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Research Article

Optimal Control of Renewable Resources Based on the Effective Utilization Rate

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The effective utilization rate of exploited renewable resources affects the final total revenue and the further exploitation of renewable resources. Considering the effective utilization rate, we propose an optimal control model for the exploitation of the renewable resources in this study. Firstly, we can prove that the novel model is nonsingular compared with the singular basic model. Secondly, we solve the novel model and obtain the optimal solution by Bang-Bang theory. Furthermore, we can determine the optimal total resources and the maximal total revenue. Finally, a numerical example is provided to verify the obtained theoretical results.

1. Introduction

Renewable resources (such as fisheries resources) are considered to be "inexhaustible" at all times, but excessive exploitation will actually exhaust them. Thus, it is useful to study the reasonable exploitation of renewable resources and their effective utilization to obtain the maximum revenue.

Various optimum exploitation schemes for renewable resources have been studied in recent decades. In a pioneering study, Gordon [1, 2] proposed the Gordon-Schaefer bioeconomic model by introducing the concept of economic efficiency and cost management. Although this model included a large number of unrealistic assumptions, it exhibited a certain degree of concordance with the histories of empirical fisheries [3]. Subsequently, further economic control models of renewable resources [4–10] were proposed based on the work of Gordon. Clark and Munro [11] extended the static version of the fisheries economics model to nonautonomous and nonlinear cases. Indeed, the models proposed by Gordon and other researchers were all static until the 1980s when Clark [12] established a dynamic bioeconomic model based on the Gordon-Schaefer biological model. In addition to the classical model of Clark, another study [13] provided a nonlinear optimal control bioeconomic model that used the variation in the fishing effort rate as the control. In the 1990s, Defeo and Seijo [14] developed a bioeconomic model using a yield-mortality model. Later, based on the Beverton-Holt age structure model [15], Beverton and Holt [16] constructed a dynamic bioeconomic model that considered interactions among populations, which was not based on the Gordon-Schaefer bioeconomic model.

In recent years, various bioeconomic models of fisheries resources have also been proposed. Smith [17] established a Bayesian bioeconomic dynamic model by introducing the Bayesian statistical method. Domíguez-Torreiro and Surís-Regueiro [18] introduced game theory into a bioeconomic model of fishery resources and proposed a management strategy that addressed fishery issues. Das et al. [19] proposed a new bioeconomic model that combined a predator-prey ecological model with marine environmental factors. Other studies [20–22] introduced the fundamentals of control parameterization, which is a popular numerical technique for solving optimal control problems. At the same time, a switched autonomous system was proposed to formulate a fed-batch culture process where the switching instants bet ween the feed and batch processes were used as control variables, which is similar to the method proposed in the present study. The Food and Agriculture Organization also produced a series of bioeconomic models [23, 24] to provide a theoretical basis for policy to facilitate the sustainable use of fisheries resources.

However, all of these bioeconomic models and optimal exploitation schemes did not consider the effective utilization rate, especially for renewable resources. Indeed, if the

exploited renewable resources are not utilized fully, they will be wasted, but they may also pollute the environment. Given the impact on the expected revenue of the effective utilization rate for renewable resources, we introduce the concept of effective utilization into a renewable resources development model and we propose an optimal control model to ensure that it approximates the actual situation.

In Section 2, we introduce the basic bioeconomic model and propose an optimal control bioeconomic model based on the effective utilization rate. In Section 3, the singularity of this novel model is analyzed based on the maximal principle. Using the Bang-Bang theory, we obtain the optimal exploitation scheme and the optimal total renewable resources subject to the maximum total revenue. In Section 4, a numerical example is given that verifies the results.

2. Optimal Control Model Based on the Effective Utilization Rate

2.1. The Basic Model. It is well known that renewable resources have their own life cycles, even if they are not exploited or consumed. In general, we expect that the natural growth rate is greater than the natural mortality rate for renewable resources. However, the maximum amount of resources cannot exceed the environmental carrying capacity. In general, it is considered that the growth of resources satisfies the following logistic equation [4]:

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{N}\right), \tag{1}$$

where $x(t)$ denotes the resource biomass, N is the carrying capacity of the ecosystem, and r is the intrinsic growth rate of resources. If we let $u(t)$ be the exploitation amount, then model (1) becomes

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{N}\right) - u(t). \tag{2}$$

