Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2015, Article ID 414596, 11 pages
http://dx.doi.org/10.1155/2015/414596

Research Article

Outer Synchronization of Complex Networks with Nondelayed and Time-Varying Delayed Couplings via Pinning Control or Impulsive Control

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Received 9 July 2014; Accepted 15 August 2014

Academic Editor: Sakthivel Rathinasamy

The outer synchronization problem between two complex networks with nondelayed and time-varying delayed couplings via two different control schemes, namely, pinning control and impulsive control, is considered. Firstly, by applying pinning control to a fraction of the network nodes and using a suitable Lyapunov function, we obtain some new and useful synchronization criteria, which guarantee the outer synchronization between two complex networks. Secondly, impulsive control is added to the nodes of corresponding response network. Based on the generalized inequality about time-varying delayed different equation, the sufficient conditions for outer synchronization are derived. Finally, some examples are presented to demonstrate the effectiveness and feasibility of the results obtained in this paper.

1. Introduction

As an important and typical collective behavior of complex networks, synchronization has been studied extensively in various research fields since it has been observed widely in potential applications in many different areas such as population dynamic, power system, chemical process simulation, and automatic control [1–7]. In real worlds, a great number of complex networks cannot achieve synchronization by themselves. Therefore, many kinds of effective control methods, for instance, adaptive control [8, 9], pinning control [10–13], impulsive control [14], and so forth, have been developed to drive complex networks to reach synchronization. Some control schemes based on imposing the controllers on all the nodes in the networks, which is difficult to implement and impractical for some large scale networks. Considering these drawbacks, pinning control is proposed as a powerful technique because it is effective and relatively easily realized by controlling a small percentage of the nodes instead of all the nodes in whole network. As a result, some authors have devoted themselves to investigating different pinning control schemes for various complex dynamical networks. Chen et al. [10] investigated both specific and random pinning schemes for linearly and diffusively coupled networks. Xiang and Zhu proved that a single controller can pin a coupled complex network to homogeneous solutions in [11]. Compared with existing continuous or discrete pinning control schemes, impulsive control, another type of control methods, has attracted lots of researchers because of its potential advantage over general continuous control schemes [14–17]. For instance, Lu et al. in [15] proposed a new approach for analyzing pinning stability in a complex dynamical network via impulsive control. In [16], the authors addressed a directed dynamical network with impulsive coupling by a single impulsive controller.

It should be noted that the aforementioned works on network synchronization have mainly focused on analyzing the synchronization behavior within a network, which is regarded as "inner synchronization." In fact, there is another type of network synchronization for networks, which is known as "outer synchronization." Generally speaking, it is a type of synchronization between two or more coupled networks [14, 18–22], which means the corresponding nodes of coupled networks will achieve synchronization regardless

of synchronization of the inner networks. A typical example is the acquired immune deficiency syndrome, AIDS for brief, which originally outbroke among gorillas and afterwards was contagious to human beings unexpectedly, where cows groups and human beings could be regarded as two different networks in terms of network language. In recent years, some researchers began to put their interests on synchronization between two coupled complex networks. In [18], the authors first analyzed synchronization between two interacting populations of different phase oscillators. In [20], Sun et al. proved that outer synchronization can be asymptotically reached by using arbitrary coupling strength for two networks with balanced structure topology. Moreover, mixed outer synchronization between two complex dynamical networks with nonidentical nodes and output coupling is investigated via impulsive hybrid control in [23]. However, these research works exclude time-delay couplings, which cannot be ignored since they are ubiquitous in the real world. Considering these reasons, Zheng and Shao discussed the inner-outer synchronization between two complex networks with nondelayed and delayed coupling in [24] by applying the pinning control method. It should be mentioned that the time delay in [24] is fixed. For the varying time delay, in [22], Zheng investigates the problem of outer synchronization between two complex networks with the same topological structure and time-varying coupling delay. The authors of [25] investigated the outer synchronization problem of complex networks with multiple coupling time-varying delays. However, in order to fit with the real world, complex networks with nondelayed and delayed coupling when the delay time is varying should be considered. To the best of our knowledge, there are few (if any) results concerning outer synchronization of complex networks with nondelayed and coupling time-varying delays until now. Therefore, how to solve the outer synchronization problem for complex networks with nondelayed and coupling time-varying delays still remains largely challenging.

Motivated by the above considerations, this paper aims to analyze the outer synchronization between two coupled complex networks with nondelayed and time-varying delays by two different control schemes. The main contribution in our work is as follows: firstly, the configuration matrices are not required to be assumed symmetric which means the networks can be either undirected or directed in the networks. Secondly, we deal with the situation of the networks with nondelayed and time-varying delayed couplings, which is more practical and can describe the actual applications better. Thirdly, based on Lyapunov stability theorem and linear matrix inequality (LMI), we investigate the networks and derive some criteria for the outer synchronization by using some pinning control scheme. Finally, we discuss the networks where the nondelayed coupling matrix does not need to be irreducible, and some sufficient conditions are obtained for achieving the outer synchronization by imposing the impulsive controllers.

The outline of this paper is organized as follows. In Section 2, we present the complex dynamical network model and introduce some necessary definitions, lemmas, and assumptions. In Section 3, we present the main theoretical analysis for outer synchronization by two methods. First, we discuss outer mixed synchronization through pinning control, and, then, we discuss outer synchronization via impulsive control. Numerical simulations to show the validity of the obtained theoretical results are also presented in Section 4. Finally, this paper is concluded in Section 5.

2. Model Description and Preliminaries

In this paper, we consider two coupled complex dynamical networks consisting of linearly coupled N identical dynamical nodes, with each node being an n-dimensional dynamic system, respectively.

