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Editorial

Variational Analysis, Optimization, and Fixed Point Theory 2014

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In the last two decades, the theory of variational analysis including variational inequalities (VI) emerged as a rapidly growing area of research because of its applications in nonlinear analysis, optimization, economics, game theory, and so forth; see, for example, [1] and the references therein. In the recent past, many authors devoted their attention to studying the VI defined on the set of fixed points of a mapping, called hierarchical variational inequalities. Very recently, several iterative methods have been investigated to solve VI, hierarchical variational inequalities, and triple hierarchical variational inequalities. Since the origin of the VI, a tool has been used to study optimization problems. Hierarchical variational inequalities are used to study the bilevel mathematical programming problems. A triple level mathematical programming problem can be studied by using triple hierarchical variational inequalities. Several abstract results in nonlinear analysis are of special interest and applicability in the theory of variational problems, optimization, and mathematical economics. We point out here three of them (we refer to [2] for other methods or approaches).

Ekeland's variational principle provides the existence of an approximate minimizer of a bounded below and lower semicontinuous function. It is one of the most important results from nonlinear analysis and it has applications in different areas of mathematics and mathematical sciences, namely, fixed point theory, optimization, optimal control theory, game theory, nonlinear equations, dynamical systems, and so forth, for example, [3–8] and the references therein. During the last decade, it has been used to study the existence of solutions of equilibrium problems in the setting of metric spaces, for example, [3, 4] and the references therein.

Banach's contraction principle is remarkable in its simplicity, yet it is perhaps the most widely applied fixed point theory in all of the analyses. This is because the contractive condition on the mapping is simple and easy to verify and because it requires only completeness of the metric space. Although, the basic idea was known to others earlier, the principle first appeared in explicit form in Banach's 1922 thesis where it was used to establish the existence of a solution to an integral equation.

Caristi's fixed point theorem [9, 10] has found many applications in nonlinear analysis. It is shown, for example, that this theorem yields essentially all the known inwardness results of geometric fixed point theory in Banach spaces. Recall that inwardness conditions are the ones which assert that, in some sense, points from the domain are mapped toward the domain. This theorem is an amazing equivalent to Ekeland's variational principle. We refer to the recent monograph [11].

This special issue is concerned with the most recent development on the topic.

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