Let the total revenue from the exploited resources in unit time be expressed as

$$(p - c)\, u(t)\, e^{-\rho t}, \tag{3}$$

where p is the revenue per unit, c is the cost per unit, and ρ is the instantaneous social rate of discount. In this case, the objective function of the total revenue can be stated as follows:

$$J = \int_0^T (p - c)\, u(t)\, e^{-\rho t} dt. \tag{4}$$

Thus, the optimal control problem involving the basic model of renewable resources can be expressed as follows:

$$\begin{gathered} \max J = \int_0^T (p - c)\, u(t)\, e^{-\rho t} dt, \\ \dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{N}\right) - u(t), \\ 0 \le u(t) \le \overline{u}, \\ x(0) = x_0, \qquad x(T) = x_T. \end{gathered} \tag{5}$$

2.2. The Proposed Model Based on the Effective Utilization Rate. Let $s(t)$ be the effective utilization rate at time t; then $s(t)$ should satisfy the following three assumptions.

(A1) The effective utilization rate $s(t)$ will increase gradually with respect to t (i.e., with the development of technology); that is, $ds(t)/dt > 0$.

(A2) The increase in $s(t)$ will become more difficult after it reaches a certain level; that is, $s(t)$ must satisfy $d^2s(t)/dt^2 < 0$.

(A3) The ideal or the best utilization of resources is achieved completely; that is, $\lim_{t\to\infty} s(t) = 1$.

By these assumptions, we take $s(t) = 1 - ae^{-bt}$ $(a, b > 0)$ as our effective utilization rate, which satisfies the preceding assumptions. Furthermore, let the initial effective utilization rate be $s(0) = s_0$ and the ultimate effective utilization rate be $s(T) = s_T$. Then, parameter a, b in $s(t)$ can be obtained as follows:

$$\begin{gathered} a = 1 - s_0, \\ b = \frac{1}{T} \ln \frac{1 - s_0}{1 - s_T}. \end{gathered} \tag{6}$$

Thus, our improved objective function for optimal control associated with the effective utilization rate can be written as

$$J = \int_0^T (p - c)\, u(t)\, e^{-\rho t} \left(1 - ae^{-bt}\right) dt \tag{7}$$

and our proposed optimal control model can be established as follows:

$$\begin{gathered} \max J = \int_0^T (p - c)\, u(t)\, e^{-\rho t} \left(1 - ae^{-bt}\right) dt, \\ \dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{N}\right) - u(t), \\ 0 \le u(t) \le \overline{u}, \\ x(0) = x_0, \qquad x(T) = x_T, \end{gathered} \tag{8}$$

where J represents the management objective, $\overline{u}$ is the maximum amount of exploitation, and the initial and the terminal populations of renewable resources x_0, x_T are assumed to be known. In model (8), the meanings of N, r, p, c, ρ are similar to those in model (5).

3. Solutions to the Model

In this section, we first analyze the singularity of the proposed model (8) and we then apply the Bang-Bang approach to obtain its solution.

3.1. Solution of the Hamiltonian Formulation. The Hamiltonian formulation to the optimal control problem (8) can be written as follows:

$$\begin{aligned} H(x, \lambda, u, t) = {} & (p - c)\, u(t)\, e^{-\rho t} \left(1 - ae^{-bt}\right) \\ & + \lambda(t)\left(rx(t)\left(1 - \frac{x(t)}{N}\right) - u(t)\right). \end{aligned} \tag{9}$$

Then, we have

$$H(x,\lambda,u,t) = \left[(p-c)e^{-\rho t}\left(1-ae^{-bt}\right)-\lambda(t)\right]u(t) + \lambda(t)\left(rx(t)\left(1-\frac{x(t)}{N}\right)\right). \tag{10}$$

By the maximal principle, the Hamiltonian $H(x,\lambda,u,t)$ will obtain the maximal value with respect to $u(t)$ if the objective function obtains the maximal value. The linear relationship between the Hamiltonian function and control $u(t)$ changes the computation of the maximization problem (10) into a Bang-Bang optimal control problem, as follows:

$$u^*(t) = \begin{cases} 0 & \lambda(t) > (p-c)e^{-\rho t}\left(1-ae^{-bt}\right) \\ \overline{u} & \lambda(t) < (p-c)e^{-\rho t}\left(1-ae^{-bt}\right). \end{cases} \tag{11}$$