The drive coupled complex network is characterized by

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + c_1\sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t)\\ &\quad + c_2\sum_{j=1}^{N} b_{ij}\Gamma_2 x_j(t-\tau(t)), \quad i = 1,2,\ldots,N.\end{aligned} \tag{1}$$

The response complex dynamical network with controllers is given as follows:

$$\begin{aligned}\dot{y}_i(t) &= f(y_i(t)) + c_1\sum_{j=1}^{N} a_{ij}\Gamma_1 y_j(t)\\ &\quad + c_2\sum_{j=1}^{N} b_{ij}\Gamma_2 y_j(t-\tau(t)) + u_i(t), \quad i = 1,2,\ldots,N,\end{aligned} \tag{2}$$

where $x_i(t) = (x_i^1(t), x_i^2(t), \ldots, x_i^n(t))^T \in R^n$ is the drive state variables of the ith node and $y_i(t) = (y_i^1(t), y_i^2(t), \ldots, y_i^n(t))^T \in R^n$ is the response state variables of the ith node, $t \in [0, +\infty)$. The constants $c_1 > 0$ and $c_2 > 0$ denote the nondelayed and delayed coupling strength, respectively. The intrinsic function $f(\cdot) : \mathbf{R}^n \to \mathbf{R}^n$ $(i = 1,2,\ldots,N)$ is continuous, which describes the local dynamics of nodes i, and $\Gamma_1 = \mathrm{diag}(\gamma_1^1, \gamma_1^2, \ldots, \gamma_1^n)$ and $\Gamma_2 = \mathrm{diag}(\gamma_2^1, \gamma_2^2, \ldots, \gamma_2^n)$ are positive definite diagonal inner coupling matrices. $A = (a_{ij}) \in R^{N\times N}$ and $B = (b_{ij}) \in R^{N\times N}$ are the nondelayed and delayed weight configuration matrices, respectively. If there is a connection from the node i to the node j $(j \neq i)$, $a_{ij} \neq 0$; otherwise, $a_{ij} = 0$. So is the case with matrix B. Here, A and B are not required to be symmetric, which correspond to the direct network in the real world. $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, and $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$, $(i = 1,2,\ldots,N)$. The coupling time-varying delay $\tau(t)$ is a bounded and continuously differentiable function. Suppose that there exist positive constants α and τ satisfying $0 \le \dot{\tau}(t) \le \alpha < 1$ and $0 \le \tau(t) \le \tau$. $u_i(t)$ $(i = 1,2,\ldots,N)$ is the linear controller for i to design later.

Suppose $C([-\tau, 0], R^n)$ is the Banach space of continuous vector-valued functions mapping the interval $[-\tau, 0]$ into R^n with the norm $\|\phi\| = \sup_{-\tau<s<0}\|\phi(s)\|$. For the functional differential equation (1), its initial conditions are given by $x_i(t) = \phi_i(t) \in C([-\tau, 0], R^n)$. It is assumed that (1) has a

unique solution with respect to these initial conditions. For the functional differential equation (2), its initial conditions are given by $y_i(t) = \psi_i(t) \in C([-\tau, 0], R^n)$. And, at least, there exists a constant i $(i = 1, 2, \ldots, N)$ such that $\phi_i(t) \neq \psi_i(t)$ for $t \in [-\tau, 0]$. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm.

In the following, some preliminaries such as definitions, lemmas, and assumptions will be given, which would be used throughout the paper.

Definition 1. The drive networks (1) and the response networks (2) are said to be achieving complete outer synchronization, if, for any initial sates $\phi_i(t)$ and $\psi_i(t)$,

$$\lim_{t\to\infty} \|y_i(t, \psi_i) - x_i(t, \phi_i)\| = 0, \qquad i = 1, 2, \ldots, N. \tag{3}$$

Definition 2. Matrix $A = (a_{ij}) \in R^{N\times N}$ is said to belong to class $A1$, denoted as $A \in A1$, if

(1) $a_{ij} \geq 0, i \neq j, a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, i = 1, 2, \ldots, N,$

(2) A is irreducible.

If $A \in A1$ is symmetrical, then one says that A belongs to class $A2$, denoted as $A \in A2$.

Lemma 3 (see [25]). *If matrix $A \in A1$, then the following are valid.*

(1) *Real parts of the eigenvalues of A are negative except an eigenvalue 0 with multiplicity 1.*

(2) *A has right eigenvalues $(1, 1, \ldots, 1)^T$ corresponding to the eigenvalue 0.*

(3) *Let $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ be the left eigenvector of A corresponding to the eigenvalue 0 satisfying $\sum_{i=1}^{N} \xi_i = 1$, and then one can let $\xi_i > 0$ hold for all $i = 1, 2, \ldots, N$.*

Lemma 4 (see [15]). *The following linear matrix inequality (LMI):*

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix} < 0, \tag{4}$$

is equivalent to the following conditions:

$$\begin{aligned} &(1)\ S_{11} < 0, \qquad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0; \\ &(2)\ S_{22} < 0, \qquad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0, \end{aligned} \tag{5}$$

where S_{11} and S_{22} are symmetric matrices and S_{12} is a matrix with suited dimensions.

Lemma 5. *Let $u(t) : [t_0 - \tau, \infty) \to [0, \infty)$ satisfy the scalar impulsive differential inequality:*

$$\begin{aligned} \dot{u}(t) &\leq pu(t) + qu(t - \tau(t)), \quad t \neq t_k,\ t \geq t_0, \\ u(t_k) &\leq \alpha_k u(t_k^-), \quad t = t_k,\ k = 1, 2, \ldots, \\ u(t) &= \varphi(t), \quad t \in [t_0 - \tau, t_0], \end{aligned} \tag{6}$$

where $p, q > 0$, $\alpha_k > 0$, $u(t)$ is continuous at $t \neq t_k$, $t \geq t_0$, $u(t_k) = u(t_k^+) = \lim_{t\to t_k^+} u(t)$, $u(t_k^-) = \lim_{t\to t_k^-} u(t)$ exists, and $\varphi \in C([t_0 - \tau, t_0], R^+)$. $\tau(t)$ is a bounded and continuously differentiable function, and $0 \leq \dot{\tau}(t) \leq \alpha < 1$ and $0 \leq \tau(t) \leq \tau$. Then

$$u(t) \leq \left(\prod_{i=1}^{k} \alpha_i\right) e^{(p+q/(1-\alpha))(t-t_0+k\tau)} \left[\sup_{t_0-\tau\leq s\leq t_0} \varphi(s)\right] \tag{7}$$

for $t \in [t_k, t_{k+1})$.

Proof. For $t \in [t_k, t_{k+1})$, integrating both sides of inequality (6) from t_k to t, we can obtain

$$u(t) - u(t_k) \leq \int_{t_k}^{t} (pu(s) + qu(s - \tau(s)))\, ds. \tag{8}$$

It is easy to get

$$u(t) \leq u(t_k) + \int_{t_k-\tau}^{t} \left(p + \frac{q}{1-\alpha}\right) u(s)\, ds. \tag{9}$$

We will prove the conclusion of Lemma 5 by induction. From inequality (6), when $t \in [t_0, t_1)$, we can get

$$\begin{aligned} \dot{u}(t) &\leq pu(t) + q \sup_{t-\tau\leq s\leq\tau} u(s) \\ &\leq pu(t) + \frac{q}{1-\alpha} \sup_{t-\tau\leq s\leq\tau} u(s). \end{aligned} \tag{10}$$

By Lemma 3 in [26], for $t \in [t_0, t_1)$, we have

$$u(t) \leq \left(\sup_{t_0-\tau\leq s\leq t_0} \varphi(s)\right) e^{(p+q/(1-\alpha))(t-t_0)}. \tag{11}$$

This implies that the conclusion of Lemma 5 holds for $k = 0$.

