

(19)

The derivative of \(V_1\) along the trajectory of (18) is

\[
\dot{V}_1 = u_1 \dot{u}_1 + u_2 \dot{u}_2
\]

\[
= u_1 (-au_1 - u_1^k) + u_2 (-au_2 - u_2^k)
\]

\[
\leq -u_1^{k+1} - u_2^{k+1}
\]

From Lemma 2, system (18) is finite-time stable. That means that there is a \(T_1 > 0\) such that \(u_1 \equiv 0\) and \(u_2 \equiv 0\), for any \(t \geq T_1\).

When \(t > T_1\), the last three parts of system (11) become

\[
\dot{u}_3 = -u_3 + \mu_3,
\]

\[
\dot{u}_4 = -u_4 + \mu_4,
\]

\[
\dot{u}_5 = -u_5 + \mu_5.
\]

(21)
Choose the following Lyapunov function for system (21):

$$V_2 = \frac{1}{2} (u_3^2 + u_4^2 + u_5^2).$$  \hspace{1cm} (22)$$

The derivative of $V_2$ along the trajectories of (21) is

$$\dot{V}_2 = u_3 \dot{u}_3 + u_4 \dot{u}_4 + u_5 \dot{u}_5$$

$$= u_3 (-u_3 + \mu_3) + u_4 (-u_4 + \mu_4)$$

$$+ u_5 (-u_5 + \mu_5).$$  \hspace{1cm} (23)$$

Substituting the controllers $\mu_3, \mu_4, \mu_5$ in (17) into the above equation yields

$$\dot{V}_2 = u_3 (-u_3 - u_3^k) + u_4 (-u_4 - u_4^k)$$

$$+ u_5 (-u_5 - u_5^k)$$

$$\leq -u_3^{k+1} - u_4^{k+1} - u_5^{k+1}.$$  \hspace{1cm} (24)$$

Then from Lemma 2, the states $u_3, u_4$, and $u_5$ will converge to zero at a finite time $T_2$. Then, after $T_2$, the states of system (11) will stay at zero; that is, the trajectories of the controlled system (11) converge to zero in a finite time. \hspace{1cm} $\square$

**Remark 6.** Strategy 1 is easier to implement than Strategy 2, but the controllers (12) obtained by Strategy 1 are more complicated than the controllers (17) obtained by Strategy 2.
2. The controllers (17) are obtained by two steps. Simpler as they are, but the stabilization time with controllers (17) will be longer than that with controllers (12).

5. Numerical Simulations

In this section, two numerical examples are presented to illustrate the theoretical analysis. In the following numerical simulations, the fourth-order Runge-Kutta method is employed with time step size 0.001. The system parameters are selected as \( a = 11 \) and \( b = 20 \), so that the complex PMSM system (3) exhibits chaotic behavior. The initial conditions for this system are given as \((z_1(0), z_2(0), z_3(0)) = (1 + 2i, 3 + 4i, 5)\); that is, \((u_1(0), u_2(0), u_3(0), u_4(0), u_5(0)) = (1, 2, 3, 4, 5)\).

Example 7. Consider Strategy 1 with the controllers (12). We choose \( k = 7/9 \). Figure 1 shows the result of the numerical simulation. From Figure 1, we can see that it takes only a very short time to stabilize the controlled system (11) at zero. So system (11) achieves chaos control in a finite time.

Example 8. Consider Strategy 2 with the controllers (17). We still choose \( k = 7/9 \). Figure 2 shows that the controlled system (11) achieves finite-time chaos control. From Figures 1 and 2, we can see that the stabilization time of the controlled system (11) in Figure 2 is longer than that in Figure 1.

6. Conclusions

Nowadays, the complex modeling of phenomena in nature and society has been the object of several investigations based on the methods originally developed in a physical context. In this paper, a complex PMSM system has been considered and the fast stabilization problem of this system has been investigated. Based on the finite-time stability theory, two kinds of simple and effective controllers for the complex PMSM system have been proposed to guarantee the global exponential stability of the controlled systems. During the past decades, the \( H_\infty \) control strategy [24–26] has been widely celebrated for its robustness in counteracting uncertainty perturbations and external disturbances. Consequently, we will investigate the finite-time \( H_\infty \) control problem of switched PMSM systems in our future work [27–29].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References