However, from the solution of (11), we cannot obtain any information about the optimal control $u^*(t)$ by $\lambda(t) = (p-c)e^{-\rho t}(1-ae^{-bt})$. Indeed, there are two possible solutions for $\lambda(t) = (p-c)e^{-\rho t}(1-ae^{-bt})$. Firstly, if there is only a countable time set $t_j = \{t_1, t_2, t_3, \ldots\} \in [t_0, t_f]$ that satisfies $\lambda(t) = (p-c)e^{-\rho t}(1-ae^{-bt})$, we can still use the Bang-Bang theory to solve this model. Secondly, if there is an interval $I \subset [0,T]$ that satisfies $\lambda(t) = (p-c)e^{-\rho t}(1-ae^{-bt})$ for $\forall t \in I$, the problem will become more complicated and difficult to solve. Thus, to obtain a better solution to the model, we first need to analyze whether such an interval I exists or not. Thus, we need to discuss the singularity [25, 26] of the optimal control model (8).

3.2. Singularity Analysis of the Model. Based on the preceding discussion, we can see that the singularity of model (8) will affect the choice of method used to solve this model. In this subsection, we discuss the singularity of model (8) by reduction to absurdity.

Assuming that the optimal control is a singularity, that is, there is an interval I for all $t \in I \subset [0,T]$ that satisfies

$$\lambda(t) = (p-c)e^{-\rho t}\left(1-ae^{-bt}\right), \tag{12}$$

the derivative of $\lambda(t)$ with respect to t can be expressed as

$$\dot{\lambda}(t) = (p-c)\left[-\rho e^{-\rho t}\left(1-ae^{-bt}\right)+abe^{-\rho t}e^{-bt}\right], \quad \forall t \in I; \tag{13}$$

by the costate equation of model (8)

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = -\lambda(t)\left(r-\frac{2rx(t)}{N}\right) \tag{14}$$

we obtain the following equation:

$$(p-c)\left[-\rho e^{-\rho t}\left(1-ae^{-bt}\right)+abe^{-bt}\right] = -\lambda(t)\left(r-\frac{2rx(t)}{N}\right), \quad \forall t \in I. \tag{15}$$

By substituting (12) into (15), we obtain

$$\left[-\rho\left(1-ae^{-bt}\right)+abe^{-bt}\right] = -\left(1-ae^{-bt}\right)\left(r-\frac{2rx(t)}{N}\right), \quad \forall t \in I. \tag{16}$$

Equation (16) admits the positive root given by

$$x(t) = \frac{N}{2r}\left(r-\rho+\frac{abe^{-bt}}{1-ae^{-bt}}\right) \tag{17}$$

which indicates that if we want (16) to hold at time $t \in I$, then $x(t)$ must satisfy (17). By taking the derivative of (17) with respect to t, we obtain

$$\dot{x}(t) = \frac{N}{2r}\frac{-ab^2e^{-bt}}{\left(1-ae^{-bt}\right)^2}. \tag{18}$$

In fact, (18) does not equal (2). In other words, $x(t)$ that satisfies the condition given above does not exist. Compared with the basic model (5), which is a singularity, our proposed model (8) involving the effective utilization rate is a normal model. Therefore, the optimal control of model (8) can be solved using the Bang-Bang approach.

3.3. Existence of the Switching Time t_s. In this subsection, we discuss the existence of the switching time t_s. First, we consider $\lambda(t) \le 0$, which indicates that the inequality

$$\lambda(t) < (p-c)e^{-\rho t}\left(1-ae^{-bt}\right) \tag{19}$$

will hold. In this case, the optimal strategy is $u^* \equiv \overline{u}$ by the Bang-Bang method.

However, this is not a reasonable method for exploiting renewable resources because the renewable resources will be destroyed. Therefore, we make the following assumption: $\lambda(t) > 0$. Then, (14) is obtained if the condition that $N > 2x(t)$ is satisfied and $\lambda(t)$ is monotonically decreasing for $t \in [0,T]$. In addition, we discuss the monotonicity of the following function:

$$f(t) = (p-c)e^{-\rho t}\left(1-ae^{-bt}\right). \tag{20}$$

By taking the derivative of (20) with respect to t

$$\frac{df(t)}{dt} = (p-c)\left[-\rho e^{-\rho t}\left(1-ae^{-bt}\right)+abe^{-\rho t}e^{-bt}\right]. \tag{21}$$

Then,

$$\frac{df(t)}{dt} = (p-c)e^{-\rho t}\left(\rho ae^{-bt}-\rho+abe^{-bt}\right). \tag{22}$$