Under the inductive assumption that the conclusion (6) holds for some $k > 0$, we will show that (6) still holds for $k+1$. For $t \in [t_{k+1}, t_{k+2})$, without loss of generality, we assume that there are l first class intermittent points, and then (9) can be rewritten as

$$\begin{aligned} u(t) \leq{}& u(t_{k+1}) + \int_{t_{k+1}-\tau}^{t_{k-l+1}} \left(p + \frac{q}{1-\alpha}\right) u(s)\, ds \\ &+ \sum_{i=1}^{l} \int_{t_{k-i+1}}^{t_{k-i+2}} \left(p + \frac{q}{1-\alpha}\right) u(s)\, ds \\ &+ \int_{t_{k+1}}^{t} \left(p + \frac{q}{1-\alpha}\right) u(s)\, ds. \end{aligned} \tag{12}$$

Using the same method as the proof of Lemma 1 in [13], we can get, for $t \in [t_{k+1}, t_{k+2})$,

$$u(t) \leq u(t_{k+1})\, e^{(p+q/(1-\alpha))(t-t_{k+1}+\tau)} \tag{13}$$

and by the inductive assumption and the second inequality of (6), we have

$$\begin{aligned} u(t) &\leq \alpha_{k+1} u(t_{k+1}^-)\, e^{(p+q/(1-\alpha))(t-t_{k+1}+\tau)} \\ &\leq \left(\prod_{i=1}^{k+1} \alpha_i\right) \left[\sup_{t_0-\tau\leq s\leq t_0} \varphi(s)\right] e^{(p+q/(1-\alpha))(t-t_0+(k+1)\tau)}. \end{aligned} \tag{14}$$

Hence, by induction, the conclusion of the lemma is attained for all $k \geq 0$. □

Assumption 6. Suppose there exists a positive constant L such that

$$\|f(y) - f(x)\| \leq L\|y - x\|, \quad i = 1, 2, \ldots, N, \tag{15}$$

for any $x, y \in R^n$, and the norm $\|x\| = \sqrt{x^T x}$.

Remark 7. In fact, there are many classical chaotic systems, such as Lorenz system, Chen system, Lü system, and Chua's circuit system, whose corresponding dynamical functions all satisfy the above assumption.

3. Outer Synchronization Analysis

3.1. Complete Outer Synchronization of Complex Networks by Pinning Control. In this subsection, we first investigate outer synchronization issue for two linear coupled complex networks with delay and nondelay by pinning control; here, the nondelay coupled matrix A is not assumed to be symmetric, but it has to be irreducible. Then, we introduce some notation employed throughout this paper.

The error vector is as follows:

$$\begin{aligned} e_i(t) &= y_i(t) - x_i(t), \\ e^k(t) &= \left(e_1^k(t), e_2^k(t), \ldots, e_N^k(t)\right)^T, \\ e^k(t - \tau(t)) &= \left(e_1^k(t - \tau(t)), e_2^k(t - \tau(t)), \ldots, e_N^k(t - \tau(t))\right)^T, \end{aligned} \tag{16}$$

and we design the negative feedback controllers corresponding to the response network (2) as

$$u_i(t) = \begin{cases} -c_1 d_i \Gamma_1 e_i(t), & 1 \leq i \leq l < N, \\ 0, & 1 + l \leq i \leq N, \end{cases} \tag{17}$$

where $d_i > 0$, $i = 1, 2, \ldots, l$, are the feedback control gains that can be suitably chosen. Without loss of generality, we can control the first l $(l < N)$ nodes.

Transforming the system yields the following error dynamical system:

$$\begin{aligned} \dot{e}_i(t) &= f(y_i(t)) - f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma_1 e_j(t) \\ &\quad + c_2 \sum_{j=1}^{N} b_{ij} \Gamma_2 e_j(t - \tau(t)) - c_1 d_i \Gamma_1 e_i(t), \\ &\qquad i = 1, 2, \ldots, l, \\ \dot{e}_i(t) &= f(y_i(t)) - f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma_1 e_j(t) \\ &\quad + c_2 \sum_{j=1}^{N} b_{ij} \Gamma_2 e_j(t - \tau(t)), \\ &\qquad i = 1 + l, \ldots, N. \end{aligned} \tag{18}$$

Then we have the following results.

Theorem 8. *Suppose that Assumption 6 holds and the drive network* (1) *with response network* (2) *can realize outer synchronization if the following condition is satisfied:*

$$L\Xi + \frac{1}{1-\alpha} I_N + c_1 \gamma_1^k \left(\Xi \widetilde{A}\right)^s + \frac{c_2^2 \left(\gamma_2^k\right)^2 \Xi B (\Xi B)^T}{4} < 0 \tag{19}$$

for $k = 1, 2, \ldots, n$, *where* $\Xi = \operatorname{diag}\{\xi_1, \ldots, \xi_N\}$, $\widetilde{A} = A - D$, $D = \operatorname{diag}\{d_1, \ldots, d_l, 0, \ldots, 0\} \in R^{N \times N}$, *and* $(\Xi \widetilde{A})^s = (\Xi \widetilde{A} + \widetilde{A}^T \Xi)/2$.

Proof. Construct a Lyapunov-Krasovskii function in the form

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} \xi_i e_i^T(t) e_i(t) + \frac{1}{1-\alpha} \sum_{k=1}^{n} \int_{t-\tau(t)}^{t} \left(e^k(s)\right)^T e^k(s)\, ds. \tag{20}$$

Then the derivative of $V(t)$ along the trajectories of (18) is

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} \xi_i e_i^T(t) \dot{e}_i(t) + \frac{1}{1-\alpha} \\ &\quad \times \sum_{k=1}^{n} \Big[\left(e^k(t)\right)^T e^k(t) - (1 - \dot{\tau}(t)) \\ &\qquad \times e^k(t - \tau(t))^T e^k(t - \tau(t)) \Big] \\ &\leq \sum_{i=1}^{N} \xi_i e_i^T(t) \Big[(f(y_i(t)) - f(x_i(t))) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma_1 e_j(t) \\ &\qquad + c_2 \sum_{j=1}^{N} b_{ij} \Gamma_2 e_j(t - \tau(t)) \Big] \\ &\quad - c_1 \sum_{i=1}^{l} \xi_i e_i^T(t) d_i \Gamma_1 e_i(t) \\ &\quad + \sum_{k=1}^{n} \Big[\frac{1}{1-\alpha} \left(e^k(t)\right)^T e^k(t) \\ &\qquad - \left(e^k(t - \tau(t))\right)^T e^k(t - \tau(t)) \Big]. \end{aligned} \tag{21}$$