It can be seen that $df(t)/dt > 0$ if and only if t satisfies the following condition:

$$t < -\frac{1}{b}\ln\frac{1}{a}\frac{\rho}{\rho+b}. \tag{23}$$

However, this condition cannot be obtained from (22). Thus, (20) is monotonically decreasing in time period $[0, T]$, which indicates that there will be an appropriate intersection between $f(t)$ and $\lambda(t)$; that is, the switching time t_s exists. Then we can obtain that

$$\begin{aligned} &\lambda(t) > (p-c)e^{-\rho t}\left(1-ae^{-bt}\right), \quad t \in [0, t_s), \\ &\lambda(t) < (p-c)e^{-\rho t}\left(1-ae^{-bt}\right), \quad t \in [t_s, T]. \end{aligned} \tag{24}$$

3.4. Solution to Model (8). After obtaining the switching time t_s and proving the nonsingularity of model (8), the optimal control strategy to (8) can be expressed by the Bang-Bang approach as follows:

$$u^*(t) = \begin{cases} 0 & 0 \le t < t_s \\ \overline{u} & t_s \le t \le T. \end{cases} \tag{25}$$

Next, we find the expressions of t_s and the optimal renewable resources function $x^*(t)$. First, if we let $0 \le t < t_s$ and $u^*(t) = 0$, we obtain

$$\begin{aligned} &\dot{x}(t) = rx(t)\left(1-\frac{x(t)}{N}\right), \\ &x(0) = x_0, \quad 0 \le t < t_s. \end{aligned} \tag{26}$$

By solving the ordinary differential equation (26), we obtain

$$x^*(t) = \frac{N}{1+Nc_1e^{-rt}}, \tag{27}$$

where c_1 is expressed as follows:

$$c_1 = \frac{1}{x_0} - \frac{1}{N}. \tag{28}$$

Second, if we let $t_s \le t \le T$ and $u^*(t) = \overline{u}$, we can obtain

$$\begin{aligned} &\dot{x}(t) = rx(t)\left(1-\frac{x(t)}{N}\right) - \overline{u}, \\ &x(T) = x_T \quad t_s \le t \le T. \end{aligned} \tag{29}$$

Unlike (26), we cannot solve the ordinary differential equation (29) directly using the Bernoulli equation. However, if there are suitable parameters p, q, the following formula holds; that is,

$$\frac{d[x(t)-p]}{dt} = -\frac{r}{N}[x(t)-p]^2 + q[x(t)-p]. \tag{30}$$

Then, we can solve (29) by the Bernoulli equation. In fact, by (29) and (30), we have

$$\begin{aligned} &\frac{2r}{N}p + q - r = 0, \\ &-\frac{r}{N}p^2 - qp + \overline{u} = 0; \end{aligned} \tag{31}$$

that is, the following quadratic equation holds:

$$\overline{u} + \frac{r}{N}p^2 - pr = 0. \tag{32}$$

Irrespective of whether (32) is positive or 0, that is,

$$\Delta = r^2 - 4\frac{r}{N}\overline{u} = r^2\left(1-4\frac{\overline{u}}{Nr}\right) \ge 0, \tag{33}$$

the suitable parameters p, q will be obtained, which indicates that (29) can be solved by the Bernoulli equation. It is known that N is the environmental carrying capacity, which is a very large number; thus the formula $1-4(\overline{u}/Nr) > 0$ holds. Then, we can obtain

$$\Delta \ge 0. \tag{34}$$

Therefore, we can solve (31), where the parameters p, q are given as follows:

$$\begin{aligned} p &= \frac{N}{2}\left(1-\sqrt{1-\frac{4\overline{u}}{Nr}}\right), \\ q &= r - r\left(1-\sqrt{1-\frac{4\overline{u}}{Nr}}\right). \end{aligned} \tag{35}$$

Moreover, we can obtain the solution of (29) by the Bernoulli equation:

$$(x(t)-p)^{-1} = \frac{r}{Nq} + c_2e^{-qt}. \tag{36}$$

That is,

$$x^*(t) = \frac{Nq}{r+Nqc_2e^{-qt}} + p, \tag{37}$$

where c_2 is expressed as

$$c_2 = \left(\frac{1}{x_T-p} - \frac{r}{Nq}\right)e^{qT}. \tag{38}$$

To summarize, in the interval $[0, T]$, the optimal renewable resources function can be expressed as

$$x^*(t) = \begin{cases} \dfrac{N}{1+Nc_1e^{-rt}} & 0 \le t < t_s \\ \dfrac{Nq}{r+Nqc_2e^{-qt}} + p & t_s \le t \le T. \end{cases} \tag{39}$$