According to Assumption 6, one can obtain

$$\sum_{i=1}^{N}\xi_i e_i^T(t)\left(f\left(y_i(t)\right)-f\left(x_i(t)\right)\right)\le L\sum_{i=1}^{N}\xi_i e_i^T(t)\,e_i(t)$$
$$=L\sum_{k=1}^{n}\left(e^k(t)\right)^T\Xi e^k(t). \tag{22}$$

Note that

$$c_1\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i e_i^T(t)\,a_{ij}\Gamma_1 e_j(t)=c_1\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\xi_i\left[\sum_{k=1}^{n}e_i^k(t)\,\gamma_1^k e_j^k(t)\right]$$
$$=c_1\sum_{k=1}^{n}\gamma_1^k\left[\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\xi_i e_i^k(t)\,e_j^k(t)\right]$$
$$=\sum_{k=1}^{n}\left(e^k(t)\right)^T c_1\gamma_1^k(\Xi A)^s e^k(t). \tag{23}$$

Then, it follows that

$$c_1\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i e_i^T(t)\,a_{ij}\Gamma e_j(t)-c_1\sum_{i=1}^{l}\xi_i d_i e_i^T(t)\,\Gamma_1 e_i(t)$$
$$=\sum_{k=1}^{n}\left(e^k(t)\right)^T c_1\gamma_1^k\Xi\,(A-D)\,e^k(t) \tag{24}$$
$$=\sum_{k=1}^{n}\left(e^k(t)\right)^T c_1\gamma_1^k\left(\Xi\widetilde{A}\right)^s e^k(t).$$

Using the same method as (23), we get

$$c_2\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i e_i^T(t)\,b_{ij}\Gamma e_j(t-\tau(t))$$
$$=c_2\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}\xi_i\left[\sum_{k=1}^{n}e_i^k(t)\,\gamma_1^k e_j^k(t-\tau(t))\right] \tag{25}$$
$$=\sum_{k=1}^{n}\left(e^k(t)\right)^T c_2\gamma_2^k\Xi B e^k(t-\tau(t)).$$

Substituting inequalities (22), (24), and (25) into (21), we obtain

$$\dot V(t)\le\sum_{k=1}^{n}\left[\left(e^k(t)\right)^T\left(L\Xi+\frac{1}{1-\alpha}I_N+c_1\gamma_1^k\left(\Xi\widetilde{A}\right)^s\right)e^k(t)\right.$$
$$+\left(e^k(t)\right)^T c_2\gamma_2^k\Xi B e^k(t-\tau(t))$$
$$\left.-\left(e^k(t-\tau(t))\right)^T I_N e^k(t-\tau(t))\right]$$
$$=\sum_{k=1}^{n}\left[\left(e^k(t)\right)^T,\left(e^k(t-\tau(t))\right)^T\right]\Xi_1\begin{pmatrix}e^k(t)\\ e^k(t-\tau(t))\end{pmatrix}, \tag{26}$$

where

$$\Xi_1=\begin{pmatrix}L\Xi+\dfrac{1}{1-\alpha}I_N+c_1\gamma_1^k\Xi\widetilde{A} & \dfrac{c_2\gamma_2^k\Xi B}{2}\\ \dfrac{c_2\gamma_2^k(\Xi B)^T}{2} & -I_N\end{pmatrix}, \tag{27}$$

from Lemma 4 and the condition of Theorem 8, is equating to $\Xi_1<0$. So we obtain $\dot V_i(t)\le 0$. This implies

$$\lim_{t\to\infty}\left\|e_i(t)\right\|=\lim_{t\to\infty}\left\|y_i(t)-x_i(t)\right\|=0,\qquad i=1,2,\ldots,N. \tag{28}$$

According to the Lyapunov stability theorem, the outer synchronization of network is achieved. This completes the proof. □

Remark 9. In a large body of the existing literature, the coupling matrix is supposed to be symmetric, which is not practical. However, in this paper, the configuration matrix need not be symmetric. This means that the networks (1) and (2) are directed networks. The complex network structure in this paper is general and this theorem can be applied to a great many complex networks in the real world.

When the coupling delay is absent in the complex network, that is, $B=0$, the drive coupled complex network is characterized by

$$\dot x_i(t)=f\left(x_i(t)\right)+c\sum_{j=1}^{N}a_{ij}\Gamma_1 x_j(t),\quad i=1,2,\ldots,N, \tag{29}$$

and the response coupled complex dynamical network is as follows:

$$\dot y_i(t)=f\left(y_i(t)\right)+c\sum_{j=1}^{N}a_{ij}\Gamma_1 y_j(t)+u_i(t),\qquad i=1,2,\ldots,N, \tag{30}$$

where $u_i(t)$ is the same as (17).

Corollary 10. *Under Assumption 6, if the condition*

$$L\Xi+c\gamma_1^k\left(\Xi\widetilde{A}\right)^s<0 \tag{31}$$

holds, then the drive systems (29) *with response system* (30) *can achieve outer synchronization.*

For the proof of Corollary 10, we choose the Lyapunov function as

$$V(t)=\sum_{i=1}^{N}\xi_i e_i^T(t)\,e_i(t). \tag{32}$$

Then, we can follow the proof of Theorem 8 to get the results above. We omitted details here.

Remark 11. The results in Corollary 10 correspond to Theorem 1 in [23] when $H=I$, which means the problem

investigated in [23] reaches complete outer synchronization. The results we obtained here are more general since the dynamical behaviors in our complex networks (29) and (30) are comprehensive. In some sense, the results in [23] could be seen as a special case of our results.

When the nondelay coupled matrix A is symmetric and irreducible, the left eigenvector of A corresponding to eigenvalue 0 is $\xi = (1/N, 1/N, \ldots, 1/N)$, and we could have the following results.

Corollary 12. *Under Assumption 6, if A is symmetric and irreducible and the condition*

$$\left(L + \frac{1}{1-\alpha}\right) I_N + c_1 \gamma_1^k \widetilde{A} + \frac{c_2^2 \left(\gamma_2^k\right)^2 BB^T}{4} < 0 \tag{33}$$

holds, then the networks achieve outer synchronization.

3.2. Outer Synchronization of Complex Networks by Impulsive Control. In this section, we discuss outer synchronization for two linear coupled complex networks with delay and nondelay by impulsive control. It should be mentioned that the coupling matrix A is irreducible in Theorem 8 when we use pinning control to get outer synchronization in the subsection above. However, in this subsection, the nondelay coupled matrix A is not necessarily assumed to be symmetric and irreducible when we use impulsive control strategy.

For achieving outer synchronization of two networks, the impulsive controllers can be designed as follows:

$$u_i(t) = \sum_{k=1}^{\infty} B_{ik} \left(y_i\left(t^-\right) - x_i\left(t^-\right)\right) \delta\left(t - t_k\right), \tag{34}$$

where the impulsive instant sequence $\{t_k\}_{k=1}^{+\infty}$ satisfies $t_{k-1} < t_k$ and $\lim_{k\to\infty} t_k = +\infty$. Matrix $B_{ik} \in R^{n\times n}$ is the states impulses gain matrix at the moment t_k. And $\delta(\cdot)$ is the Dirac impulsive function; that is,

$$\delta\left(t - t_k\right) = \begin{cases} 1, & t = t_k, \\ 0, & t \neq t_k. \end{cases} \tag{35}$$

The response network with impulsive control can be expressed as follows:

$$\begin{aligned} \dot{y}_i(t) &= f\left(y_i(t)\right) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma_1 y_j(t) \\ &\quad + c_2 \sum_{j=1}^{N} b_{ij} \Gamma_2 y_j\left(t - \tau(t)\right), \quad t \neq t_k,\ t \geq t_0, \\ \Delta y_i &= y_i\left(t_k^+\right) - y_i\left(t_k^-\right) \\ &= B_{ik}\left(y_i\left(t_k^-\right) - x_i\left(t_k^-\right)\right), \quad t = t_k,\ k = 1, 2, \ldots, \\ y_i(t) &= \psi_i(t), \quad t \in [-\tau, 0], \end{aligned} \tag{36}$$

where $y_i(t_k^+) = \lim_{t\to t_k^+} y_i(t)$ and $y_i(t_k^-) = \lim_{t\to t_k^-} y_i(t)$. Here, we suppose that $y_i(t_k) = y_i(t_k^+)$, which means that the solution $y(t)$ of (36) is right-hand continuous at the impulsive moment t_k.

Let $e_i(t) = y_i(t) - x_i(t)$, and then the error system can be written as follows:

$$\begin{aligned} \dot{e}_i(t) &= f\left(y_i(t)\right) - f\left(x_i(t)\right) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma_1 e_j(t) \\ &\quad + c_2 \sum_{j=1}^{N} b_{ij} \Gamma_2 e_j\left(t - \tau(t)\right), \quad t \neq t_k,\ t \geq t_0, \\ \Delta e_i &= e_i\left(t_k^+\right) - e_i\left(t_k^-\right) = B_{ik} e_i\left(t_k^-\right), \quad t = t_k,\ k = 1, 2, \ldots, \\ e_i(t) &= \psi_i(t) - \phi_i(t), \quad t \in [-\tau, 0], \end{aligned} \tag{37}$$

where the goal in this section is to set the control gains matrix B_{ik} and the impulsive distance $\Delta t_k = t_k - t_{k-1}$, $(k = 1, 2, \ldots)$ such that the error $\|e_i(t)\|$ converges to 0, which implies the impulsive control synchronization of (37) is completed for arbitrary initial conditions.

Theorem 13. *If Assumption 6 is satisfied, the drive system* (1) *and the response system* (2) *can achieve outer synchronization under the impulsive controller* (34) *if there exists a positive constant η, such that*

$$\left(\beta + \frac{c_2}{1-\alpha}\right)\left(1 + \frac{\tau}{T_{\min}}\right) + \frac{2\ln\left|\sigma_k\right|}{T_{\max}} < -\eta \tag{38}$$

holds, where

$$\beta = 2L + \lambda_m\left(2c_1\left(A \otimes \Gamma_1\right)^s + c_2\left(B \otimes \Gamma_2\right)\left(B \otimes \Gamma_2\right)^T\right), \tag{39}$$

and $\sigma_k = \max_{1\leq i\leq N}\|I_n + B_{ik}\|$, $k \in Z^+$, $T_{\min} = \min\{t_k - t_{k-1} \mid k \in Z^+\}$, $T_{\max} = \max\{t_k - t_{k-1} \mid k \in Z^+\}$, and $\lambda_m(\cdot)$ is the largest eigenvalue of a matrix.

Proof. Let $e(t) = \left(e_1^T(t), e_2^T(t), \ldots, e_N^T(t)\right)^T$. Choose the following Lyapunov function:

$$V(t) = e^T(t)\, e(t). \tag{40}$$

For $t \neq t_k$, $k \in N$, taking the time derivative of the Lyapunov function (40) along the trajectories of (37), we have

$$\begin{aligned} \dot{V}(t) &= 2\sum_{i=1}^{N} e_i^T(t)\, \dot{e}_i(t) \\ &= 2\sum_{i=1}^{N} \Bigg[e_i^T(t)\left(f\left(y_i(t)\right) - f\left(x_i(t)\right)\right) \\ &\qquad + c_1 \sum_{j=1}^{N} a_{ij} e_i^T(t)\, \Gamma_1 e_j(t) \\ &\qquad + c_2 \sum_{j=1}^{N} b_{ij} e_i^T(t)\, \Gamma_2 e_j\left(t - \tau(t)\right) \Bigg]. \end{aligned} \tag{41}$$

From Assumption 6, one can obtain

$$2\sum_{i=1}^{N} e_i^T(t)\left(f\left(y_i(t)\right)-f\left(x_i(t)\right)\right) \le 2\sum_{i=1}^{N} L e_i^T(t)\, e_i(t) = 2LV(t). \tag{42}$$

Notice that

$$2c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)\, a_{ij}\Gamma_1 e_j(t) = 2c_1 e^T(t)\left(A\otimes\Gamma_1\right)e(t) \le 2c_1 e^T(t)\left(A\otimes\Gamma_1\right)^s e(t). \tag{43}$$

The third term in (41) can be rewritten as

$$\begin{aligned} &2c_2\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij} e_i^T(t)\,\Gamma_2 e_j(t-\tau(t)) \\ &\quad = 2c_2 e^T(t)\left(B\otimes\Gamma_2\right)e(t-\tau(t)) \\ &\quad \le c_2\left[e^T(t)\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T e(t) \right. \\ &\qquad \left. + e^T(t-\tau(t))\, e(t-\tau(t))\right]. \end{aligned} \tag{44}$$