The total amount of resources is continuous with respect to time t, even with $t = t_s$; that is,

$$\frac{1}{1+Nc_1e^{-rt_s}} = \frac{Nq}{r+Nqc_2e^{-qt_s}} + p. \tag{40}$$

We can determine the time t_s by (40). Thus, we obtain the optimal exploitation and the optimal level of total resources under the maximum revenue:

$$\begin{aligned} u^*(t) &= \begin{cases} 0 & 0 \le t < t_s \\ \overline{u} & t_s \le t \le T, \end{cases} \\ x^*(t) &= \begin{cases} \dfrac{N}{1+Nc_1e^{-rt}} & 0 \le t < t_s \\ \dfrac{Nq}{r+Nqc_2e^{-qt}} + p & t_s \le t \le T. \end{cases} \end{aligned} \tag{41}$$

Table 1: Initial parameters for model (8).

Parameter	Parameter value	Unit
N	100,000	Tons
r	4.4	/
p	38	Dollars per ton
c	20	Dollars per ton
ρ	0.1	/
$\overline{u}$	20000	Tons
T	1	Year
$x(T)$	70000	Tons
S_0	0.6	/
S_T	0.62	/

In summary, the optimal management strategy can be interpreted as follows. At the beginning of a time period $[0, t_s)$, we should not exploit the renewable resources to reach a certain number. At time $[t_s, T]$, we can exploit the renewable resources at the maximum capacity. Finally, we obtain the total benefit as follows:

$$J^* = \int_0^{t_s} (p-c) \times 0 \times e^{-\rho t} \left(1 - ae^{-bt}\right) dt + \int_{t_s}^{T} (p-c)\, \overline{u} e^{-\rho t} \left(1 - ae^{-bt}\right) dt. \tag{42}$$

4. Numerical Example

Based on the solution to the proposed model (8), we now present an example that verifies the model. The initial parameters for the fisheries resources are specified in Table 1, that is, the carrying capacity of the ecosystem N, the intrinsic growth rate of the renewable resources r, the revenue per unit p, the cost per unit c, the instantaneous social rate of discount ρ, the maximum amount of exploitation $\overline{u}$, the time interval $[0, T]$, the ultimate amount of renewable resources $x(T)$, the initial effective utilization rate s_0, and the ultimate effective utilization rate s_T. Thus, by (6), (28), (35), and (38), we can obtain the remaining parameters: $a = 0.4$, $b = 0.05$, $c_1 = 1.9\times10^{-4}$, $p = 4775$, $q = 3.9799$, and $c_2 = 2.288\times10^{-4}$, respectively. By substituting parameters N, r, c_1, c_2, p, q into (38), the switching time $t_s = 0.33999 \approx 0.34$ is obtained (note that in another example $t_s = 0.80487$ for $\overline{u} = 70000$ tons based on the same calculation used for $\overline{u} = 20000$). Finally, we obtain the expressions for optimal exploitation $u^*(t)$ and the optimal level $x^*(t)$ of total resources under the maximum revenue as follows (we omit the case where $\overline{u} = 70000$):

$$u^*(t) = \begin{cases} 0 & 0 \le t < 0.34 \\ 20000 & 0.34 \le t \le 1, \end{cases}$$

$$x^*(t) = \begin{cases} \dfrac{100000}{1 + 19e^{-4.4t}} & 0 \le t < 0.34 \\ \dfrac{90450}{1 + 20.6969e^{-3.9799t}} + 4775 & 0.34 \le t \le 1, \end{cases} \tag{43}$$

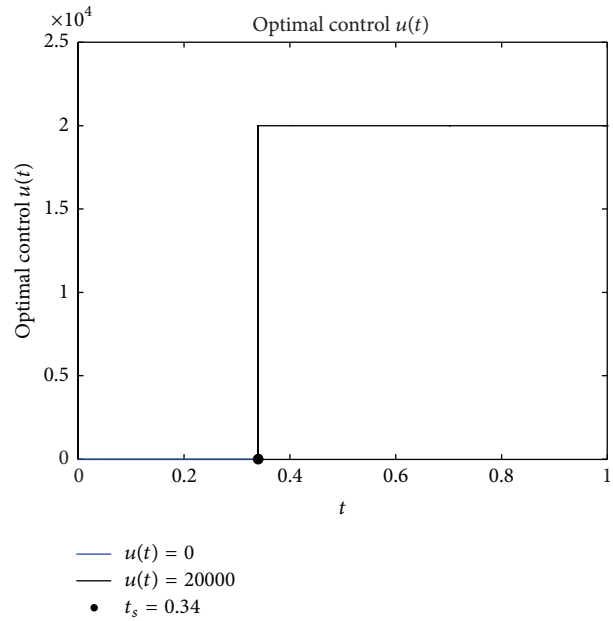

Figure 1: Optimal control $u(t) = 20000$.