Combining (43) and (44), we have

$$\begin{aligned} &2\sum_{i=1}^{N}\sum_{j=1}^{N}\left[c_1 e_i^T(t)\, a_{ij}\Gamma_1 e_j(t) + b_{ij} e_i^T(t)\,\Gamma_2 e_j(t-\tau(t))\right] \\ &\quad = e^T(t)\left(\left(2c_1 A\otimes\Gamma_1\right) + c_2\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T\right)e(t) \\ &\qquad + c_2 V(t-\tau(t)) \\ &\quad \le \lambda_m\left(2c_1\left(A\otimes\Gamma_1\right)^s + c_2\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T\right)V(t) \\ &\qquad + c_2 V(t-\tau(t)). \end{aligned} \tag{45}$$

Referring to the inequalities (42) and (45), for $t \ne t_k$, $k \in Z^+$, it follows from (41) that

$$\begin{aligned} \dot V(t) &\le \left[2L \right. \\ &\qquad \left. + \lambda_m\left(2c_1\left(A\otimes\Gamma_1\right)^s + c_2\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T\right)\right]V(t) \\ &\qquad + c_2 V(t-\tau(t)) \\ &= \beta V(t) + c_2 V(t-\tau(t)), \end{aligned} \tag{46}$$

where

$$\beta = 2L + \lambda_m\left(2c_1\left(A\otimes\Gamma_1\right)^s + c_2\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T\right). \tag{47}$$

When $t = t_k$, $k \in Z^+$, one has

$$\begin{aligned} V(t_k) &= \sum_{i=1}^{N} e_i^T(t_k^+)\, e_i(t_k^+) \\ &= \sum_{i=1}^{N} e_i^T(t_k^-)\left(I_n + B_{ik}\right)^T\left(I_n + B_{ik}\right)e_i(t_k^-) \\ &\le \max_{1\le i\le N}\left\|I_n + B_{ik}\right\|^2 \sum_{i=1}^{N} e_i^T(t_k^-)\, e_i(t_k^-) \\ &= \sigma_k^2 V(t_k^-), \end{aligned} \tag{48}$$

where $\sigma_k = \max_{1\le i\le N}\|I_n + B_{ik}\|$.

Thus, employing Lemma 5, from (45) and (46), for $t \in [t_k, t_{k+1})$, we have

$$V(t) \le \left[\sup_{t_0-\tau\le s\le t_0} V(s)\right]\left(\prod_{i=1}^{k}\sigma_k^2\right)e^{(\beta+c_2/(1-\alpha))(t-t_0+k\tau)}. \tag{49}$$

Let $T_{\min} = \min\{t_k - t_{k-1} \mid k \in Z^+\}$ and $T_{\max} = \max\{t_k - t_{k-1} \mid k \in Z^+\}$, and then

$$\begin{aligned} V(t) &\le \left[\sup_{t_0-\tau\le s\le t_0} V(s)\right] e^{(\beta+c_2/(1-\alpha))(t-t_0+k\tau)+2k\ln|\sigma_k|} \\ &\le \left[\sup_{t_0-\tau\le s\le t_0} V(s)\right] e^{[(\beta+c_2/(1-\alpha))(1+\tau/T_{\min})+2(\ln|\sigma_k|/T_{\max})](t-t_0)}. \end{aligned} \tag{50}$$

From condition (38) of Theorem 13, we get

$$V(t) \le \left[\sup_{t_0-\tau\le s\le t_0} V(s)\right] e^{-\eta(t-t_0)}. \tag{51}$$

Thus, we can obtain that

$$\|e_i(t)\| \le \left[\sup_{t_0-\tau\le s\le t_0} V(s)\right]^{1/2} e^{-(\eta/2)(t-t_0)}, \quad i = 1, 2, \ldots, N. \tag{52}$$

This means the impulsive outer synchronization between complex network (1) and network (2) is realized. Thus, we complete the proof work. □

When the nondelayed coupling matrix $A = 0$,

$$\begin{aligned} &\dot x_i(t) = f\left(x_i(t)\right) + c\sum_{j=1}^{N} b_{ij}\Gamma_2 y_j(t-\tau(t)), \\ &\qquad i = 1, 2, \ldots, N, \\ &\dot y_i(t) = f\left(y_i(t)\right) + c\sum_{j=1}^{N} b_{ij}\Gamma_2 y_j(t-\tau(t)), \\ &\qquad t \ne t_k,\ t \ge t_0, \\ &\Delta y_i = y_i(t_k^+) - y_i(t_k^-) = B_{ik}\left(y_i(t_k^-) - x_i(t_k^-)\right), \\ &\qquad t = t_k,\ k = 1, 2, \ldots, \\ &y_i(t) = \psi_i(t), \quad t \in [-\tau, 0]. \end{aligned} \tag{53}$$

Corollary 14. *Suppose Assumption 6 is satisfied, and the network* (53) *can achieve outer synchronization if there exists a positive constant η, such that*

$$\left(\beta + \frac{c}{1-\alpha}\right)\left(1 + \frac{\tau}{T_{\min}}\right) + \frac{2\ln\left|\sigma_k\right|}{T_{\max}} < -\eta \tag{54}$$

holds, where

$$\beta = 2L + \lambda_m\left(c\left(B\otimes\Gamma_2\right)\left(B\otimes\Gamma_2\right)^T\right), \tag{55}$$

and $\sigma_k = \max_{1\le i\le N}\|I_n + B_{ik}\|$, $T_{\min} = \min\{t_k - t_{k-1} \mid k \in Z^+\}$, $T_{\max} = \max\{t_k - t_{k-1} \mid k \in Z^+\}$, *and* $\lambda_m(\cdot)$ *stands for the largest eigenvalue of a matrix.*

Remark 15. Mentions should be made on Theorem 2 in [23]. In fact, (13) in [23] corresponds to (53) in this paper if we set $H = I_n$ in [23]. Both negative feedback controllers and impulsive controllers are used in [23] to make the complex network (53) to achieve outer synchronization. Compared with [23], only impulsive controllers are made use of. As a result, the criteria we presented in this paper are easier than those in [23].

4. Numerical Simulations

In this section, we give numerical simulation to verify and demonstrate the effectiveness of the proposed method. In order to verify our results, we consider the driving complex network (1) as

$$\begin{aligned}\dot{x}_i(t) &= f\left(x_i(t)\right) + c_1\sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t)\\ &\quad + c_2\sum_{j=1}^{N} b_{ij}\Gamma_2 x_j\left(t-\tau(t)\right), \quad i = 1,2,3,4.\end{aligned} \tag{56}$$

Example 16. We take the Lorenz system as the local node dynamics, which is shown in Figure 1. Its dynamical equations in a dimensionless form are as follows:

$$\begin{aligned}\dot{x}_1 &= 10\left(x_2 - x_1\right),\\ \dot{x}_2 &= 28x_1 - x_2 - x_1x_3,\\ \dot{x}_3 &= x_1x_2 - \frac{8}{3}x_3,\end{aligned} \tag{57}$$

where $(x_1, x_2, x_3)^T \in R^3$ is the state variables group of unified chaotic system. The initial condition is randomly chosen. We choose $\tau(t) = 0.01 - 0.01e^{-t}$, $\alpha = 0.01$, $c_1 = 40$, $c_2 = 0.01$, $\Gamma_1 = \mathrm{diag}\{5,5,5\}$, and $\Gamma_2 = \mathrm{diag}\{0.2, 0.2, 0.2\}$. Similar to the verification in [23] with $L = 150$, clearly, Assumption 6 is verified. Choosing the asymmetric coupling configuration matrices

$$\begin{aligned}A &= \begin{pmatrix}-2 & 1 & 0 & 1\\ 1 & -3 & 2 & 0\\ 0 & 3 & -4 & 1\\ 2 & 0 & 0 & -2\end{pmatrix},\\ B &= \begin{pmatrix}-2 & 1 & 1 & 0\\ 0 & -2 & 1 & 1\\ 1 & 0 & -2 & 1\\ 1 & 1 & 0 & -2\end{pmatrix}\end{aligned} \tag{58}$$

by calculation, the left eigenvector of A corresponding to the eigenvalue 0 is $\xi = (3/8, 2/8, 1/8, 2/8)^T$. We control the second node of the system with coupling strength 52 and the third node 10 with coupling strength 48 by pinning control, and then all the conditions in Theorem 8 are satisfied, so the asymmetric coupled network (21) can achieve outer synchronization. The simulation results are given in Figure 2, and we can observe that outer synchronization state is realized.

Example 17. Now, let us consider the outer exponential synchronization of complex networks. Again, the network model is the same as Example 16. Hence $\lambda_m(2c_1(A\otimes\Gamma_1)^s + c_2(B\otimes\Gamma_2)(B\otimes\Gamma_2)^T) = 6.2435$, $T_{\max} = T_{\min} = 0.01$, let

$$B_{ik} = \begin{pmatrix}-1.04 & 0 & 0\\ 0 & -1.04 & 0\\ 0 & 0 & -1.04\end{pmatrix}, \tag{59}$$

and then $\sigma = 0.04$. Then all the conditions in Theorem 13 are satisfied, and $\eta = 31.0662$, so the asymmetric coupled network (21) can achieve outer synchronization. The simulation results are given in Figure 3. It can be seen clearly from Figure 3 that outer synchronization state is realized.

5. Conclusion

In this paper, we considered the outer synchronization between two complex networks with time nondelayed and time-varying delayed couplings under different control laws. First of all, we handled the outer synchronization by pinning control and derived some sufficient criteria based on Lyapunov stability theorem and LMI. Particularly, we obtain two corollaries, which are single nondelay coupled networks cases, and the coupling matrixes A and B are symmetric and irreducible. On the other side, we discuss the outer synchronization between two networks by impulsive control. Some sufficient conditions are derived by imposing the impulsive controllers to the nodes. In addition, we also present some corollaries. What is more, numerical simulations for two coupled complex networks which are composed of unified chaotic systems are given to demonstrate the effectiveness and feasibility of the schemes.

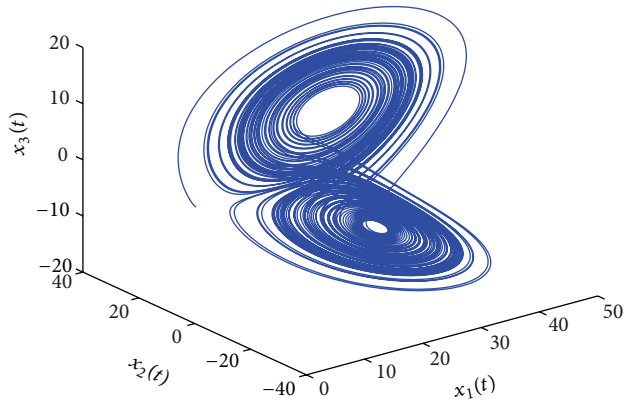

Figure 1: The attractor of the Lorenz system.