which are shown in Figures 1 and 2, respectively. The total maximum revenue for the cases where $\overline{u} = 20000$ and $\overline{u} = 70000$, respectively, can also be obtained from (42) as follows:

$$\begin{aligned} J^*_{\overline{u}=20000} &= \int_0^{t_s} (p-c) \times 0 \times e^{-\rho t} \left(1 - ae^{-bt}\right) dt + \int_{t_s}^{T} (p-c)\, \overline{u} e^{-\rho t} \left(1 - ae^{-bt}\right) dt \\ &\approx 136254.85, \\ J^*_{\overline{u}=70000} &= \int_0^{t_s} (p-c) \times 0 \times e^{-\rho t} \left(1 - ae^{-bt}\right) dt + \int_{t_s}^{T} (p-c)\, \overline{u} e^{-\rho t} \left(1 - ae^{-bt}\right) dt \\ &\approx 138753.7. \end{aligned} \tag{44}$$

In Figure 2, for the case where $\overline{u} = 20000$, the switching time is $t_s = 0.34$, which indicates that we should allow the renewable resources to grow naturally for about 4 months of 1 year. Subsequently, the renewable resources can be exploited optimally where $\overline{u} = 20000$ and the final total maximum revenue is 136,254 dollars. Figure 2 also shows that the renewable resources retain approximately the same growth rate as before. However, when the optimal exploitation is $\overline{u} = 70000$ with the development of exploitation technology, we should delay the exploitation time; that is, $t_s = 0.80487$, which is about 10 months of 1 year. In this case, the total maximum revenue is about 138,753 dollars, which is greater than that for $\overline{u} = 20000$ and the renewable resources continue to grow, but at a slower growth rate.

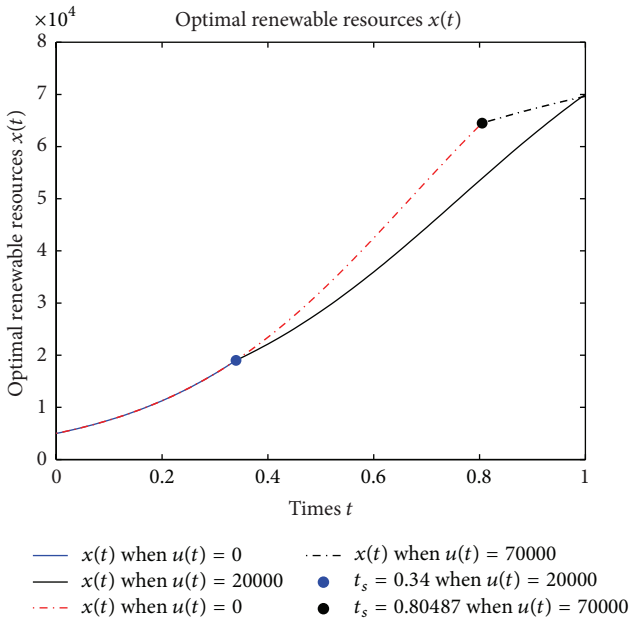

Figure 2: Optimal renewable resources $x(t)$.

5. Conclusion

In this study, we considered a model for the optimal control of renewable resources, which is affected by the effective utilization rate. We analyzed the singularity of the model. If the model is a singularity, the problem is difficult because the maximal principle cannot identify an optimal candidate explicitly. Indeed, we proved that our proposed optimal control model involving the effective utilization rate is normal and we solved the model with the aid of the maximal principle. Finally, we determined the optimal exploitation rate and the optimal level of the total renewable resources under the maximum revenue. This study may provide reference values to facilitate the development of renewable resources. However, it is clear that $u(t)$ is not related directly to $x(t)$ according to the present study. Thus, further research is required to address the case where $u(t)$ and $x(t)$ have a linear relationship.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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