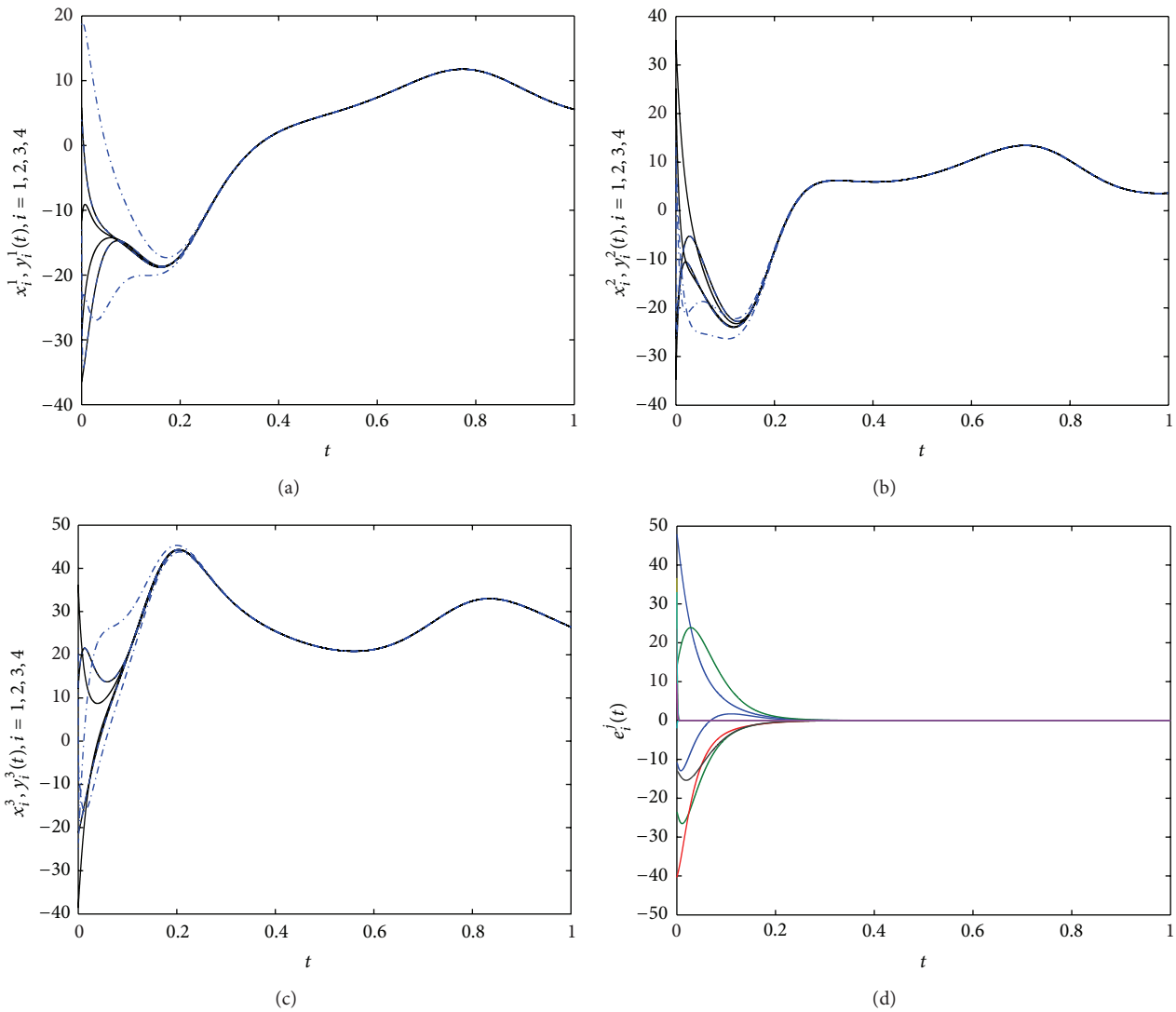

Figure 2: Time evolution between node x_i^k and node y_i^k, $i = 1, 2, 3, 4$, $k = 1, 2, 3$, and error evolution between drive and response networks under pinning control.

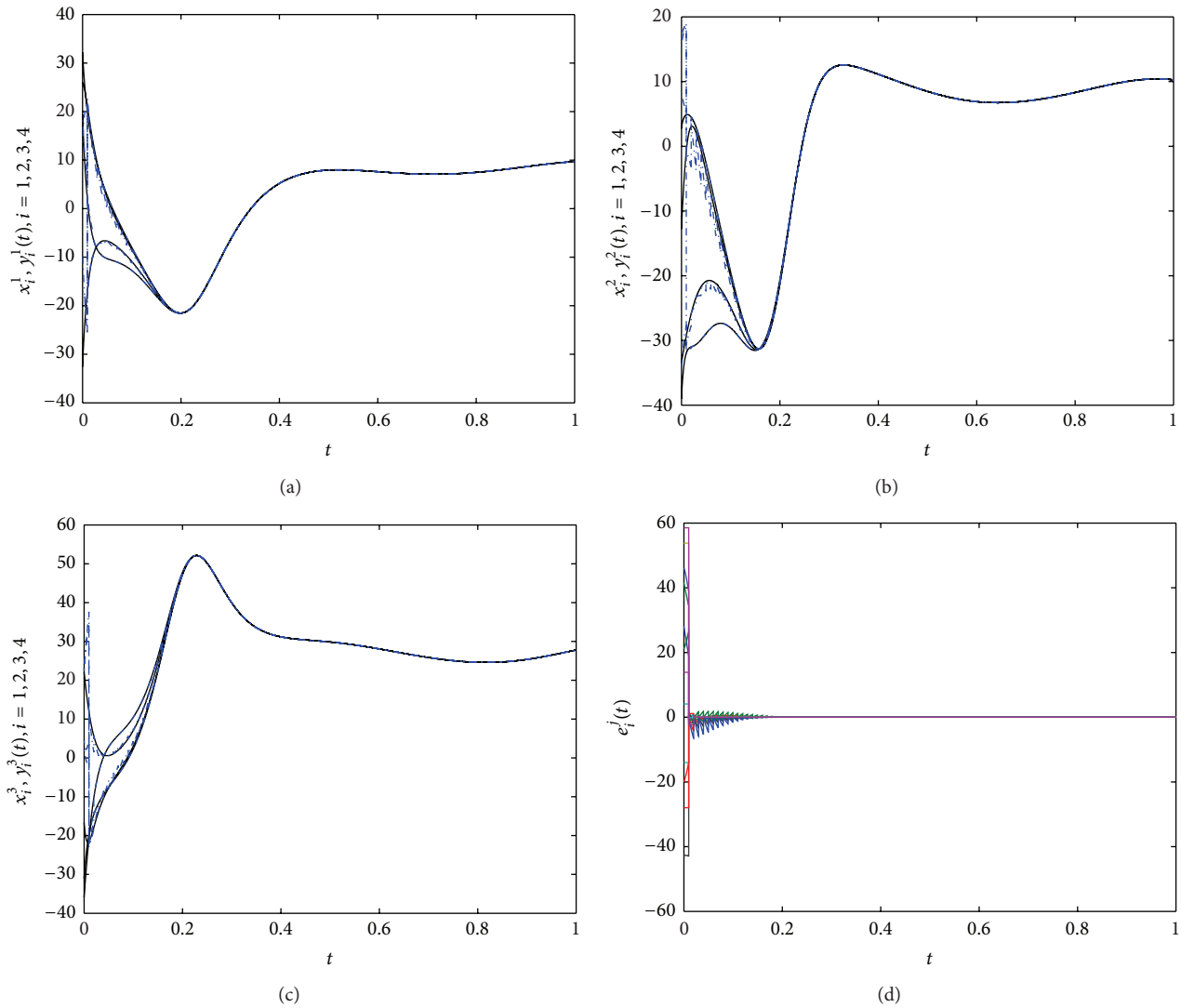

Figure 3: Time evolution between node x_i^k and node y_i^k, $i = 1, 2, 3, 4$, $k = 1, 2, 3$, and error evolution between drive and response networks under impulsive control.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors thank the referees and the editor for their valuable comments on this paper. This work is supported by the National Natural Science Foundation of China (Grants nos. 61273220 and 61373087) and the Shenzhen Basic Research Project (JCYJ20120613105730482).

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